Generation of a residual current by interaction between the coastal boundary layer and the Ekman layer in a tidal motion

Tidal currents
Boundary layer
Ekman layer
Residual current

Courants de marée
Couche limite
Couche d'Ekman
Courant résiduel

Denis AELBRECHT [a], **Gabriel CHABERT D'HIERES** [a] **and Xiuzhang ZHANG** [b]

[a] Laboratoire des Écoulements Géophysiques et Industriels, Institut de Mécanique de Grenoble, URA CNRS n° 1509, B.P. 53 X, 38041 Grenoble Cedex, France.

[b] Arizona State University, Department of Mechanics and Aerospace Engineering, Tempe, Arizona 85287-6106, USA.

ABSTRACT

Some experiments have been made with the large rotating tank at the Institut de Mécanique de Grenoble to reproduce a sinusoidal flow along a vertical wall, over a flat bottom, with or without slope, in a homogeneous fluid. The objective of these experiments was to simulate a tidal motion, restricted to its main harmonic component M_2, parallel to a vertical coast; and to demonstrate a particular mode of generation of residual circulation near the coast as a result of the interaction between friction, background rotation of the system (Ekman pumping) and oscillatory motion (Stokes effects). Moreover, the stabilizing (or otherwise) effects of the Earth's rotation on this shear flow may modify the rectification process. The configuration is in a good agreement with the general topographic and hydrographic situation along the French coast of the Pas-de-Calais, near Boulogne-sur-Mer.

In the Northern hemisphere, the bottom current is diverted to the left of the main current due to the balance between the Coriolis force and the pressure gradient. This imposes a dissymetric behaviour of the flow between flux and ebb, as well as the generation of an alternative component perpendicular to the coastline, together with lateral transport (vertical close to the coasts) of horizontal momentum which modifies the coastal boundary layer. A rectified (residual) current is observed with the coast to its right. Experiments were systematically made on a flat bottom moving in the rotating tank by varying the main parameters of the flow; and we measured the velocity field with ultrasonic velocimetric probes and photographs of surface floats. The results (amplitude and width of the rectified current) show a good correlation with the few observations at sea and with the scaling analysis we provide.

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RÉSUMÉ

Formation d'un courant résiduel par interaction entre une couche limite côtière et une couche d'Ekman dans un écoulement de marée

Des expériences ont été menées sur la grande plaque tournante de l'Institut de Mécanique de Grenoble afin de reproduire un écoulement alternatif sinusoïdal en fluide homogène ou stratifié, sur un fond plat incliné ou non, le long d'un mur rectiligne vertical. L'objectif est de simuler un écoulement de marée parallèle à une côte, et de mettre en évidence un mode particulier de génération d'un courant résiduel confiné à la côte, résultat de l'interaction entre les effets du frottement, de la rotation terrestre (effet de pompage d'Ekman) et de l'écoulement oscillant (effet de Stokes). De plus, les effets stabilisants ou non de la rotation terrestre sur cet écoulement cisaillé peuvent modifier ce processus d'échange de quantité de mouvement. Les résultats de cette étude sont applicables aux courants de marée au voisinage de Boulogne-sur-Mer, sur les côtes françaises de la Manche.

Dans l'hémisphère Nord, l'équilibre entre la force de Coriolis et le gradient de pression dévie le courant de fond vers la gauche de l'écoulement principal. Pour un écoulement alternatif, cela engendre une composante alternative perpendiculaire à la côte. Il s'ensuit un transport latéral (et vertical près de la côte) de quantité de mouvement horizontale, modifiant la couche limite latérale suivant que l'écoulement laisse la côte sur sa droite ou sur sa gauche. Un courant résiduel laissant la côte à droite est alors observé. Les expériences sont réalisées sur un fond plat oscillant dans la cuve tournante, en faisant varier les principaux paramètres de l'écoulement. Les mesures du champ des vitesses sont obtenues à l'aide de sondes vélocimétriques à ultra-son et par chronophotographie de flotteurs de surface. Les résultats sur l'amplitude et la largeur du courant résiduel observé montrent une bonne corrélation avec quelques observations en mer et avec notre estimation théorique.

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INTRODUCTION

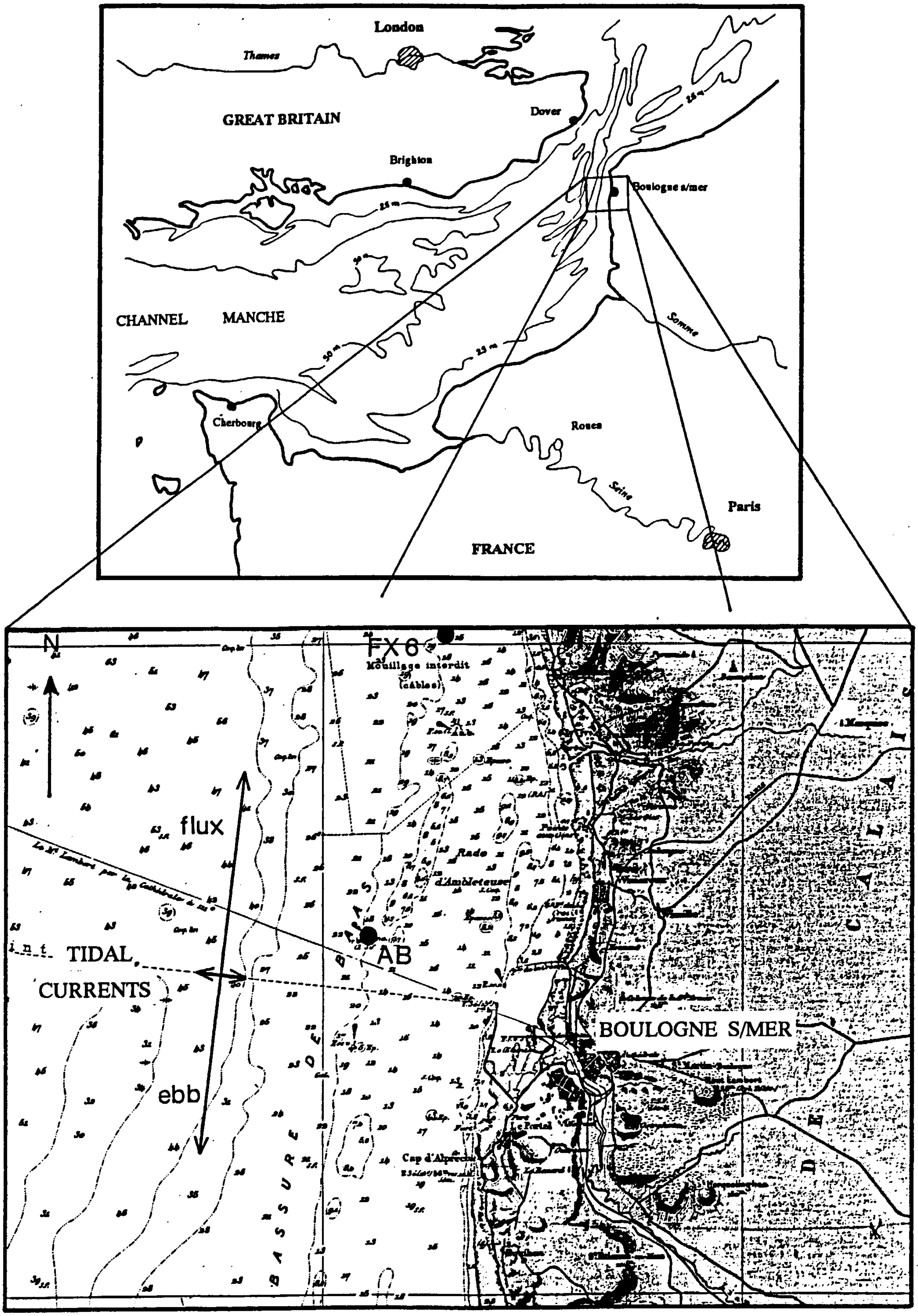
The general objectives of this study are to describe the structure of the residual current resulting from an alternative flow, parallel to a rectilinear wall, over a flat bottom; and to understand the physical process which tends to generate this rectified current. The first experiments were performed on a horizontal bottom. A better similarity with nature was obtained with a sloping bottom. In order to realize such a flow, we used the large rotating tank of the Institut de Mécanique de Grenoble to create the oscillation of a flat plate in homogeneous water. We do not reproduce exactly the tide and all its effects, and neglect, for example, the variation of water depth which imposes a variation of vorticity that becomes dissymetric due to friction; possible gravity currents due to longitudinal slope of the free surface; the specific nonlinearity of the tidal wave, *etc*. Nor, of course, do we take into account the effects of wind-forcing, sand-banks, *etc*. The physical model provides some information concerning the particular process known as Ekman Pumping. Moreover, the dynamics in nature and on the model are different, especially in regard to the pressure gradient, due to the fact that in our experiments the bottom is moving and not the water. But the general process of Ekman pumping remains valid. In nature, and particularly close to the French coastline at Boulogne-sur-Mer in the Channel (*see* Fig. 1), the residual circulation reflects the combined effect of tide, wind, irregular bathymetry, and other factors. The regular shape of the coastal topography near Boulogne-sur-Mer allows us to extend our results to the particular case of this region: the French coast is oriented in a virtually rectilinear north-south direction, the average depth some ten kilometres offshore is about 30 m, and the tidal currents observed there appear to be quite parallel to the coast, with an average amplitude of about 1 m/s. For these reasons, our experiments can bear a good resemblance to nature. Because of the real impact of the fresh water contribution in the region studied, we carried out some specific experiments in stratified fluid by introducing fresh water into the salt water at the coast. Particular attention was paid to the evolution of the density front which occurs. But the observation results of this phenomenon lie outside the scope of the present article.

Numerous papers have been written on the subject of tidal motions and particularly on the generation of residual currents in such flows. Salomon and Breton (1991) computed a 2-D hydrodynamic model showing the general structure of residual currents throughout the Channel, according to the different weather conditions (notably of wind) which modify the residual circulation near the coast. Tee (1979 and 1980) analyzed the tidal motion in terms of oscillating currents on the one hand, and residual currents on the other. He first computed a depth-averaged velocity field which permitted determination of the vertical velocity gradient. But he failed to take into account the effects of vertical variations of tidal currents in the lateral boundary layer. Prandle (1982) studied the vertical structure of tidal currents in the case of an infinite flat bottom (merely in terms of oscillating currents) and proposed some analytical solutions for different forms of eddy viscosity (constant or linear depth dependence). Maas and Van Haren (1987) obtained a large number of observations at

Figure 1 ➡

a) general map of the channel; b) bathymetric map near Boulogne-sur-Mer (from "Carte SHOM 5107, September 1943").

a) carte générale de la Manche ; b) carte bathymétrique au large de Boulogne-sur-Mer (tirée de «Carte SHOM 5107, septembre 1943»).


London
Thames
GREAT BRITAIN
Dover
Brighton
Boulogne s/mer
CHANNEL
MANCHE
Somme
Cherbourg
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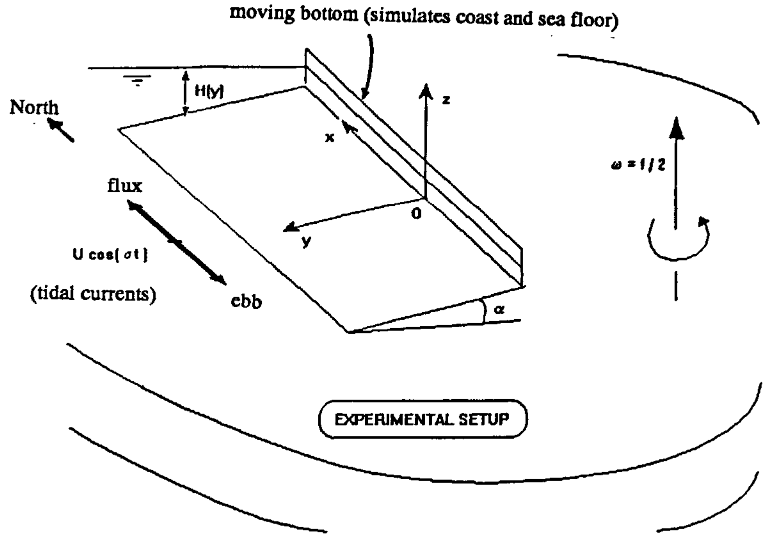
Seine
Paris
FRANCE
N
FX6
Mouillage interdit
(câbles)
Rade d'Ambleteuse
flux
ebb
TIDAL
CURRENTS
AB
BOULOGNE S/MER
Cap d'Alprech

Figure 2

Experimental setup.

Dispositif expérimental.



sea of tidal currents (in the middle of the North Sea) and compared their observations with the theoretical model of Prandle concerning the vertical structure of the currents (configuration without any coast).

EXPERIMENTS

Experimental setup (*see* Fig. 2)

We use Cartesian co-ordinates (Oxyz), rotating around the z-axis with angular velocity f/2, the x-axis being parallel to the rectilinear coastline, so that "y" is the offshore distance. The relative movement of the water is produced by oscillating the flat plate. This plate has a rectangular shape, 8 m long and 2 m wide. The parameters of the flow are: the depth, H(y) which can be constant or varying linearly with the offshore coordinate y: $H(y) = H$ or $H(y) = H_1 + y \tan \alpha$; the slope of the oscillating plate, α; the Coriolis parameter, $f = 2\ \Omega$ (period of rotation $T = 2\pi/\Omega$); the oscillation frequency of the plate σ (period of oscillation $t = 2\pi/\sigma$); and the amplitude of the oscillating current, U_o. We also assume the flow to be independent of the "x" coordinate, and we study what happens through the (x = 0, y, z)-plane. Note that disturbance of frequency σ could drive shallow-water gravity waves with a wavelength of l_w: $l_w = 2\pi/\sigma * (gH)^{1/2}$. Typical values are: H = 5 cm, τ = 60 s, so that l_w = 42 m; this indicates that the effects of gravity waves are negligible and that the Froude number is not an important parameter of the flow.

We can show, by writing the equations governing the dynamics (conservation of momentum and mass), that the flow characteristics are governed by the following non-dimensional parameters:

$R_{ot} = \sigma/f$ = inertia/Coriolis temporal Rossby number (1)

$E_k = \nu/fH^2$ = friction/Coriolis Ekman number (2)

$R_e = U_oH/\nu$ = inertia/friction Reynolds number (3)

The respect of the Reynolds number is not systematic, but we chose values for R_e in the range of fully turbulent flows. In the definition of the Rossby number, we selected a typical horizontal distance L being the elongation of the tide (*i.e.* $L = U_o/\sigma$). Thus, $R_o = U_o/fL = U_o/ f(U_o/\sigma) = \sigma/f = R_{ot}$: temporal Rossby number.

Similitude

We deal merely with the similitude of the flow in homogeneous fluid. The important point to consider here is the difficulty of realizing and measuring a significative eddy viscosity. Chabert d'Hières (1991) chose a typical value of 0.04 $cm^2\ s^{-1}$ for the eddy viscosity. We carried out a large number of preliminary experiments (*i.e.* Ekman flow, uniform motion in a rotating frame; Stokes flow, oscillating motion in a fixed frame) in order to determine a significant value of the eddy viscosity. These gave a typical value of 0.03 $cm^2\ s^{-1}$ (by assessing Ekman layer thickness; experiment using a Reynolds number of approximately 3000). Of course, this value should vary in time and space. For nature, Maas and Van Haren (1987) found a good representation with a constant eddy viscosity of 1.4 $10^{-3}\ m^2\ s^{-1}$. We retain a value of $10^{-3}\ m^2\ s^{-1}$ for the nature scale. In the following, the index "N" will specify natural values, the index "m" model values. [X] will be the scale of quantity X, *i.e.* the ratio between model value of X and natural value of X.

Then, the above considerations impose $[\nu] = 3\ 10^{-3}$.

An expression of Ekman thickness may be (*see* Pedlosky, 1987):

$$\delta_e = \pi\ (2\nu/f)^{1/2} \quad \text{or} \quad \delta_e = \pi\ H\ (2\ E_k)^{1/2} \qquad (6)$$

Since the nature values are: H_N = 30 m, U_{oN} = 1 m s^{-1} (mean amplitude of tidal current), $f_N = 2\Omega\sin\lambda = 1.1\ 10^{-4}\ s^{-1}$, where Ω is the earth angular velocity and λ the latitude (λ = 51° N), τ_N=12 h 25 mn = 44700 s (tidal period), $\nu_N = 10^{-3}\ m^2\ s^{-1}$, so $(\delta_e)_N$ = 13 m. In nature (near Boulogne-sur-Mer), the thickness of the Ekman layer may be estimated to be half the total depth. This ratio will be conserved on the physical model: $(\delta_e)_m = H_m/2$. We have to select one scale to solve the problem.

We first choose: H_m = 5 cm, then [H] = 0.05/30 = $1.67.10^{-3}$. We have consequently $(\delta_e)_m$ = 2.5 cm. Since $[\nu] = 3\ 10^{-3}$, equation (2) gives [T] = 1/1076. Equation

Table

Summary of all the parametric experiments. Experiments Nos. 1 to 5 are for the homogeneous case. Experiments Nos. 6 to 8 are for the stratified case, always without bottom roughness. For No. 9 onwards, all the experiments were performed with bottom roughness.

Résumé des expériences. Les expériences n° 1 à 5 sont effectuées avec un fluide homogène. Les expériences 6 à 8 sont réalisées dans le cas stratifié (sans rugosité de fond). Pour les expériences n° 9 et suivantes, on utilise une rugosité de fond.

N°	T (s)	τ (s)	H (cm)	tan α	U_0 (cm/s)	Q (l/mn)
1	300	200	5	0.00	3.9	0.0
2	150	60	5	0.00	17.7	0.0
3	150	60	1	0.05	17.1	0.0
4	150	30	1	0.05	18.0	0.0
5	75	60	1	0.05	16.0	0.0
6	150	60	1	0.05	17.7	1.0
7	150	60	1	0.05	11.0	1.0
8	140	56	1	0.05	16.3	1.0
9	∞ ($\Omega = 0$)	∞ ($\sigma = 0$)	1	0.05	12.0	0.0
10	130	∞ ($\sigma = 0$)	1	0.05	12.0	0.0
11	130	52	1	0.05	12.0	1.0
12	130	52	1	0.05	6.0	1.0
13	130	52	1	0.05	12.0	0.5
14	130	52	1	0.05	6.0	0.5
15	130	52	1	0.05	6.0	4.0

(1) gives $[\sigma] = 1076$. We can choose [U] independently if no respect of Froude similarity is assumed. However, the condition of fully turbulent flow incited us to the velocity scale of [U] = 0.12. We decided on model values of: $H_m = 5$ cm, $\Omega_m = 0.042$ s^{-1} ($f_m = 0.084$ s^{-1}, and $T_m = 150$ s), $\sigma_m = 0.10$ s^{-1} ($\tau_m = 60$ s (tidal period)), $U_{om} = 12$ cm s^{-1}.

The horizontal length is then given by [L] = [U] [T] = 1.2 10^{-4}, which means that 10 cm on the model represents approximatively 1 km in nature. In the case of the sloping sea bottom, we have $[\alpha]=[H][L]^{-1}=14$. The slope on the model should be fourteen times larger than in nature. Near Boulogne-sur-Mer, we can estimate that $\alpha_N = 30$ m/10 km = 0.003, which gives $\alpha_m = 0.042$. We chose the value: $\alpha_m = 5$ %.

Remark

Non-respect for Froude similarity keeps acceptable limit: $Fr_N = 0.058, Fr_m = 0.17$ (Table).

Measurements

The velocity field (*i.e.* horizontal components u and v of the Eulerian velocity, in the x and y directions respectively) in the (x = 0, y, z) is investigated with a system of ultrasonic probes, disposed in the x = 0 plane, and fixed to the oscillating bottom. Then, we analyzed the signals of u and v in terms of decomposition into harmonic components:

$$u\,(x = 0, y, z, t) = u_o + u_1 \cos(\sigma t - \phi_1) + u_2 \cos(2\sigma t - \phi_2) + ... \quad (5)$$

$$v\,(x = 0, y, z, t) = {}_o + v_1 \cos(\sigma t - \psi_1) + v_2 \cos(2\sigma t - \psi_2) + ... \quad (5')$$

We retained only the two first terms (mean component with index o and fundamental harmonic term with index 1) to recompose the signals after filtering (harmonic analysis).

We also used some floats to visualize Lagrangian trajectories (*see* Fig. 8).

The methods used to study the effect of the contribution of fresh water comprised investigation of the density field with conductivity probes and visualization of the front with a laser sheet and video tape recorder.

RESULTS

First harmonic components

In appendix A, we give the theoretical solution of the first-order harmonic components u_1 and v_1 [*see* equations (5) and (5')] for the case of an infinite horizontal bottom with: a) a no-slip boundary condition; or b) a stress condition on the bottom; no stress being the boundary condition at the surface. The problem involves the possible estimation of the temporal characteristics of the flow field by theories assuming an infinite horizontal plate. We consider this question in relation to the analyses of the vertical structure of tidal currents by Prandle (1982), and the observations of Maas and Van Haren (1987). Figure 3 plots the vertical structure of the normalized amplitude of the streamwise component u_1/U_0 at two different lateral locations (near the vertical wall and far from it). The symbols represent the measured data and the dashed line the theoretical predic-

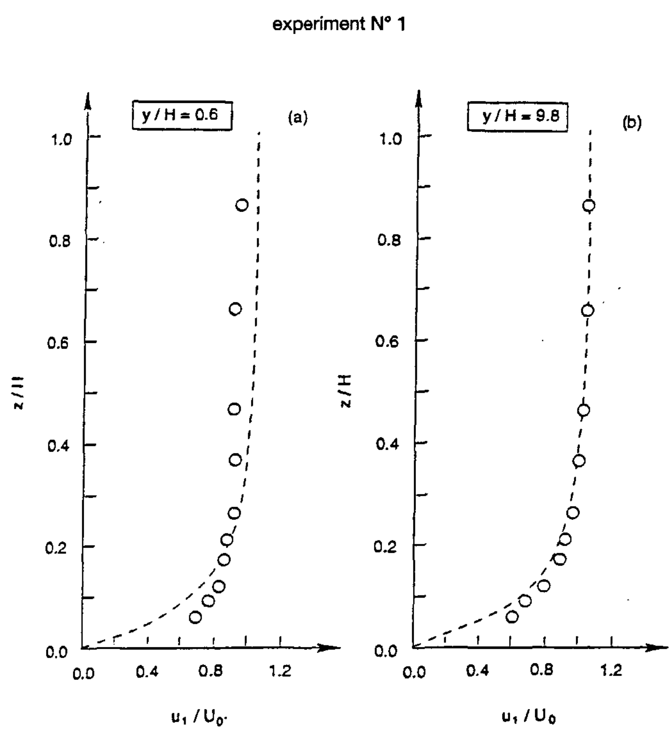


Figure 3

Vertical profiles of experimental (circles) and theoretical data from Appendix A (dashed line) for the first harmonic amplitude u_1 (streamwise component) at two different distances from the wall.

Profils verticaux des résultats expérimentaux (cercles) et théoriques (lignes pointillées) pour l'amplitude de la première harmonique u_1 à deux distances du mur.

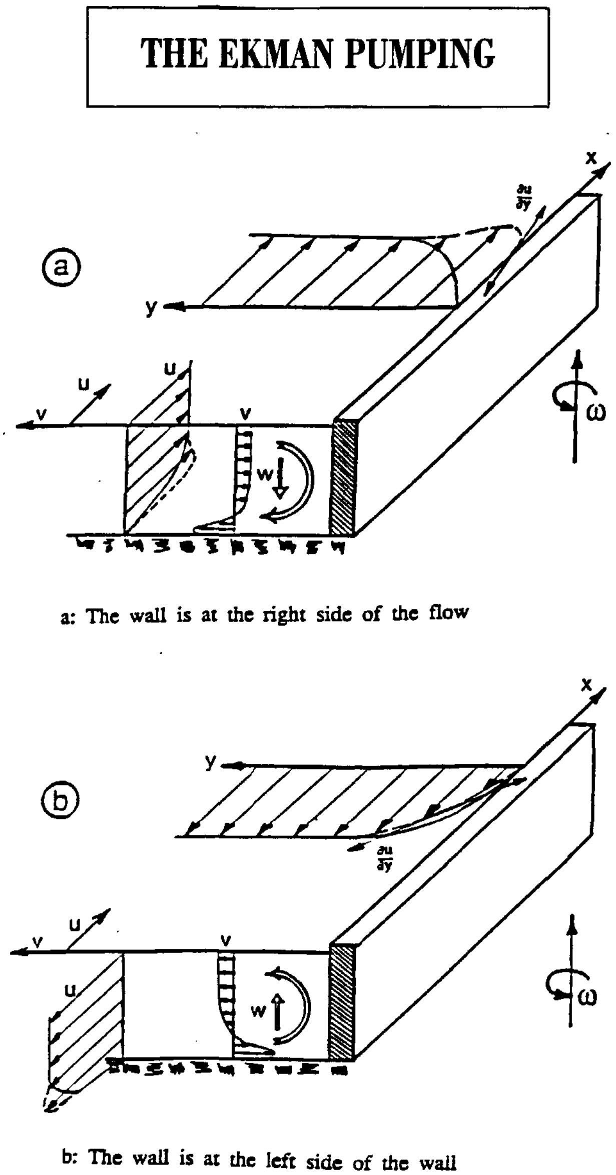


Figure 4

Schematic representation of Ekman pumping when: a) the wall is to the right of the flow; b) the wall is to the left of the flow.

Représentation schématique du pompage d'Ekman : a) le mur est à droite de l'écoulement ; b) le mur est à gauche de l'écoulement.

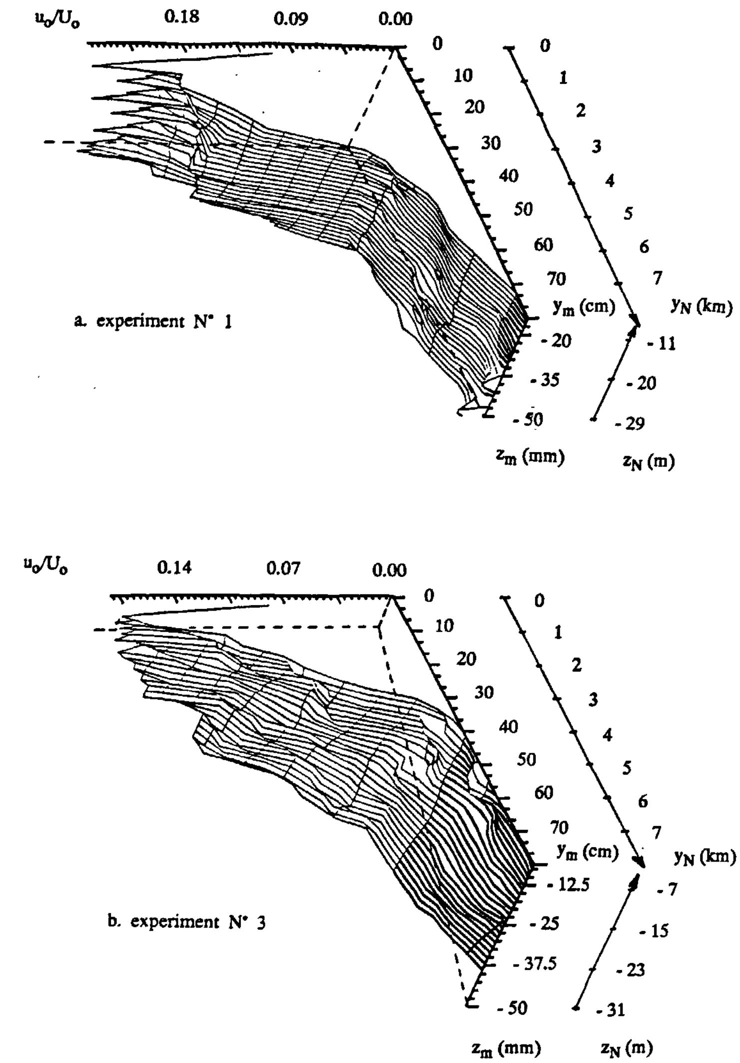


Figure 5

3-D profile of the streamwise residual current u_0. For y and z coordinates, the laboratory and nature (using similitude) scales are represented: a) for experiment No. 1; b) for experiment No. 3.

Profil 3-D du courant résiduel u_0. Les échelles en y et en z de laboratoire et en nature sont représentées : a) pour l'expérience No. 1 ; b) pour l'expérience No. 3.

tion (case of no-slip condition). For the location far from the wall, the measured data are in good agreement with the prediction, although close to the wall there appears to be a deficit of momentum in the upper layer, while the momentum in Ekman layer seems to increase. This leads to the conclusion that there is a vertical transport of horizontal momentum very near the coast.

Rectified current

The present physical system leads to a rectified current in the streamwise direction with the coastline on its right for the Northern hemisphere rotation. When the flow has the coastline to its right (*see* Fig. 4 *a*), the parcels of fluid in the Ekman layer move away from the wall. This implies that the momentum in the upper layer is advected towards the coast and modifies (decreases) the lateral boundary layer. Conversely, when the streamwise flow has the coast to its left (*see* Fig. 4 *b*), the parcels in the Ekman layer first head towards the coast and move close to the vertical wall [in the so-called Stewartson layer (*see* Pedlosky, 1987)]. Then, weaker momentum near the wall is advected away from the coastline, and modifies (increases) the lateral boundary layer. As a result, a rectified (residual) current is generated with the coast on its right-hand side. The vertical upward and downward transports occur in a very close lateral region, the Stewartson layer (*see* Pedlosky, 1987), whose thickness is δ_S. It should be noted that for the present experimental conditions, $\delta_S/H = 0.2$. During the half-period described on Figure 4 *b*, fluid parcels are carried along the plate floor towards the wall in the Ekman layer at a characteristic lateral speed $u = 2U_0/\pi$. By conservation of mass, this transport must be balanced by an adjustment flow through an interior of thickness H - ($\delta e/\pi$) which is about H, with a characteristic cross-stream speed v, so that: vH = u ($\delta e/\pi$). For v > 0 (Fig. 4 *b*), low-momentum particles are transported into the fluid interior for a characteristic time t = π/σ (half a cycle). The characteristic lateral distance dy to which such parcels move during this half-cycle is taken as a typical width of the rectified current: δ_y = v.t = ($\delta_e/(\pi H)$) . u.π/σ, δ_y/H is scaled by: $E_k^{1/2}.R_o.R_{ot}^{-1}$, by writing $R_o = U_o/$ (f H) (a typical Rossby number). This rectified aspect of the motion can be clearly demonstrated by tracking free-surface floats (Lagrangian point of view, *see* Fig. 8), as well as by analyzing the mean of the velocimetric signal over a cycle of motion (Eulerian point of view). Analysis of the ultrasonic signals shows that for the standard experiment (No. 3, sloping bottom) in similarity with nature, the schematic structure of the depth-averaged mean current extrapolated to nature is (*see* Fig. 5 *b*):

* u_o/U_o = 20 % at a distance of about 1 km from the coast
* u_o/U_o = 12 % " 2 km " ;
* u_o/U_o = 7 % " 3 km " ;
* u_o/U_o = 0 " 4 km " .

This structure has to be compared with the nature value of 6 cm/s (6 % of the tidal amplitude) for residual current measured at Approche Boulogne station in 1989 during one month (*see* point AB on Fig. 1). Figures 6 *a* and 6 *b* clearly show the presence of residual circulation because of the drift of the ellipse current centres near the coast. Moreover, Figure 6 *b* shows that the Ekman effect is completely modified near the coast (see the size of the minor axis on the ellipse current). Figures 7 *a* and 7 *b* depict the vertical structure of the current. We observe on Figure 7 *b* (far from the coast) the inversion of rotation sense with depth and the increasing of v component towards the bottom. Figure 8 shows surface floats trajectories for experiment No. 1. This demonstrates the occurrence of a Lagrangian mean current, but Eulerian component and Stokes drift cannot be disconnected. We do not deal with this problem in the present paper.

A numerical study of the laboratory experiment is currently underway. It will provide some general information on the circulation in the cross-stream section, particularly concerning the presence of cells of circulation (*see* Zhang *et al.*, submitted).

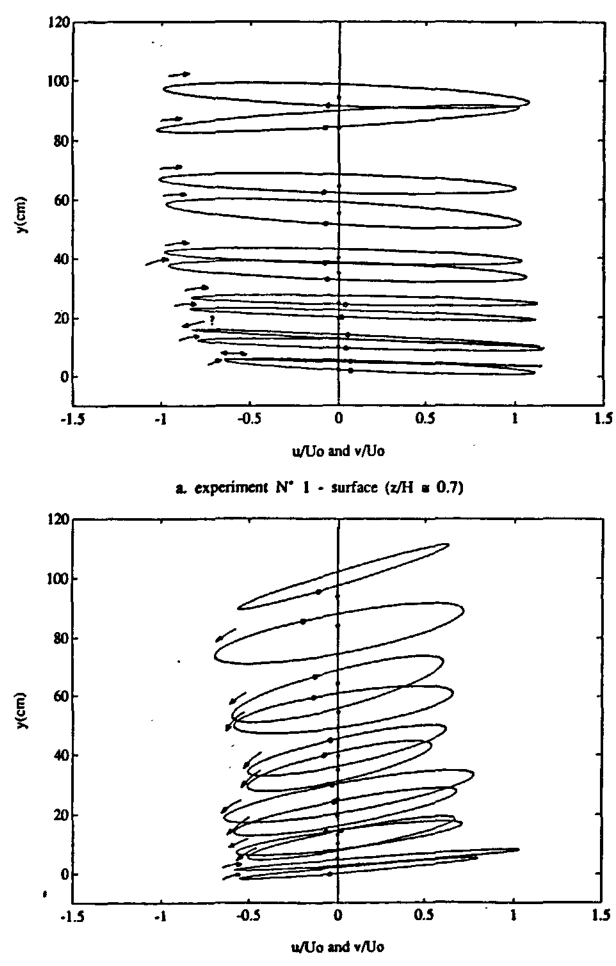


Figure 6

Horizontal hodographs (ellipse current) for experiment No. 1: a) at surface; b) near bottom.

Hodographes horizontaux (ellipses de courant) pour l'expérience No. 1 : a) en surface ; b) près du fond.

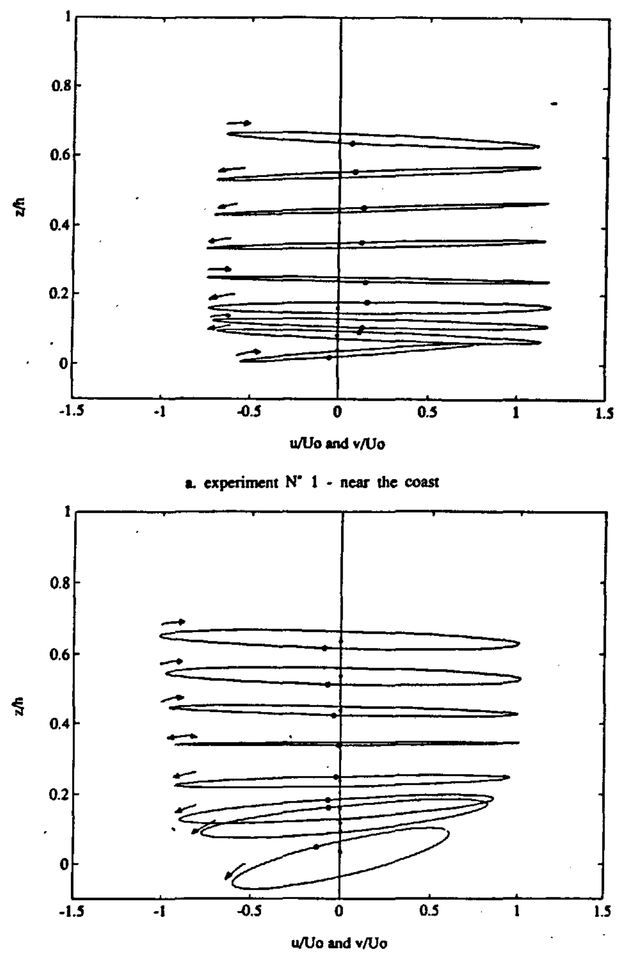


Figure 7

Vertical hodographs (ellipse current) for experiment No. 1: a) near the coast; b) far from the coast.

Hodographes verticaux (ellipses de courant) pour l'expérience No. 1 : a) près de la côte ; b) loin de la côte.

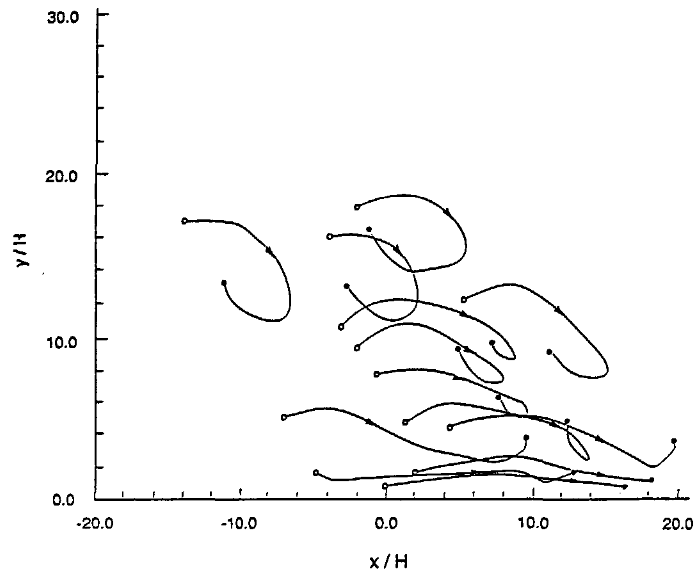


Figure 8

Surface float trajectories during one period of oscillation. ○ *: beginning of the period;* ● *: end of the period.*

Trajectoires de flotteurs de surface durant une période d'oscillation : ○ : début de la période ; ● : fin de la période.

Effects of sloping bottom, roughness

The previous explanation of Ekman pumping is still valid, but the general effect of sloping bottom is an increase in the width of rectified current: the slope pushes the rectified current in the offshore direction. Beginning with No. 9, our experiments were done with bottom roughness. The impact of this roughness is a heightened degree of turbulence, without modifying strongly the typical value of eddy viscosity (according to our preliminary findings). Because of friction, the rectified current is found to be weaker than in the smooth case. Some experiments have been performed with a coastal input of fresh water to reproduce the impact of the contribution of a river (for example, the *Somme* along the French coastline). We observed the possible stabilization of a density front at a certain distance from the wall; and the variation of this front according to the tidal strength (*i.e.* according to the tidal velocity amplitude).

In standard conditions (experiment No. 11), we carried out some experiments involving the input of fresh water at the coast (simulation of a river discharge). Although this is not directly related to the process involved there, we clearly observed a density front whose the distance from the coast is estimated at 50 cm (about 5 km in nature). Moreover, the influence of the tidal motion is such that the front becomes vertical when the tidal currents increase. This is in a very good agreement with the observations of Brylinski and Lagadeuc (1990). During neap tide, the front becomes almost horizontal; and particularly during the reverses, some salt water may flow under the fresh water and move close to the coast. This thesis is confirmed by oceanographers in Boulogne-sur-Mer. It seems that the buoyancy force has no significant effect on the dynamics described before, whereas dynamics obviously modify the structure of the stratification (*see* remarks above).

CONCLUSION

Shallow coastal waters call for intensive study, because they constitute the location of a large number of interactions between ocean processes and human activities. The coastal process considered here leads to the generation of a residual circulation due to Ekman pumping along a vertical, rectilinear coastline. A coastal residual current is observed, with the coastline on its right-hand side (Northern hemisphere). The net flux of coastal water occurring in nature due to this process is estimated from our model to average 10000 m^3/s. The intensity of this residual current accounts for some 10 % of the mean tidal currents (20 % very close to the coast). It generates movements of water masses and materials (sediments) and thus contributes to the dynamic local balance between offshore and coastal waters in the region (*see* point FX 6 on Fig. 1) where the variation in concentration of metals, sediments, *etc*, is important. The formation of a density front is also observed, whose evolution corresponds well with observations at sea. Other phenomena are observed, notably the instabilities of the flow in the Ekman layer (possible occurrence of Faller waves, *see* Faller, 1963). These waves appear to break down when the currents are reversing, perhaps generating powerful turbulence from bottom to surface at a particular moment during the tidal period. This matter is to be studied in the near future.

Acknowledgements

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APPENDIX A

Analytical solution in the case of an infinite horizontal plate (without any coastal boundary)

We consider a homogeneous, incompressible fluid limited below by an infinite horizontal plate, without any coastal boundary. The fluid is rotating about the z-axis with origin on the plate (Coriolis parameter f). The turbulence is represented by the assumption of a constant eddy viscosity ν. Near the surface, the fluid is assumed to have an oscillatory motion along the x-axis at velocity $u = U_o \cos(\sigma t)$. The equations of motion in this system may be written:

$$u_t - fv = \nu\, u_{zz} - g\, \zeta_x \quad (A.1)$$

$$v_t - fu = \nu\, v_{zz} - g\, \zeta_y \quad (A.2)$$

$$0 = -(1/\rho)\, p_z - g \quad (A.3)$$

(in A.1 to A.3, indexes represent partial derivative operator: for ex. $u_t = \partial u/\partial t$).

The continuity equation being automatically verified because of the independence in x and y (no coastal boundary), and negligible vertical velocity component. Following Maas and Van Haren (1987), we look for solutions of (A.1), (A.2) and (A.3) with the form of $u(z, t) = \mathrm{Re}[\exp(i\sigma t).f(z)]$ and similarly for v, assuming two cases of boundary conditions.

First case

* $(u, v) = (0, 0)$ at $z = 0$ (no-slip condition on the bottom)
* $(u_z, v_z) = (0, 0)$ at $z = H$ (no-stress at the surface).

We obtain in this case:

$u = U_o [\cos(\sigma t) - 1/2\, \mathrm{Re}\, \{\exp(i\sigma t)(\cosh \alpha_+(z - H) / \cosh \alpha_+H + \cosh \alpha_- (z - H) / \cosh \alpha_-H)\}]$

$v = U_o [1/2\, \mathrm{Im}\, \{\exp(i\sigma t) (\cosh \alpha_+(z - H) / \cosh \alpha_+H - \cosh \alpha_-(z - H) / \cosh \alpha_-H)\}]$

where $\alpha_+ = (1+i) [(\sigma + f)/2\nu]^{1/2}$ and $\alpha_- = (1 + i) [|\sigma - t|/2\nu]^{1/2}$

Second case

* $(u_z, v_z) = (s.u, s.v)$ with $s = u_* H/\nu$ at $z = 0$ (linearized stress condition on the bottom, u_*: bottom friction velocity)
* $(u_z, v_z) = (0, 0)$ at $z = H$ (no-stress at the surface).

This time, the terms "$\cosh \alpha_{+-}(z-H)/\cosh \alpha_{+-}H$" found in the first case become:

$\cosh \alpha_{+-}(z - H)/(\cosh \alpha_{+-}H + (\alpha_{+-}H/s) \sinh \alpha_{+-}H)$.

In the case of no residual current, these solutions correspond to the first harmonic components u_1 and v_1.

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