# Choosing altimeter orbits as a problem in experiment design

Altimetry
Fourier decomposition
Experiment design
Optimization
Sampling perspective

Altimétrie
Décomposition Fourier
Expérience
Optimisation
Théorie de l'échantillonnage

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## ABSTRACT

A quantitative approach based on sampling theory and experiment design was used to evaluate configurations of satellite borne altimeters. A simple 2D representation of the experiment design problem, retaining most of the major design issues was implemented. Two different oceanic regions - a large/slow (equatorial Pacific) and a small/fast (Atlantic mid-latitude) region - were selected. The sampling characteristics of configurations of two or three altimeters - all constrained to be of the Topex/Poseidon (T/P) type - *vis à vis* the two regions were investigated under changes in phase angle between the orbital planes of the altimeters. The preferred arrangement for two T/P altimeters is a separation of 180°. This arrangement simultaneously samples the large/slow region nearly optimally; the small/fast region less so. Under these constrained conditions - and for these two regions - the two orbiter configuration essentially are as good as, or outperform any configuration of three T/P altimeters. This conclusion holds in the two cases of noiseless (perfect measurements) and extreme noise-to-signal levels, and hence would seem likely to hold for all intermediate realistic noise-to-signal regimes as well. With a different representation and fewer constraints however, these conclusions could change. Heterogeneous configurations of two altimeters - one of type T/P, the other, one of the 364 repeating orbits (with an even number of nodal days per repeat) with altitudes between 1 200 and 3 000 km - were also briefly examined. For the limited set of phase angles evaluated, no heterogeneous configurations outperformed two T/P satellite borne altimeters.

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## RÉSUMÉ

### Le choix des orbites en altimétrie

Des configurations d'altimètres satellitaires ont été évaluées par une approche quantitative fondée sur la théorie de l'échantillonnage et sur une expérience en préparation. Une simple représentation bidimensionnelle prend en compte les principaux aspects de l'expérience. Deux régions océaniques ont été retenues, l'une grande/lente dans le Pacifique, au voisinage de l'équateur ; l'autre petite/rapide dans l'Atlantique, à une latitude moyenne. Les caractéristiques de l'échantillonnage par les configurations à deux ou trois altimètres - de type Topex/Poséidon (T/P) - ont été étudiées pour chaque région en variant l'angle de phase entre les plans d'orbite des altimètres. Pour deux altimètres T/P, la meilleure disposition est obtenue avec un écart de 180° ; elle permet d'échantillonner la région grande/lente de manière presque optimale, et la région petite/rapide un peu moins bien. Dans ces conditions, et pour les régions considérées, la configuration à deux orbites est équivalente ou meilleure que toute configuration à trois altimètres T/P. Ce résultat est valable lorsque le bruit de fond est nul (mesures parfaites) et lorsqu'il est maximal ; elle s'appliquerait donc aussi à tous les cas intermédiaires. Cependant, avec une autre représentation et moins de contraintes, les conclusions pourraient être différentes. Des configurations hétérogènes à deux altimètres ont été examinées brièvement, l'une de type T/P, l'autre parmi les 364 orbites répétitives (avec le même nombre de jours nodaux par cycle) à des altitudes comprises entre 1 200 et 3 000 km. Pour le petit nombre d'angles de phase considéré, les performances des configurations hétérogènes ne sont pas meilleures que celles des deux altimètres satellitaires T/P.

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## INTRODUCTION

This paper is based on a poster given at the JASO conference held in Toulouse, France in Autumn of 1991. It is meant to be a preliminary look at a quantitative approach to the problem of matching the space/time sampling characteristics of satellite borne altimeters to the differing space/time scales of the sea-surface height field found in the ocean.

With the actual placement of the Topex/Poseidon (T/P) altimeter in orbit expected in 1992, and ongoing interest in climate and global change questions, it is natural to ask how a configuration of several satellite borne altimeters might be used to indirectly measure the general circulation of the world ocean, and thereby monitor the major heat reservoir of the planetary climate system.

There are different perspectives from which to approach this experiment design problem. In 1983 Bretherton discussed orbit choice based on a "sampling" perspective (Bretherton, 1983). Parke *et al.* (1987) have shown that careful and astute reasoning from both an orbit dynamics and sampling perspective is nearly sufficient to completely determine what orbit a single altimeter should have. However a configuration of altimeters poses a more difficult problem in experiment design. Chase and Mundt (1989) have also taken a sampling approach although, as will become clear, their qualitative measures are not sufficient to take into account the complexity of matching the sampling characteristics of an ensemble of altimeters to the many space and time scales found in the ocean.

There is also the "modelling perspective". For example, the ability of such data to be usefully assimilated in models - as a function of orbit parameters - has been investigated by, among others, Hurlburt (1986), Kindle (1986), Thompson (1986), Holland and Malanotte-Rizzoli (1989) and Verron (1990). Other important references can be found in each of these.

The experiment design approach can make use of either of these, or other, perspectives. What distinguishes it from them however, is that it is a quantitative approach which seeks to actually determine optimal experiment designs. Sensitivity studies are quantitative in that they investigate the performance under changes in various design parameters. These in turn give important qualitative information which can help motivate particular design choices. What is missing however is the combination of both quantitative evaluation of design performance, and the actual optimization of this performance.

Here a quantitative approach from the sampling perspective, integrating the work of Barth and Wunsch (1990) and Wunsch (1989) will be pursued. It seeks to determine optimal orbits based on a quantitative measure of the performance of particular orbits.

The model used here is that of a simple 2D representation of the experiment design problem (*see* Fig. 1). It retains most of the important design issues. Specifically: two different oceanic regions - a large/slow (equatorial Pacific) and a small/fast (Atlantic mid-latitude) region were selected as "regions of interest". They were represented by one spatial and one temporal dimension (rather than two spatial and one temporal as in the full 3D problem). Configurations of two or three altimeters were then sought which "best" sampled these regions. In one series of investigations, the altimeters in all configurations were assumed to have the same

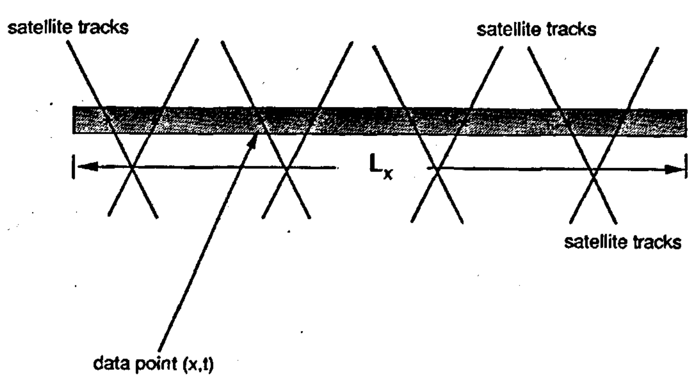



Figure 1

*A simple 2D representation of how an altimeter samples a region with a spatial scale $L_q$ and temporal scale $L_t$.*

orbit parameters: a repeat of ten nodal days, 127 orbits per repeat, an inclination of 66.0°, and excentricity of 1.0 (that is circular orbits). What is left to optimize then is the angle between the orbital planes of the individual satellites. Arrangements for such configurations have already been speculated upon in the document "The future of space borne altimetry: oceans and climate change", produced by the Topex/Poseidon Scientific Working Group (1992). In a final, though preliminary investigation, the sensitivity of the problem to heterogeneous configurations of two altimeters with differing orbital parameters was investigated.

Use of global optimization methods [such as in Barth and Wunsch (1990) or Barth (1992)] on what is a potentially difficult non-linear problem was not attempted. Rather the search space was constrained to be small yet still crafted to include what are hopefully interesting results. This search space was then exhaustively evaluated. In future work these constraints on the search space will be lifted.

The sampling characteristics of an altimeter form complex spatial and temporal patterns (Wunsch, 1989). Depending on the choice of orbit parameters, both can change significantly, although not independently of each other, due to the fact that an altimeter is constrainted to have orbits consistent with Newton's Laws of Motion. As an altimeter orbits the rotating Earth, it traces out a series of tracks on the Earth's surface. It is along these tracks that the altimeter makes measurements of the sea-surface height. This pattern of tracks determines the sampling characteristics of the altimeter *vis-à-vis* the sea-surface height field. Given a region with particular spatial/temporal scales an approach to choosing an altimeter orbit can be based on attempting to maximize the ability of an altimeter to sample these particular space/time scales and give estimates of the sea-surface height field which have a minimum sensitivity to fluctuations in measurement errors or altimeter orbit.

In the next section the experiment design approach is explained and a version of the satellite borne altimeter orbit "problem" adapted to it. The third section gives important computational details and a summary of the simulations performed. Finally in the last section the results are discussed.

## EXPERIMENT DESIGN AND SAMPLING A FIELD USING A SATELLITE

The entire mathematical treatment of the experiment design problem will not be given here. Barth and Wunsch (1990) give some of it, but this cannot be said to be complete. Here I will give a brief overview and motivation of the main ideas.

Consider an experiment characterized by the equation:

$$Ax = b + \delta b \qquad (1)$$

where b is a vector of N observations, $\delta b$ is an N vector whose elements are related to some noise level, x a vector of M unknowns, and A is a matrix of M columns and N rows which - for the experiment design problem - typically contains information about where and when the observations were made, and their relationship to the unknowns. Indeed the matrix A can be considered to represent a particular measurement strategy.

The estimation problem for x in equation (1) can take one of three forms: overdetermined ($N > M$), underdetermined ($N < M$), or exactly determined ($N = M$). The exactly determined case will not interest us here. Although other norms may be more appropriate in different situations, the L2 norm will be used in obtaining the estimate $\hat{x}$ by demanding that

$$||A\hat{x} - (b + \delta b)|| \qquad (2)$$

be minimized over $\hat{x}$. The estimate $\hat{x}$ is unique in the overdetermined case, and in the underdetermined case various estimators for $\hat{x}$ will differ in how the associated null space of $\hat{x}$ is treated (*see* for example Lawson and Hanson (1974).

Associated with any estimate of $\hat{x}$ is its covariance:

$$C(x) = \langle (x - \hat{x})^T \rangle \qquad (3)$$

defined over some suitable expectation procedure $\langle\ \rangle$. This matrix contains a wealth of information regarding the relationships between the observations, and the unknowns as well as how well the unknowns have been estimated. The goal of any experiment is to make measurements of a quantity (or field, *etc.*) such that it is determined as well as possible. The covariance matrix contains this information. As it stands however, it does not give a single value with which to characterize the soundness of the estimate $\hat{x}$. The trace of $C(\hat{x})$ - *i. e.* the sum of the variances of each estimated parameter - does provide one such characterization. Certainly there are many other choices, but this is the one which will be used here as a measure of how well a design A performs.

The trace of equation (3) can be written as:

$$\mathrm{Tr}\, C(\hat{x}) = \langle (x - \hat{x})(x - \hat{x})^T \rangle = \langle ||(x - \hat{x})^2|| \rangle \qquad (4)$$

This characterization of performance of a design A essentially measures the sensitivity of the estimate $\hat{x}$ to perturbations in the observations b due to the noise $\delta b$. The minimization of this performance index, or objective function, is what leads to experiment designs.

Perturbations in the matrix A, denoted $\delta A$ (ships, satellites, and experimenters do not always make their observations in the exact precise space/time location desired), can also lead to changes in the covariance matrix. Equation (1) is now:

$$(A + \delta A)x = b + \delta b \qquad (5)$$

and the problem is cast as one of "total least squares" (*e. g.* van Huffel and Joos Vandewalle, 1991). Whether the perturbations in the matrix A are significant enough to be given special treatment is a decision to be made in each individual case. Setting $\delta A$ equal to zero in what follows recovers the more well-known equation (1).

Following Golub and van Loan (1989), a quantitative expression for the sensitivity of the trace of the covariance matrix to these perturbations - for the overdetermined case ($N > M$) - is given by:

$$\mathrm{Tr}\, C(\hat{x}) = \langle \varepsilon ||x_p|| \{2\kappa/\cos(\theta) + \tan(\theta)\,\kappa^2\} \rangle^2 + O(\varepsilon^2) \qquad (6\,a)$$

where:

$$\varepsilon = \max\left\{\frac{||\delta A||}{||A||}, \frac{||\delta B||}{||B||}\right\} < \frac{\sigma_N(A)}{\sigma_1(A)} = \frac{1}{\kappa} \qquad (6\,b)$$

$$\rho = ||Ax_p - b|| = \text{minimum} \qquad (6\,c)$$

$$\sin(\theta) = \rho/||b|| \neq 1 \qquad (6\,d)$$

$$\kappa = \sigma_1(A)/\sigma_N(A) \qquad (6\,e)$$

and where $\sigma_i(A)$ equals the $i^{th}$ singular value of A, and $x_p$ corresponds to the case of perfect measurements [*i. e.* $\delta b = 0$ in equation (1)] [it should be noted that the singular values $\sigma_i$ are conventionally arranged in decreasing order; $\sigma_1$ being the largest]. The norm used in equation (6) is understood to be the L2 norm.

It is also assumed that equation (6 *b*), which might be called the "noise-to-signal" level, is satisfied for the tabulated designs found in Tables 2, 3, and 4. It is well known that singular values which are of the same order of magnitude as the noise in the observations - noise dominated singular values - lead to unstable estimates **x**. Truncation (*i. e.* choosing a subset of singular values having larger values) of the singular value spectra of A can always be used so that equation (6 *b*) does hold. This is the "regularization" problem (Tikhonov, 1963). To avoid making a specific choice of what an appropriate noise level might be and truncating the spectra of A accordingly, it is assumed that designs with very well conditioned A will minimize the need to confront the regularization problem, or to truncate A's spectrum. In other words, the philosophy pursued here is that the better the spectrum of A due to optimal design of the experiment, the less likely the need for any truncation. This question of noise level in the measurements will be discussed again in the fourth section, but in any case from equation (6) an ill conditioned A is clearly undesirable.

Equation (6) shows how the upper bound of Tr C(**x**) varies with perturbations to either the observations b or the matrix A. A similar relation to equation (6) holds for the underdetermined case, and can also be found in Golub and van Loan (1989).

The only part of equation (6) over which the experiment designer has control in order to reduce the upper bound on Tr C $(\hat{x})$ is the condition number $\kappa$. Thus by choosing an appropriate matrix A the upper bound of Tr C$(\hat{x})$ can be reduced. This is the basis of experiment design.

What needs to be done now, is to cast satellite sampling characteristics into the form (1). The matrix A will then be related to where and when an altimeter makes observations. The better the condition number of A, the better its associated orbit.

This recast into form (1) of the orbit problem has already been done in Wunsch (1989). Consider a function f(q) supposed periodic on the interval $[0, L_q]$. If there are $M_q$ samplings of f, then it is possible to expand each f $(q_m)$ in terms of a discrete Fourier decomposition:

$$f(q_m) = 1/N_q \sum_{k=0}^{(N_q = 1)} c_k \exp\left[i\,(2\pi k/L_q)\,q_m\right] \qquad (7)$$

where $N_q$ determines the number of frequencies used in the expansion (1), and the coefficients $c_k$ can be obtained *via*

$$c_k = \sum_{m=0}^{(M_q = 1)} f(q_m) \exp\left[-i\,(2\pi k/L_q)\,q_m\right] \qquad (8)$$

If f(q) is band limited to frequencies on $[-(\pi/2L_q)N_q, +(\pi/2L_q)N_q]$, and $M_q = N_q$, then equation (7) is exact; otherwise higher frequency components of f(q) will be aliased into this interval.

Although there are well known "conventional" methods for obtaining the coefficients $c_k$ in (7) - especially for equally spaced measurements - for the purposes here the coefficients of (7) are obtained by minimizing the following:

$$\sum_m \left[\, f(q_m) - 1/N_q \sum_{k=0}^{(N_q = 1)} c_k \exp\left(i\,(2\pi k/L_q)q_m\right)\right] \qquad (9)$$

which can be written as a matrix equation:

$$A\alpha = b \qquad (10)$$

where the $k^{th}$ element of the vector a is given by:

$$\alpha_k = c_k \qquad (k = 0,\ldots, N_q \sim 1) \qquad (11\,a)$$

$$\beta_j = f(q_j) \qquad (j = 0,\ldots, M_q \sim 1) \qquad (11\,b)$$

and

$$A_{jk} = 1/N_q\,[\cos(2\pi/L_q)kq_j] + i \sin(2\pi/L_q)\,kq_j)] \qquad (12)$$

Notice that the $A_{jk}$ depend only on where the measurement $q_j$ is made but not on the actual value of $f(q_j)$ at that point. Thus not only can A be calculated before any experiment is performed, it also contains the information of what is meant by an experiment design: where the measurements are to be made.

The formalism above has recast the discrete Fourier decomposition of (7) and (8) as an explicit estimation problem: given $M_q$ measurements of the function f(q) estimate the $N_q$ coefficients $\alpha$.

Altimeters make measurements over the 2D surface of the Earth as they orbit, and as the Earth rotates. The full altimeter design problem is therefore 3D (two spatial and one temporal). However, there are several reasons why only the 2D version of this experiment design problem will be considered here.

Computationally the 2D problem is significantly less intensive. The matrix A grows exponentially in the number of dimensions, and the singular value decomposition of A - used to evaluate its condition number - also scales as the cube of the smaller of its two dimensions. A few evaluations of the spectrum of A may be possible even if it is very large, but the experiment design problem necessarily demands that many configurations be investigated. Furthermore, although mathematically a 2D region (*see* Fig. 1) is not the same as a 3D region, certainly the nearby ocean regions will be sampled in a similar way and if the 2D region is well sampled, the neighboring ocean will be as well. Arguments based on the Rossby radius of deformation can be used to give some idea of just how far this region might extend.

Indeed, because the ocean's temporal and spatial scales are different in different regions (*e. g.* Fu, 1983 and Le Traon *et al.*, 1990), posing the 3D problem in a reasonable way requires that one attempt to "image" all of the time and length scales everywhere, even if they are appropriate only in the two different regions individually; a waste of computational effort if the individual regions can be imaged separately. Considering different 2D regions - *i. e.* "2 + 1" D - of the ocean is just such an approach and allows the different time and length scales of the ocean to be accommodated. Thus using several 2D regions (as will be described in the next section) mimics the 3D design problem although in a more computationally efficient manner.

It must also not be forgotten that in fact the ocean does not have periodic boundary conditions, and in some way the Fourier decomposition used is not appropriate, although it has become very much a standard tool in data analysis. Extending this formalism too far does not necessarily import more "realism", or does so only superficially.

Lastly, the work presented here is very much a "proof of concept"; an attempt to cast the choice of orbit for a satellite borne altimeter as a quantitative problem in experiment design. Hence an easily understood - simple - approach to this design problem is appropriate.

For all of these reasons a 2D version - one spatial, one temporal - of the design problem was considered. Thus equation (7) is now written:

$$f(p_n, q_m) = (1/N_p)\,(1/N_q) \sum_{k=0}^{(N_p=1)} \sum_{j=0}^{(N_q=1)} c_{kj}$$

$$\exp\left[i2\pi \left((k/L_p)\, p_n + (j/L_q) q_m\right)\right] \tag{13}$$

The variables p and q can be identified with x and t respectively. The region on which f(x, t) is supposed to be periodic is $\{x \text{ on } [0, L_x],\ t \text{ on } [0, L_t]\}$. Equations (11) and (12) generalize straightforwardly to a minimization problem of the form (1) with $N_x N_t$ unknowns (the coefficients $c_{kj}$ in (13) to be estimated.

## DETAILS OF THE SIMULATIONS, AND SUMMARY OF THE RUNS

Figure 1 diagrams the 2D representation of how an altimeter would sample the sea-surface height along a 1D spatial strip of the Earth's surface. The time at which the altimeter passes over the 1D strip is the value of second coordinate in the model. As it is the matrix A which is of primary interest here, the numerous errors and corrections associated with an altimeter measurement are not parameterized, but are assumed to make up the $\delta b$ and $\delta A$ vectors in equation (5). It is also assumed that (6 *b*) is always satisfied and hence truncating the spectrum of A - the regularization problem - was not considered. Indeed, in general the better the spectrum of A, the less likely the small singular values are to be dominated by the noise and errors associated with altimeter measurements.

To proceed with the construction of A, $L_x$ and $L_t$ in (13) need to be chosen. The ocean presents us with regions with many different length and time scales. For the purposes here, two regions were considered [*see* Fu (1983) and LeTraon *et al.* (1990)]. One was a region of high variability, a small "fast" region with $L_x$ = 3 000 km and $L_t$ = 15 days situated at 40° North and eastern edge at 40° West (to be referred to as the "small/fast" region from here on). The other is a large "slow" region in the equatorial Pacific with $L_x$ = 10 000 km and $L_t$ = 30 days situated on the equator with the eastern edge of the region at 100° West (the "large/slow" region). These two regions were chosen because they have significantly different length and time scales and geographic locations, hence provide insight into how altimeter configurations vary with different regions of interest (the thirty day time scale is still too short for the large/slow region, but the spirit of having two different regions is retained). The frequencies used in the decomposition (13) were all evenly spaced. In all runs, $N_x$ and $N_t$ were both equal to 10.

To evaluate the performance of a particular orbit *vis-à-vis* a particular region - large/slow, or small/fast - its associated A was constructed by calculating when and where it passed over-sampled - the region of interest. Then using the 2D version of (12), A is easily obtained. The spectrum of A was then obtained *via* its singular value decomposition. This allowed the condition number of A to be directly calculated. It should be noted that for underdetermined versions of (1), the rank of A can be different for different orbits. This occurred for the small/fast region. Intercomparison of the spectra then becomes problematic. The large/slow region always posed an overdetermined problem and thus was more easily evaluated. This will touched on again in the fourth section when the results are discussed.

Table 1 lists the six investigations. Runs 1 and 2 were for configurations of two altimeters. Each was considered to be of type "Topex/Poseidon" namely: 10 nodal days per repeat, 127 orbits per repeat, inclination of 66.0°. Given this, the only parameter being varied was the angle - or phase - between the orbit planes, and the relation of these planes to the region being sampled. This constitutes a modest search space. Dividing up 360° in 10° increments, and choosing one of the angles for each altimeter give 630 configurations with two satellites. Runs 3 and 4 were for configurations of three altimeters. Again, all were of type "Topex/Poseidon" and following the same scheme just above, 7 140 configurations for three satellites were evaluated.

Table 1

*List of satellite borne altimeter configurations evaluated.*

| Run | Altmtrs | Type | Region of A | Evaluations |
|---|---|---|---|---|
| 1 | 2 | both T/P | large/slow | 630 |
| 2 | 2 | both T/P | small/fast | 630 |
| 3 | 3 | all T/P | large/slow | 7 140 |
| 4 | 3 | all T/P | small/fast | 7 140 |
| 5 | 2 | T/P plus another | large/slow | 12,736 |
| 6 | 2 | T/P plus another | small/fast | 12,736 |

In the last two runs - designed to test heterogeneous configurations - one of the altimeters was of type "Topex/Poseidon". The available search space for the other satellite was reduced following Parke's *et al.* recommendation that only repeating orbits with altitudes higher than roughly 1 200 km be considered for long term altime-

Table 2

*Best configurations found in runs 1 and 2 of Table 1.*

**BEST CONFIGURATIONS FOR RUN 1 (SAMPLING OF LARGE/SLOW REGION)**

| Phases (degrees) altmtr 1 | altmtr 2 | Obs in rgn of intrst altmtr 1 | altmtr 2 | Condition number (100th) |
|---|---|---|---|---|
| 0.0 | 180.0 | 192 | 192 | 1.088 |
| 350.0 | 170.0 | 192 | 192 | 1.088 |
| 340.0 | 160.0 | 192 | 192 | 1.088 |
| 330.0 | 150.0 | 192 | 192 | 1.088 |
| 320.0 | 140.0 | 192 | 192 | 1.088 |
| 310.0 | 130.0 | 192 | 192 | 1.088 |
| 300.0 | 120.0 | 192 | 192 | 1.088 |
| 290.0 | 110.0 | 192 | 192 | 1.088 |
| 280.0 | 100.0 | 192 | 192 | 1.088 |
| 270.0 | 90.0 | 189 | 189 | 1.102 |

**BEST CONFIGURATIONS FOR RUN 2 (SAMPLING OF SMALL/FAST REGION)**

| Phases (degrees) altmtr 1 | altmtr 2 | Obs in rgn of intrst altmtr 1 | altmtr 2 | Condition number (72nd) |
|---|---|---|---|---|
| 300.0 | 50.0 | 38 | 36 | 2.508 |
| 270.0 | 90.0 | 38 | 38 | 2.510 |
| 0.0 | 250.0 | 39 | 37 | 2.521 |
| 280.0 | 100.0 | 38 | 38 | 2.525 |
| 290.0 | 110.0 | 38 | 38 | 2.526 |
| 310.0 | 130.0 | 38 | 38 | 2.526 |
| 310.0 | 60.0 | 38 | 37 | 2.526 |
| 290.0 | 40.0 | 38 | 39 | 2.528 |
| 270.0 | 20.0 | 38 | 39 | 2.532 |
| 320.0 | 70.0 | 37 | 36 | 2.532 |

| Phases (degrees) altmtr 1 | altmtr 2 | Obs in rgn of intrst altmtr 1 | altmtr 2 | Condition number (80th) |
|---|---|---|---|---|
| 210.0 | 190.0 | 40 | 40 | 57.757 |

**BEST CONFIGURATIONS FOR TRADE OFF BETWEEN RUNS 1 AND 2**

**Circled points of trade off plot for runs 1 and 2 (Fig. 4)**

| Phases (degrees) altmtr 1 | altmtr 2 | Condition number large/slow (100th) | small/fast (72nd) |
|---|---|---|---|
| 280.0 | 100.0 | 1.088 | 2.525 |
| 270.0 | 90.0 | 1.102 | 2.510 |

| Phases (degrees) altmtr 1 | altmtr 2 | Condition number large/slow (100th) | small/fast (80th) |
|---|---|---|---|
| 210.0 | 190.0 | 2.145 | 57.757 |

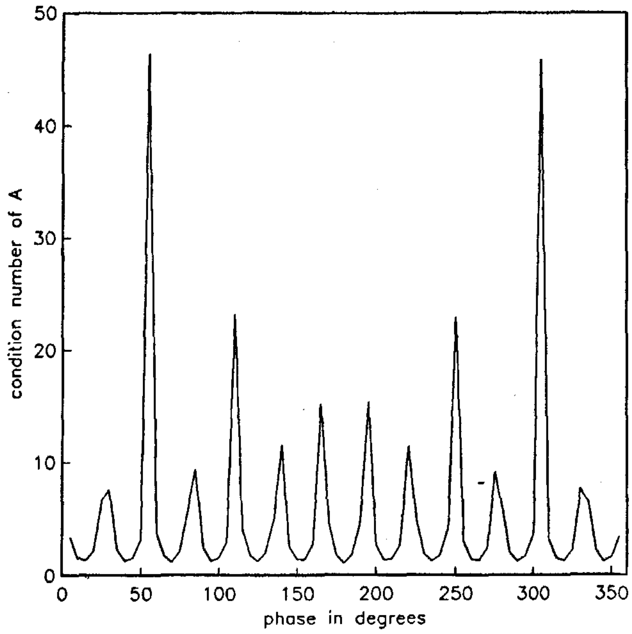



Figure 2

*Condition number of the matrix A* versus *angle between them orbital planes of two Topex/Poseidon altimeters sampling the large/slow equatorial region described in the text. One altimeter is fixed at the Greenwich Meridian, the other is free to take any angle on [0, 359.9]. The best conditioned A is associated with a separation of 180° between the orbital planes.*

ter missions. The set of orbits was further reduced by limiting them to altitudes lower than 3 000 km and by selecting only those with an even number of nodal days per repeat. In all, 364 different sets of values of the number of nodal days per repeat, and the number of orbits per repeat, were considered. They ranged from 2 (nodal) days/21 orbits per repeat to 36 days/349 orbits per repeat. All orbits were given an inclination of 66.0° and assumed circular. For each of the 364 sets of orbit parameters, 35 configurations were evaluated. These configurations were for different angles chosen on [0.0, 359.9 (the conventions used here were such that the first ascending track of a satellite with phase of 0.0 begins at the equator at the Greenwich meridian)]. The Topex/Poseidon type altimeter's phase varied in increments of 45°, and the other altimeter's phase varied in 15° increments. Together this amounted to roughly 12 700 different configurations.

## RESULTS AND DISCUSSION

Table 2 gives the best configurations for runs 1 and 2. The first two columns give the angle of the orbital plane with respect to Greenwich. The second two columns give the number of observations in the region of interest by each of the two altimeters. The last column gives the condition number associated with the design A. For the large/slow region, A always has many more observations than unknowns, and (1) is an overdetermined system. Thus rank of A was always equal to the number of unknowns (100) and hence comparison of the various configurations was straightforward. Apparently, an angle of 180° between the orbital planes gave the best values for the condition num-

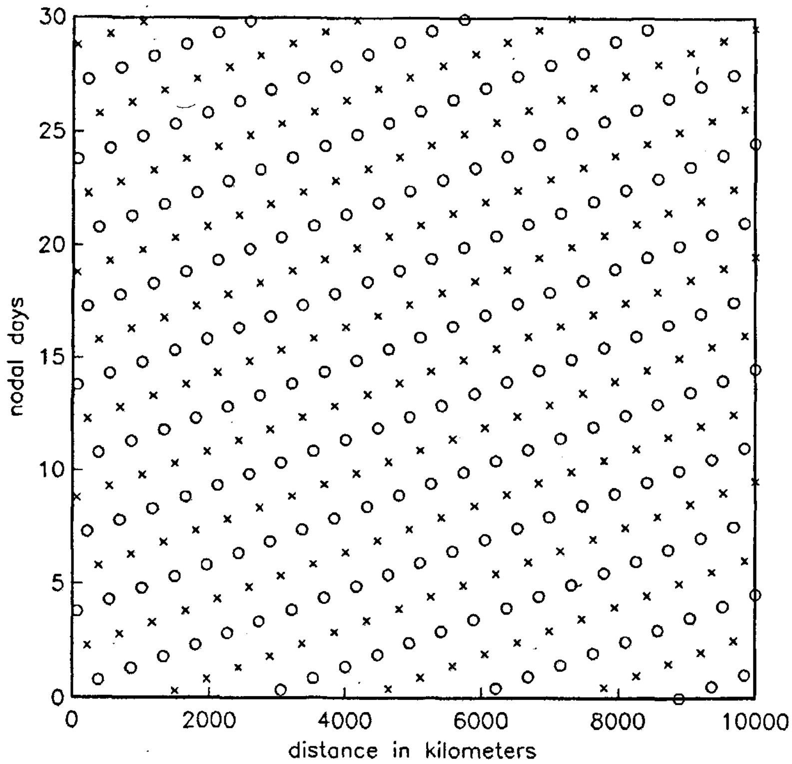



Figure 3 *a*

*Sampling of the large/slow region two Topex/Poseidon altimeters separated by 180°. The "x's" and "o's" are the space/time coordinates of the observations by the individual satellites.*

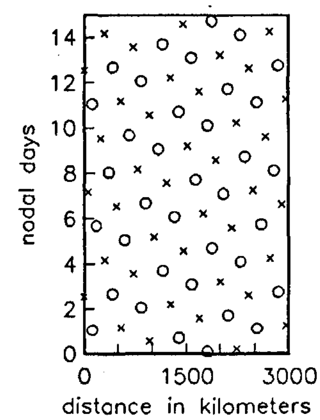
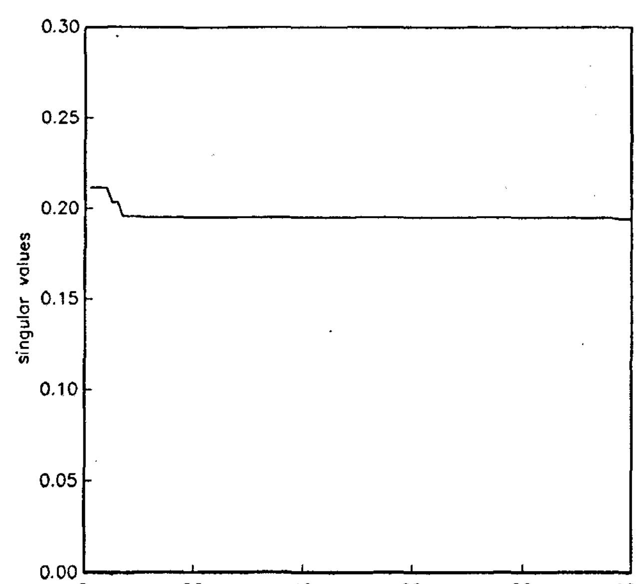



Figure 3 *b*

*The same as in Figure 3* a *except to for the small/fast region. These two figures are to scale.*

Figure 3 *c*

*The spectrum of A associated with Figure 3* a. *If the sampling was perfectly uniform, then the spectrum would be flat (see Wunsch, 1989).*




ber of A. Figure 2 gives a plot of condition number *versus* angle when one altimeter is fixed at the Greenwich meridian, and the other varies in 5° increments on [0, 359.9].

In contrast to the above, the small/fast region was always underdetermined. This makes comparison of the condition number of A for different designs more difficult, because the rank of A changed as well. Low rank A's tend to have better condition numbers than A's with a higher rank. Yet the higher the rank, the more unknowns are determined, (although in general not as well). Using the condition number as an index of performance of the design is thus ambiguous. This problem will have to be resolved before a more complex numerical optimization compared to the one used here can be undertaken. For now the issue is side stepped in the following way.

In the small/fast region the smallest singular value which was non-zero for all of the configurations evaluated was used to calculate the condition number used for comparison of the configurations. For runs 1 and 2 this was the 72$^{nd}$ singular value. All configurations had an A of at least rank 72 for the small/fast region, and hence this rank configuration will be used as a main point of comparison. As the spectrum of A is truncated, the singular values increase in magnitude. Spectra for high rank A's were necessarily truncated more than low rank A's and the resulting condition numbers tended to reflect this. Hence comparison of condition numbers - even for spectra truncated to rank 72 - tended to favor designs which were of higher rank.

For the small/fast region, the best of the rank 72 configurations have either 180 or 250° between the orbital planes of the two satellites. These configurations had condition numbers which were only about a factor of two larger than those of the best configurations for the large/slow region.

Table 2 also gives the best configuration with the highest rank possible: 80. There is only one such configuration for two Topex/Poseidon type satellites, and its condition number is an order of magnitude larger than the best configurations with truncated spectra of rank 72. Interestingly the phase angle between orbital planes is only 20°.

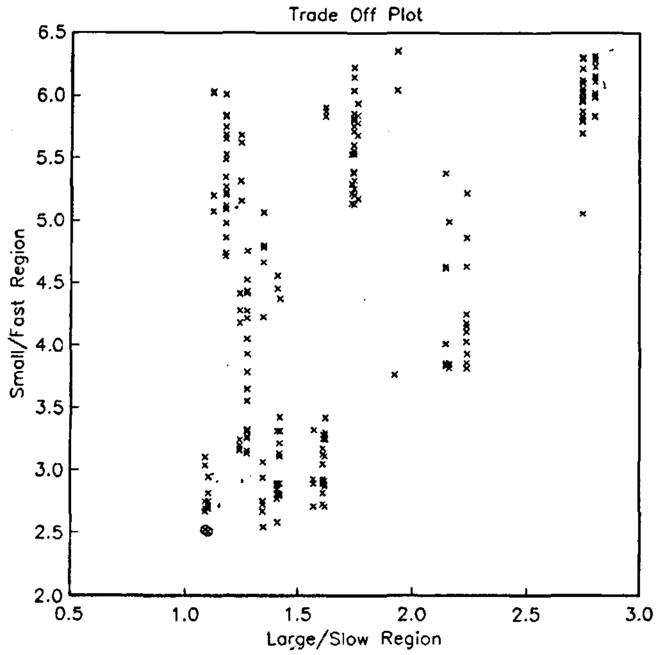



Figure 4

*Trade off plot for runs 1 and 2 (two T/P orbiters).Each x represents a particular configuration of the two altimeters. The x-axis is the condition numberfo this configuration for the large/slow region; the y-axis is the condition number for this same configuration for the small/fast region. The two configurations which best sample both regions simultaneously are circled. Table 2 gives their specifics (NB: for the small/fast region A was equal, or truncated, to 72.)*

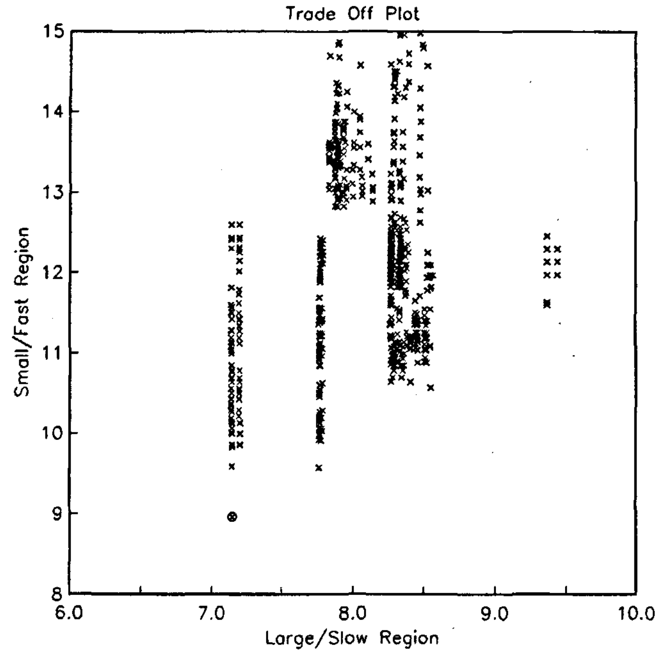



Figure 5 *a*

*Trade off plot for runs 3 and 4 (three T/P orbiters) using designs in the small/fast region equal, or truncated, to 72. Circled point particulars as in Table 3.*

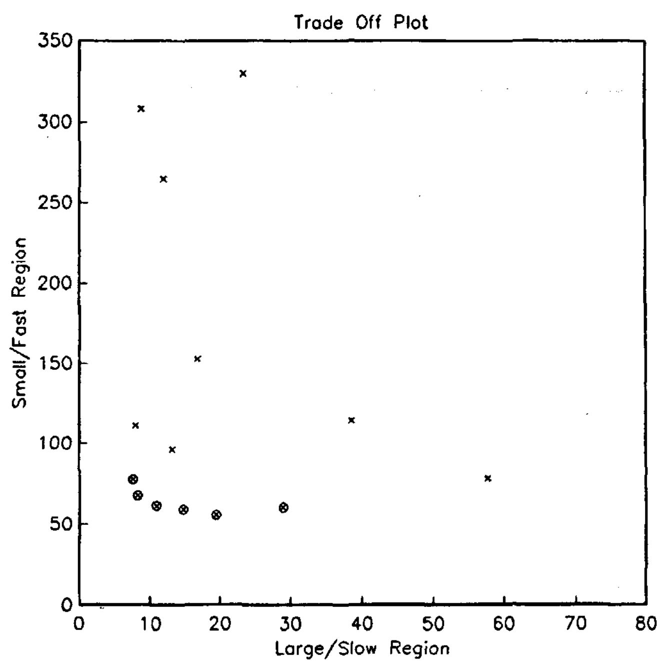



Figure 5 *b*

*Trade off plot for runs 3 and 4 using the "highest rank" designs with three T/P orbiters. Circled points particulars as in Table 3.*

Figure 3 shows how the observations points for the two regions are distributed in space and time, and the spectrum of A for the large/slow region when the separation is 180°. Had the sampling been perfectly uniform, A's spectrum would have been flat (*see* Wunsch, 1989).

Figure 4 is a "trade off" plot for the large/slow and the small/fast regions. Note well that for comparison reasons mentioned above, all spectra for the small/fast region are of rank 72. This plot is used to find which single configuration best samples both regions simultaneously. However it should be noted that the trade off plot may not be exactly as depicted. This is because when the satellite data are analyzed individual time origins can be used for each of the regions, rather than constraining them to both use the same one. This can be done without loss of generality. As the change to Figure 4 would be only slight finding optimal time origins for the two regions was not investigated.

Table 2 lists particulars for the two circled points in Figure 4. Both have phase angles of 180° and are among the best configurations for both regions. It should also be noted that even for the small/fast region's sole rank 80 configuration which has a phase angle of 20°, the large/slow region still has reasonable condition number of 2.145.

Table 3 gives the best configurations for runs 3 and 4. These runs are for configurations of three altimeters. For the large/slow region two of the altimeters are always separated by 180°. Configuration with the altimeters spaced at intervals of 120° did not perform as well as any of those in Table 3.

The best configurations of three altimeters for the small/fast region seem to have even less of a pattern. Again, for comparison with the two orbiter configurations the three orbiter configurations are truncated to be of rank 72. In all of the configurations two orbiters are separated by an angle of 180°. The value of the condition number for these configurations suggests the small/fast region is being sampled about as well as the large/slow region.

Table 3

*Best configurations found in runs 3 and 4 of Table 1.*

**BEST CONFIGURATIONS FOR RUN 3 (SAMPLING OF LARGE/SLOW REGION)**

| Phases (degrees) | | | Obs in rgn of intrst | | | Condition number |
|---|---|---|---|---|---|---|
| altmtr 1 | altmtr 2 | altmtr 3 | altmtr 1 | altmtr 2 | altmtr 3 | (100th) |
| 320.0 | 190.0 | 140.0 | 192 | 189 | 192 | 7.139 |
| 320.0 | 200.0 | 140.0 | 192 | 189 | 192 | 7.139 |
| 320.0 | 210.0 | 140.0 | 192 | 189 | 192 | 7.139 |
| 320.0 | 220.0 | 140.0 | 192 | 189 | 192 | 7.139 |
| 320.0 | 230.0 | 140.0 | 192 | 189 | 192 | 7.139 |
| 320.0 | 240.0 | 140.0 | 192 | 189 | 192 | 7.139 |
| 320.0 | 250.0 | 140.0 | 192 | 189 | 192 | 7.139 |
| 320.0 | 260.0 | 140.0 | 192 | 189 | 192 | 7.139 |
| 320.0 | 270.0 | 140.0 | 192 | 189 | 192 | 7.139 |
| 330.0 | 190.0 | 150.0 | 192 | 189 | 192 | 7.139 |
| 330.0 | 200.0 | 150.0 | 192 | 189 | 192 | 7.139 |
| 330.0 | 210.0 | 150.0 | 192 | 189 | 192 | 7.139 |
| 330.0 | 220.0 | 150.0 | 192 | 189 | 192 | 7.139 |
| 330.0 | 230.0 | 150.0 | 192 | 189 | 192 | 7.139 |
| 330.0 | 240.0 | 150.0 | 192 | 189 | 192 | 7.139 |
| 330.0 | 250.0 | 150.0 | 192 | 189 | 192 | 7.139 |
| 330.0 | 260.0 | 150.0 | 192 | 189 | 192 | 7.139 |
| 330.0 | 270.0 | 150.0 | 192 | 189 | 192 | 7.139 |
| 310.0 | 190.0 | 130.0 | 192 | 189 | 192 | 7.141 |

**BEST CONFIGURATIONS FOR RUN 4 (SAMPLING OF SMALL/FAST REGION)**

| Phases (degrees) | | | Obs in rgn of intrst | | | Condition number |
|---|---|---|---|---|---|---|
| altmtr 1 | altmtr 2 | altmtr 3 | altmtr 1 | altmtr 2 | altmtr 3 | (72nd) |
| 290.0 | 210.0 | 110.0 | 38 | 40 | 38 | 8.960 |
| 200.0 | 190.0 | 20.0 | 36 | 40 | 39 | 9.576 |
| 350.0 | 190.0 | 170.0 | 36 | 40 | 37 | 9.588 |
| 280.0 | 210.0 | 100.0 | 38 | 40 | 38 | 9.822 |
| 280.0 | 230.0 | 100.0 | 38 | 39 | 38 | 9.850 |
| 280.0 | 200.0 | 100.0 | 38 | 36 | 38 | 9.852 |
| 280.0 | 260.0 | 100.0 | 38 | 36 | 38 | 9.852 |
| 280.0 | 240.0 | 100.0 | 38 | 36 | 38 | 9.852 |
| 280.0 | 180.0 | 100.0 | 38 | 36 | 38 | 9.852 |
| 280.0 | 150.0 | 100.0 | 38 | 36 | 38 | 9.852 |
| 290.0 | 260.0 | 110.0 | 38 | 36 | 38 | 9.864 |

| Phases (degrees) | | | Obs in rgn of intrst | | | Condition number |
|---|---|---|---|---|---|---|
| altmtr 1 | altmtr 2 | altmtr 3 | altmtr 1 | altmtr 2 | altmtr 3 | (81st) |
| 300.0 | 210.0 | 190.0 | 38 | 40 | 40 | 55.051 |
| 250.0 | 210.0 | 190.0 | 37 | 40 | 40 | 55.617 |
| 350.0 | 210.0 | 190.0 | 36 | 40 | 40 | 59.169 |
| 270.0 | 210.0 | 190.0 | 38 | 40 | 40 | 60.274 |
| 320.0 | 210.0 | 190.0 | 37 | 40 | 40 | 61.399 |
| 340.0 | 210.0 | 190.0 | 37 | 40 | 40 | 67.752 |
| 260.0 | 210.0 | 190.0 | 36 | 40 | 40 | 77.737 |
| 330.0 | 210.0 | 190.0 | 36 | 40 | 40 | 78.373 |
| 280.0 | 210.0 | 190.0 | 38 | 40 | 40 | 96.279 |
| 230.0 | 210.0 | 190.0 | 39 | 40 | 40 | 111.385 |

**BEST CONFIGURATIONS FOR TRADE OFF BETWEEN RUNS 3 AND 4**

**Circled points of trade off plot for runs 3 and 4 (Fig. 5 *a*)**

| Phases (degrees) | | | Condition number | |
|---|---|---|---|---|
| altmtr 1 | altmtr 2 | altmtr 3 | large/slow (100th) | small/fast (72nd) |
| 290.0 | 210.0 | 110.0 | 7.146 | 8.960 |

**Circled points of trade off plot for runs 3 and 4 (Fig. 5 *b*)**

| Phases (degrees) | | | Condition number | |
|---|---|---|---|---|
| altmtr 1 | altmtr 2 | altmtr 3 | large/slow (100th) | small/fast (81st) |
| 250.0 | 210.0 | 190.0 | 19.442 | 55.617 |
| 350.0 | 210.0 | 190.0 | 14.715 | 59.169 |
| 270.0 | 210.0 | 190.0 | 28.984 | 60.274 |
| 320.0 | 210.0 | 190.0 | 10.899 | 61.399 |
| 340.0 | 210.0 | 190.0 | 8.269 | 67.752 |
| 260.0 | 210.0 | 190.0 | 7.610 | 77.737 |

Table 3 also lists "highest rank" configurations of three T/P orbiters. They were found to have rank equal to 81. Their condition numbers were comparable to the one rank 80 configuration for two orbiters. Similar to that configuration, these high rank three satellite arrangements have a phase angle of only 20° between two of the orbiters; the third altimeter having a phase angle always greater than or equal to this with the other two.

Figure 5*a* gives the trade off plot for the three orbiter (rank 72) configurations, with the circled point having its particulars listed in Table 3. This is among the best configuration for both the large/slow and the small/fast region indicating that both regions can be sampled simultaneously about as well as they can be individually. As indicated in Table 3, this simultaneity also holds true for the "highest rank" configurations whose trade off plot is given in Figure 5 *b*.

Comparison of the results shown in Tables 2 and 3 indicates that there are configurations of two T/P satellite borne altimeters which perform as well, or better than configurations of three T/P altimeters. The best two T/P satellite configurations always sample the large/slow region slightly better than best configurations of three T/P satellites. This is also true when comparison between two or three satellite configurations restricted to being of rank 72 is undertaken. Only if the "highest rank" configurations are compared do the three T/P satellite configurations perform slightly better, but this only because they are of rank 81 rather than rank 80. Essentially the best "highest rank" two and three satellite configurations have the same condition numbers.

This may be surprising at first because in general one expects more measurements to lead to better estimations. Apparently the space/time pattern with which three T/P altimeters sample the two regions is more sensitive to noise in the measurements, than the sampling pattern for two altimeters. In other words, there is a poor "matching" of the space/time scales of the region, and the space/time scales of the sampling for three altimeters all of the Topex/Poseidon type.

For two Topex/Poseidon altimeters however, the match is almost perfect for the large/fast region (changing the size of $L_x$ and $L_t$ slightly or the origin of the region would make it even more so) because the condition number for A is nearly 1.0 (its minimum). The small/fast region, being underdetermined, is not sampled as well, but still - for the rank 72 comparison - better than for the best configurations with three altimeters.

If there was any advantage to the three satellite configurations however, it was to be found in the way the small/fast region was sampled. There the number of measurements made with three satellites is roughly 50 % greater than the case with two, and was always greater than the number of unknowns (which was 100). Hence there were several high rank designs of three satellites (rank 81) compared to a single "highest rank" design (rank 80) with two T/P altimeters. But unless determining one extra unknown is an experimental imperative, it seems reasonable - for the model used here - to suggest that optimal configurations of two T/P altimeters perform as well optimal configurations of three such satellites do. The conditions under which this conclusion may change are mentioned below.

Before discussing the last two runs, recall that so far it has always been assumed that for all the designs considered, the "noise-to-signal" level has always satisfied (6 *b*). If it does not, then the effective rank of A will be smaller, and its spectrum must be truncated. This is most likely to be necessary for the "highest rank" designs in the small/fast region where the condition number is roughly 50.0. However, such high rank designs will be penalized by (6 *b*) in the same way for both two or three T/P orbiters configurations. Furthermore, in the most extreme case of noise in which only rank 72 designs satisfy (6 *b*), the comparison between two and three T/P altimeter configurations has been explicitly carried out and documented in Tables 2 and 3.

The conclusion remains that if there is no noise in the measurements, then the only difference between two or three T/P altimeter designs is that three orbiters can determine one more unknown in the small/fast region than two can. If there is substantial a noise-to-signal level nearly equal to 1.0, then two T/P altimeters will out perform three such altimeters by a factor of 2 to 8. Given these two extreme cases it seems unlikely that there is some intermediate noise-to-signal regime in which three T/P altimeter designs will be better than two.

The last two runs made were for heterogeneous configurations of two altimeters. One was always of type Topex/Poseidon, whereas the other was one of the 364 repeating orbits with altitudes greater than 1 200 and less than 3 000 km and with an even number of nodal days per repeat (*see* also the Introduction). Runs 5 and 6 are summarized in Table 4. For the 35 separations evaluated, the large/slow region continues to be better sampled in general when both altimeters are of type Topex/Poseidon. However two other choices for the second altimeter perform essentially as well: 16 nodal day repeat with 203 orbits, or a 22 nodal day repeat with 279 orbits. Although Run 1 shows that this region is sampled optimally when both altimeters are of the type Topex/Poseidon, it is important to remember that only 35 different separations were evaluated for each choice of orbit parameters. There may be other angles between the orbit planes, or angles between

Table 4

*Best configurations found in runs 5 and 6 of Table 1.*

**BEST CONFIGURATIONS FOR RUN 5 (SAMPLING OF LARGE/SLOW REGION)**

| Phases (degrees) altmtr 1 | Phases (degrees) altmtr 2 | Obs in rgn of intrst altmtr 1 | Obs in rgn of intrst altmtr 2 | Orbit parameters d1 | Orbit parameters n1 | Orbit parameters d2 | Orbit parameters n2 | Condition number (100th) |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 180.0 | 192 | 192 | 10 | 127 | 10 | 127 | 1.088 |
| 0.0 | 345.0 | 192 | 189 | 10 | 127 | 10 | 127 | 1.213 |
| 315.0 | 300.0 | 189 | 192 | 10 | 127 | 10 | 127 | 1.213 |
| 270.0 | 255.0 | 189 | 192 | 10 | 127 | 10 | 127 | 1.213 |
| 225.0 | 210.0 | 192 | 189 | 10 | 127 | 10 | 127 | 1.213 |
| 0.0 | 285.0 | 192 | 190 | 10 | 127 | 16 | 203 | 1.401 |
| 0.0 | 210.0 | 192 | 189 | 10 | 127 | 10 | 127 | 1.408 |
| 315.0 | 240.0 | 189 | 190 | 10 | 127 | 16 | 203 | 1.413 |
| 315.0 | 210.0 | 189 | 190 | 10 | 127 | 22 | 279 | 1.427 |
| 270.0 | 195.0 | 189 | 191 | 10 | 127 | 16 | 203 | 1.428 |

**BEST CONFIGURATIONS FOR RUN 6 (SAMPLING OF LARGE/SLOW REGION)**

| Phases (degrees) altmtr 1 | Phases (degrees) altmtr 2 | Obs in rgn of intrst altmtr 1 | Obs in rgn of intrst altmtr 2 | Orbit parameters d1 | Orbit parameters n1 | Orbit parameters d2 | Orbit parameters n2 | Condition number (smallest) |
|---|---|---|---|---|---|---|---|---|
| 315.0 | 315.0 | 39 | 29 | 10 | 127 | 30 | 293 | 3.624 |
| 315.0 | 300.0 | 39 | 29 | 10 | 127 | 30 | 293 | 3.770 |
| 315.0 | 255.0 | 39 | 28 | 10 | 127 | 6 | 61 | 3.911 |
| 0.0 | 180.0 | 39 | 30 | 10 | 127 | 24 | 251 | 3.933 |
| 0.0 | 315.0 | 39 | 27 | 10 | 127 | 20 | 193 | 4.072 |
| 225.0 | 210.0 | 38 | 29 | 10 | 127 | 20 | 193 | 4.093 |
| 0.0 | 285.0 | 39 | 27 | 10 | 127 | 14 | 137 | 4.103 |
| 0.0 | 300.0 | 39 | 30 | 10 | 127 | 14 | 143 | 4.180 |
| 0.0 | 285.0 | 39 | 36 | 10 | 127 | 10 | 127 | 4.182 |
| 270.0 | 225.0 | 38 | 30 | 10 | 127 | 10 | 107 | 4.211 |

the planes and the Greenwich Meridian, for which other orbits also perform as well. This study is not yet complete.

Table 4 shows that the small/fast region is well sampled for a larger variety of orbits for the second altimeter. The condition numbers listed here are calculated using the smallest singular value, rather than the $72^{nd}$ as in Table 2. For different arrangement of the altimeters, the rank of A changed (the rank of A varied between 72 and 80). The condition numbers listed are essentially all the same, but the second last configuration corresponds to a design with a rank nearly 9 % larger than the next highest rank design and for this reason is interesting. It involves two Topex/Poseidon altimeters, but with an angle of only 75° between their orbital planes. Notice that none of the 35 angles considered corresponded to the optimal configurations listed in Table 2. Table 2 configurations of two T/P altimeters were the best performers found, although it must be repeated that not all of the possible angles, nor all of the possible sets of heterogeneous orbit parameters, were evaluated.

Choosing optimal orbits for satellite borne altimeter is a difficult problem. The approach used here toward solving this problem was from the perspective of sampling theory. A simple 2D representation was used for two different regions characterized by large/slow (equatorial Pacific) and small/fast (Atlantic mid-latitude) space and time scales. Although a full 3D representation may be more suited to the sampling characteristics of a satellite borne altimeter it is argued here that the "2 + 1"D method used here is significantly more efficient computationally and provides an approach easily adapted toward sampling the different space time regions of ocean variability.

Having said this, the results above have shown that sampling strategies are not determined by track separation alone. The interplay of the sampling characteristics of an altimeter and different spatial and temporal scales present in the ocean is a complex one not necessarily well understood using intuition or other qualitative methods alone (Chase and Mundt, 1989). For example, although satellite tracks are always closer together at higher latitudes than they are at the equator, this does not mean that higher latitude regions are always estimated with more accuracy. The small/fast mid-latitude region used in these investigations was always underdetermined. The large/slow equatorial region was always overdetermined.

It is also interesting to note that there seem to be no configurations of three Topex/Poseidon altimeters as well matched to the two regions as for two such altimeters. Altering the space/time scales of the regions, or changing the frequencies used in the Fourier decomposition might change this conclusion. It should also be remembered that the configurations considered were quite constrained: satellites had inclinations of 66°, and the Topex/Poseidon orbit parameters. Changing any of these could change the performance of two *versus* three orbiters. However, given the above constraints, and the 2D representation in the large/slow and small/fast regions, two Topex/Poseidon orbiters essentially performed as well, or better than three. That is two orbiters with 180° between their orbit planes were able to estimate the sea-surface height field for two regions of the ocean with different spatial/temporal scales such that the estimate had minimum sensitivity to measurement and orbit errors, as well as minimum average variance for the estimated Fourier components of that field.

It must be noted however, that in a full 3D model, it seems likely that both spatial/temporal regions of the ocean considered would have posed underdetermined problems. Then, three orbiters may lead to higher ranks for the matrix A than two orbiters would. Although these (perhaps only slightly) higher rank A's would likely have poorer condition numbers, and hence lead to estimates of the sea-surface height field which are more sensitive to measurement and orbit error, more of the Fourier components of the field would be estimated (but with a higher average variance). This is a common trade off situation in experiment design.

With the given model and orbit parameters used, heterogeneous configurations (*i. e.* only one of the three is of type Topex/Poseidon) of three altimeters offer the sole possibility of superior performance to two Topex/Poseidon altimeters, although such configurations have not yet been found. Indeed, because the large/slow region is sampled nearly optimally, it seems heterogeneous configurations can be important only for improving the sampling in the underdetermined small/fast region. Even so, this will not be easy as this region is also very well sampled by two T/P altimeters. If the model is extended to a full 3D representation - despite its formidable computational burden - and the other constraints on the orbit parameters, inclination, and phase are relaxed the conclusions above may change. But the quantitative experiment design approach used here will remain, and as such provides a valuable tool toward sorting out the interplay of an orbiting altimeters sampling characteristics, and the space time variability of the different regions of the ocean.

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