Oceanic response to coastal winds with shear

Upwelling
Kelvin waves
Rossby waves
El Niño
Upwelling
Ondes de Kelvin
Ondes de Rossby
El Niño

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Received 13/10/83, in revised form 26/1/84, accepted 2/2/84.

ABSTRACT

This paper deals with the response of a stratified ocean to a wind blowing over a patch and parallel to a coast. In the f-plane case, it is found that the upwelling and the jet occurring at the coast acquire stable values after a relatively short time, because they are arrested by Kelvin wave fronts generated at the wind discontinuity, while the downwelling and the double jet occurring at the wind discontinuity continue to increase with time as the wind blows. In the case of an Eastern coast, it is found that the β effect suppresses the coastal motion via radiation of westward propagating Rossby waves. Here, the downwelling, eventually reaches a constant value which extends westward at the speed of long baroclinic Rossby waves and occupies a large surface of the ocean. This phenomenon could trigger the El Niño.

Oceanol. Acta, 1984, 7, 4, 409-416.

RÉSUMÉ

Réponse de l'océan à un cisaillement de vent côtier

On étudie la réponse d'un océan stratifié à un vent parallèle à la côte soufflant sur un rectangle. Dans le cas du plan f, on montre que l'upwelling et le jet engendrés à la côte atteignent une valeur constante relativement rapidement car leur croissance temporelle est arrêtée par des fronts de Kelvin engendrés par les discontinuités spatiales du vent, tandis que le downwelling et le double jet apparaissant au niveau de la discontinuité de vent continuent de croître aussi longtemps que le vent souffle. Dans le cas d'une côte Est, on trouve que l'effet β supprime les mouvements côtiers par radiation d'énergie vers l'Ouest via des ondes de Rossby. Le downwelling devient borné dans le temps et tend à se propager vers l'Ouest en gardant une valeur constante et en affectant une large surface de l'Océan. Ce phénomène peut avoir une influence sur El Niño.

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INTRODUCTION

It is well known that a spatially uniform wind, parallel to a coast, generates a jet and intense upwelling within a radius of deformation of the coast (Charney, 1955). In the absence of any coast, a wind with strong shear -- suppose for example a wind that is constant in the domain $x<0$ but is zero in the domain $x>0$ – induces an oceanic response very similar to that near a coast: within a radius of deformation of the shear at $x=0$ there are jets and upwelling. This paper is concerned with the oceanic response to winds with shear that blow parallel to a coast. The motivation for this study is Enfield's (1981) recent description of the winds along the coast of Peru. Far from this coast, the southeast tradewinds prevail, but within approximately 100 km of the coast there is a low level atmospheric jet, apparently driven by the land/sea temperature difference. The jet is most intense during the austral summer (February) at which time the intensity of the southeast tradewinds has a seasonal minimum. This means that the shear of the wind, in an offshore direction, is at a maximum in February. The present paper describes the oceanic response to wind shear of this type.

The model to be studied is presented in section 2. Section 3 describes general solutions, section 4 gives examples. Section 5 deals with the β effect. The principal results are summarized in the Discussion.

GOVERNING EQUATIONS

The model ocean covers the domain $x>0$ and is bounded by a coast at $x=0$. The wind blows over the region $0<x<a$ and $|y|<b$ (Fig. 1). We consider a coordinate system with x positive eastward, y positive northward. We deal with linearized flow in a two-layer fluid of constant depth (Fig. 2). We use the standard decomposition into normal modes (Crépon, Richez, 1982). The equations of motion and of continuity take the form:

$$\frac{\partial}{\partial t}\mathbf{u}_i+f\mathbf{z}\times\mathbf{u}_i=-g\nabla\zeta_i+\frac{1}{\rho \mathrm{H}_i}\tau, \qquad (2.1)$$

$$\frac{\partial}{\partial t}\zeta_i+\nabla.\mathrm{H}_i\mathbf{u}_i=0. \qquad (2.2)$$

The index i refers to one of the two possible modes. Here $\mathbf{u}_i$ is the velocity of the i-mode; ζ_i is the elevation of the i-mode; ∇ is the two-dimensional operator $\mathbf{x}\,\partial/\partial x$, $\mathbf{y}\,\partial/\partial y$; f is the Coriolis parameter; $\mathbf{z}$ is the unit vector on the z axis, positive upwards; h_i is the depth of each layer; H_i is the equivalent depth of the i-mode ($\mathrm{H}_1=h_1+h_2$; $\mathrm{H}_2=\varepsilon h_1 h_2/\mathrm{H}_1$); τ is the wind stress at the surface; g is the acceleration of gravity; $\varepsilon=(\rho_2-\rho_1)/\rho_2$ is the density ratio.

From (2.1) and (2.2), $\mathbf{u}$ can be eliminated and a equation for ζ is obtained:

$$\frac{\partial}{\partial t}\left[c_i^2\nabla^2\zeta_i^2-\left(\frac{\partial^2}{\partial t^2}+f^2\right)\zeta_i\right]=\frac{1}{\rho}\left[\frac{\partial}{\partial t}\nabla.\tau+f\mathbf{z}\nabla\times\tau\right], \qquad (2.3)$$

where $c_i^2=g\mathrm{H}_i$.

From (2.1) one obtains

$$\left(\frac{\partial^2}{\partial t^2}+f^2\right)\mathbf{u}_i=\frac{1}{\mathrm{H}_i}\left[-c_i^2\left[\frac{\partial}{\partial t}\nabla\zeta_i+\nabla\zeta_i\times f\mathbf{z}\right]+\frac{1}{\rho}\left[\frac{\partial}{\partial t}\tau+\tau\times f\mathbf{z}\right]\right]. \qquad (2.4)$$

The elevation $\bar{\zeta}_i$ at the surface ($i=1$) and at the interface ($i=2$) and the velocities $\bar{\mathbf{u}}_i$ in the upper ($i=1$) and the lower ($i=2$) layer are given in terms of the normal mode elevations ζ_i and velocities $\mathbf{u}_i$ as

$$\left.\begin{array}{l}\bar\zeta_1=\zeta_1+\varepsilon(h_2/\mathrm{H}_1)^2\zeta_2,\\ \bar\zeta_2(h_2/\mathrm{H}_1)\,(\zeta_1-\zeta_2,\end{array}\right\} \qquad (2.5)$$

$$\left.\begin{array}{l}\bar{\mathbf{u}}_1=\mathbf{u}_1+(h_2/h_1)\,(\mathrm{H}_2/\mathrm{H}_1)\,\mathbf{u}_2,\\ \bar{\mathbf{u}}_2=\mathbf{u}_1-(\mathrm{H}_2/\mathrm{H}_1)\,\mathbf{u}_2,\end{array}\right\} \qquad (2.6)$$

We assume that the motions start from rest at $t=0$, and we study their evolution with respect to time.

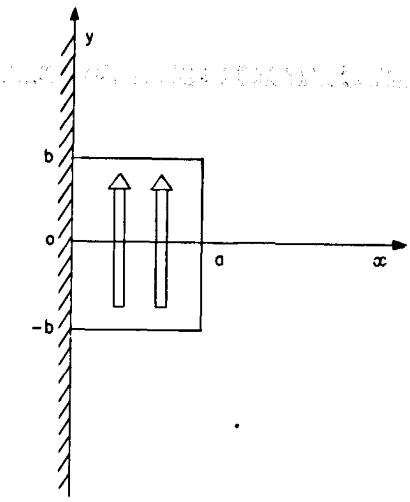

Figure 1
Wind pattern.
Schéma de vent utilisé.

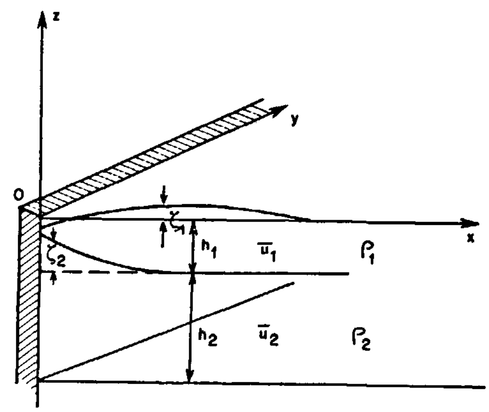

Figure 2
Definition of parameters.
Définition des paramètres.

In order to solve (2.3), we first apply a Fourier transform in y:

$$Z(t,x,k)=\int_{-\infty}^{+\infty}\exp(-i2\pi ky)\,\zeta(t,x,y)\,dy,$$

and then a Laplace transform in t:

$$\tilde{Z}(p,x,k)=\int_0^{\infty}\exp(-pt)\,Z(t,x,k)\,dt.$$

Implicitly, we suppose that the solutions sought, are generalized functions or distributions of exponential order in t, and polynomial order in y, in the sense of Lighthill (1958) and Schwartz (1966).

The Fourier-Laplace Transform (FLT hereinafter) of (2.3) is:

$$c^2\frac{\partial^2}{\partial x^2}\tilde{Z}-[p^2+f^2+(2\pi kc)^2]\tilde{Z}=\frac{1}{\rho}\left[\nabla.\tilde{\mathrm{T}}(\frac{f}{p}\mathbf{z}\nabla\times\tilde{\mathrm{T}}\right], \qquad (2.7)$$

where $\tilde{\mathrm{T}}$ is the FLT of τ. The partial differential equation (2.3) has been transformed into a linear differential equation with constant coefficients, the solution of which is well known. For simplicity, the indices are omitted in future references.

ANALYTICAL SOLUTIONS IN THE f-PLANE

The half-plane $(x>0)$ is divided into two regions (Fig. 1):

• In the region (1) $(0<x<a)$, the wind is parallel to the coast and function of y only.

$\tau = y\,\tau(y)\,\mathrm{Y}(x)\,\mathrm{Y}(a-x)\,\mathrm{Y}(t)$,

i. e.

$\mathrm{T} = y\,\mathrm{T}(k)\,\mathrm{Y}(x)\,\mathrm{Y}(a-x)/p$.

Y( ) being the step function

The solution of (2.7) may be written as

$$\mathrm{Z} = \mathrm{A}\,e^{-\gamma x} + \mathrm{B}\,e^{+\gamma x} + \mathrm{Z}_0, \qquad (3.1)$$

where

$$\gamma^2 = (p^2 + f^2 + (2\pi kc)^2)/c^2$$

and

$$\tilde{\mathrm{Z}}_0 = \frac{i 2\pi k \mathrm{T}(k)}{\rho c^2 p \gamma^2}.$$

A and B are functions of p and k, and $\mathrm{T}(k)$ is the Fourier Transform of $\tau(y)$.

• In the region (2) $(x>a)$, the wind is equal to zero and the solution of (2.7) which satisfies the radiation condition is

$$\mathrm{Z}' = \mathrm{A}'\,e^{-\gamma x}. \qquad (3.2)$$

A, B and A′ are obtained from the boundary conditions:

— at $x=0$, the x-component of the velocity in the region (1) must be equal to zero;

— at $x=a$, the elevations and the x-components of the velocities must be continuous.

The FLT of the elevation is

$$\begin{aligned}\tilde{\mathrm{Z}} = \frac{1}{2\rho c}\,\frac{\mathrm{T}(k)}{p^2\gamma^2 c^2}\Bigg[\Bigg[& 2i\pi kc\gamma + \frac{fp}{c}(1-2e^{\gamma a})\Bigg] \\ & \times \frac{(p^2+f^2)\,e^{-\gamma(x+a)}}{p\gamma - 2i\pi kf} \\ & - e^{-\gamma|x-a|}\,pc\left[\mathrm{sgn}\,(a-x)\,2i\pi k + \frac{f}{p}\gamma\right] \\ & - 4i\pi kcp\,\mathrm{Y}(a-x)\Bigg]. \qquad (3.3)\end{aligned}$$

It is difficult to Laplace invert $\tilde{\mathrm{Z}}$. But we can obtain an asymptotic expansion as t tends to infinity by using Sutton's algorithm (1934), based on algebraic expansions around the singularities of the image which have the largest real values.

As $t \to \infty$, the leading term of the asymptotic expansion of Z is

$$\begin{aligned}\underset{t\to\infty}{\mathrm{Z}} = \frac{\mathrm{T}(k)}{2\rho cf}\Bigg[& \overbrace{[\exp(-|x-a|m/r) - \exp(-|x+a|m/r]\,ft/m}^{(1)} \\ & + \frac{2i\pi kr}{m^2}\overbrace{[2\exp(-mx/r) - \exp(-|x-a|m/r) - \mathrm{sgn}\,(x-a)\exp(-|a-x|m/r) - 2\mathrm{Y}(a-x)]}^{(2)} \\ & + \frac{i}{\pi kr}\overbrace{[\exp(-|x+a|m/r) - \exp(-|x+a|/r) - \exp(-xm/r) + \exp(-x/r)]}^{(3)} \\ & - \frac{i}{\pi kr}\overbrace{[\exp(-|x+a|/r) - \exp(-x/r)]\,[\exp(i2\pi krft) - 1]}^{(4)}\Bigg],\end{aligned}$$

where Y( ) is the step function,

$m^2 = (1 + 4\pi^2 k^2 r^2)$,

$r = c/f$.

The quantity r is implicitly indexed (r_1 is the barotropic and r_2 the baroclinic radius of deformation).

The first term (n° 1) of (3.4) represents a motion similar to a coastal jet. It is generated by the discontinuity of the wind stress at $x=a$ and by its image with respect to the coast at $x=-a$. At the coast, this term is equal to zero.

The second (n° 2) and third (n° 3) terms represent Poincaré waves generated at $x=0$, $x=a$, $x=-a$, in order to satisfy the boundary conditions at the coast and at $x=a$.

The fourth (n° 4) term represents two Kelvin fronts; the first propagates along the coast, the second along the virtual line of equation $x=-a$.

By substituting (3.3) into the FLT of (2.4) and applying Sutton's algorithm, one obtains the asymptotic expansion of velocity components:

$$\begin{aligned}\underset{t\to\infty}{\mathrm{U}(t,x,k)} = \frac{\mathrm{T}(k)}{2\rho\mathrm{H}f} & \times\Bigg[ft\frac{i2\pi kr}{m}[\exp(-m|x+a|/r) - \exp(-m|x-a|/r)] \\ & + \frac{1}{m^2}[\exp(-m(x+a)/r) - 2\exp(-mx/r) \\ & - \mathrm{sgn}\,(a-x)\exp(-m|x-a|/r) + 2\mathrm{Y}(a-x)]\Bigg], \qquad (3.5)\end{aligned}$$

$$\begin{aligned}\underset{t\to\infty}{\mathrm{V}(t,x,k)} = \frac{\mathrm{T}(k)}{2\rho\mathrm{H}f}\Bigg[& ft[\exp(-m(x+a)/r) \\ & + \mathrm{sgn}\,(a-x)\exp(-m|x-a|/r)] \\ & + \frac{1}{i\pi kr}\Bigg[\frac{\exp(-m(x+a)/r) - \exp(-mx/r)}{m} \\ & - \exp(-(x+a)/r) + \exp(-x/r)\Bigg] \\ & - \frac{1}{i\pi kr}[\exp(-(x+a)/r) \\ & - \exp(-x/r)]\,[\exp(i2\pi krft) - 1]\Bigg]. \qquad (3.6)\end{aligned}$$

In these expressions the terms oscillating with the Coriolis period have been filtered out.

EXAMPLES

The solutions presented in the preceeding section are general and are valid for any function $T(k)$, the Fourier transform of the windstress. We now consider the response of a shallow-water model to different wind patterns.

Wind with no shear in presence of a coast

Consider the windstress shown in Figure 1, but assume that $a=\infty$. In this case, the problem is reduced to that solved by Allen (1976), Philander and Yoon (1982) and Crépon and Richez (1982). In the forced region ($|y|<b$) the flow evolves in two stages. Initially the wind accelerates a coastal jet and induces upwelling that constantly raises the thermocline. The passage of a southward travelling Kelvin wave excited at $y=+b$ establishes equilibrium conditions by introducing an alongshore pressure gradient to balance the windstress. In the wake of this wave, the acceleration of the jet and the upwelling cease. The elevation is given by the fourth term of (3.4):

$$\zeta=-\frac{\tau_0}{\rho cf}\exp(-x/r)\frac{(b-y)}{r} \qquad (|y|<b). \qquad (4.1)$$

In the unforced region ($y<-b$), accelerating motion is introduced by a Kelvin wave excited at $y=-b$ and is subsequently modified by a Kelvin wave excited at $y=+b$ so that the flow ultimately is steady:

$$\zeta=-\frac{\tau_0}{\rho cf}\exp(-x/r)\frac{2b}{r} \qquad (y \leftarrow b). \qquad (4.2)$$

Suppose that the wind were to stop blowing after a time t_0. In the forced region $|y|<b$, the pressure force is in the opposite direction to that of the wind and the jet, and causes the jet to decelerate until the passage of a Kelvin wave excited at time t_0 at $y=b$ stops the deceleration when the ocean is in a state of rest with the thermocline in its original position.

Wind with shear in the absence of coasts

Consider the wind blowing over an unbounded ocean with no coast:

$\tau=\tau_0 Y(x)$ for $|y|<b$,

$\tau=0$ for $|y|>b$.

This case was solved by Crepon *et al.* (1984). The response is shown schematically in Figures 3 and 4. In the forced region and far from the two West-East edges, the interface increases linearly with time and does not depend on y.

$$\zeta=\frac{\tau_0}{2\rho cf}(ft)\exp(-x/r). \qquad (4.3)$$

In geostrophic balance, there is an accelerating jet in the direction of the wind in the region $x>0$ and an accelerating jet in the opposite direction in the region $x<0$:

$$v=\mathrm{sgn}(x)\, ft\,(\tau_0/2\rho H f)\exp(-x/r). \qquad (4.4)$$

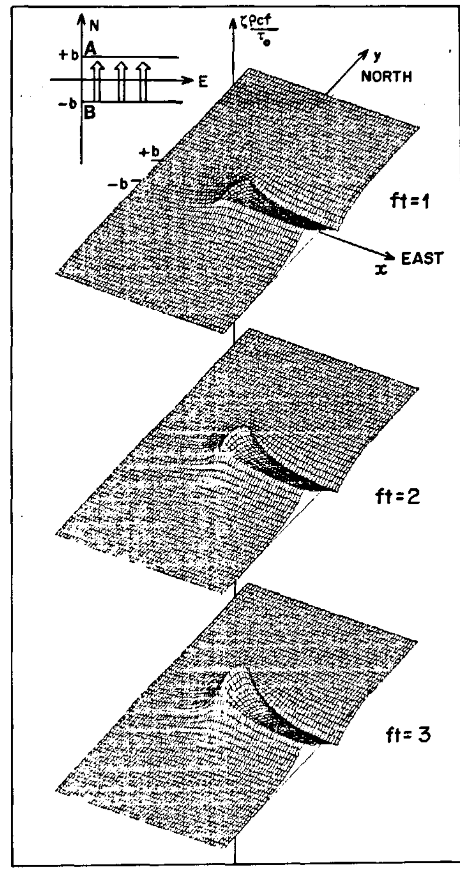

Figure 3

Wind with shear blowing over a strip on an unbounded ocean. Interface elevation at three different times ($b=2r$, $a=r$).

Vent avec cisaillement soufflant sur une bande au-dessus d'un océan infini. Élévation de l'interface à trois instants différents.

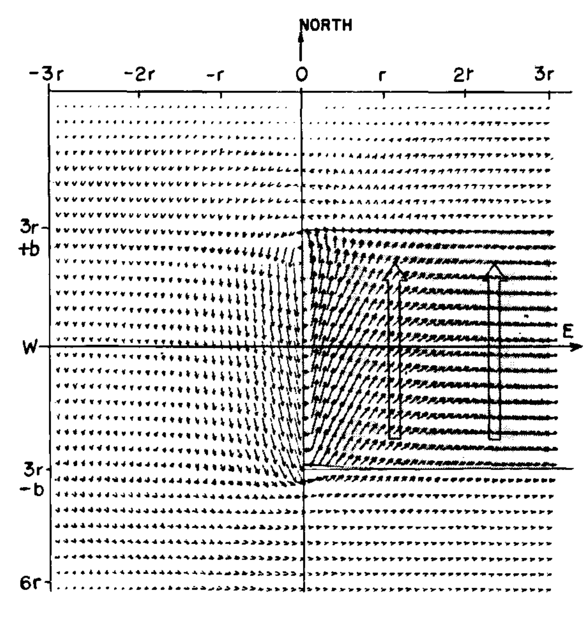

Figure 4

Modal currents corresponding to the wind pattern in Figure 3, at $ft=5$, but with $b=3r$, $a=r$. Note that the scales are different on Ox and Oy.

Champ de courant relatif à un mode normal et correspondant au schéma de vent de la figure 3, à l'instant $ft=5$, mais avec $b=3r$ et $a=r$ (les échelles sont différentes sur Ox et Oy).

The mass continuity of this double jet is ensured by a loop of a width of the order of r centered on each of the two wind corners A and B (Fig. 3). At each extremity the velocity u is of the form $K_0[(x'^2+y'^2)^{1/2}]\,ft$ where K_0 is the modified Bessel function of order zero and x' and y' represent the coordinates in a frame centered on the wind corner. This unbounded ocean, unlike the ocean in example (a) above, supports no Kelvin wave which can establish pressure gradients and arrest the acceleration. The flow therefore accelerates as long as the wind blows. Should the wind stop blowing at time t_0, then the acceleration stops and the jets persit indefinitely in this inviscid ocean. This is to be contrasted with example (a) above, where the coastal zone has no long-term memory: the coastal jet ultimately disappears when the wind stops blowing.

Wind blowing over a patch adjacent to the coast

The wind is confined to the region $0<x<a$, $|y|<b$, shown in Figure 1. The Fourier transform of the wind-stress modulus is

$$T(k)=(\tau_0/\pi k)\sin(2\pi kb).$$

At the coast ($x=0$), the first term in equation (3.4) is zero and the most important term is the fourth one:

$$\zeta_{(4)}=-\frac{\tau_0}{\rho cf}[\exp(-x/r)-\exp(-(x+a)/r)]\left[\frac{b-y}{r}\right]$$

$$\text{for}\quad |y|<b,\quad t>\frac{2b}{c}. \qquad (4.5)$$

This describes the state equilibrium condition near the coast in the wake of the Kelvin wave excited at $y=b$. The Kelvin wave has a negligible amplitude far offshore ($x>r$). In that region, specifically near the region of wind shear at $x=a$, the first term in (3.4) is dominant:

$$\begin{aligned}\zeta_{(1)}=\frac{\tau_0}{\rho cf}ft\frac{1}{2r}\int_y^{\infty} & K_0[((x-a)^2+(y-b)^2)^{1/2}r]\\ & -K_0[((x-a)^2+(y+b)^2)^{1/2}r]\\ & -K_0[((x+a)^2+(y-b)^2)^{1/2}r]\\ & +K_0[((x+a)^2+(y+b)^2)^{1/2}]]\,d(y/r). \qquad (4.6)\end{aligned}$$

According to this expression, the flow accelerates constantly near $x=a$. (At $x=0$, $\zeta(1)=0$). Figures 5 and 6 show the evolution of the elevation and currents in response to the wind. Near the coast there is upwelling until the passage of a Kelvin wave from $y=b$, whereafter steady conditions obtain. Offshore, near $x=a$, there is downwelling that continues indefinitely, and in the region $x>a$, a jet in a direction opposite to that of the wind, which accelerates constantly. The circulation is closed in the region $|y|>b$ so that a gyre forms. The velocity increases with time. In a two-layer ocean, the gyre extends in the lower layer; near the coast, the current in the upper layer is in the wind direction while in the lower layer, under the Kelvin wave front, in the region $y<-b$, it is in the opposite direction.

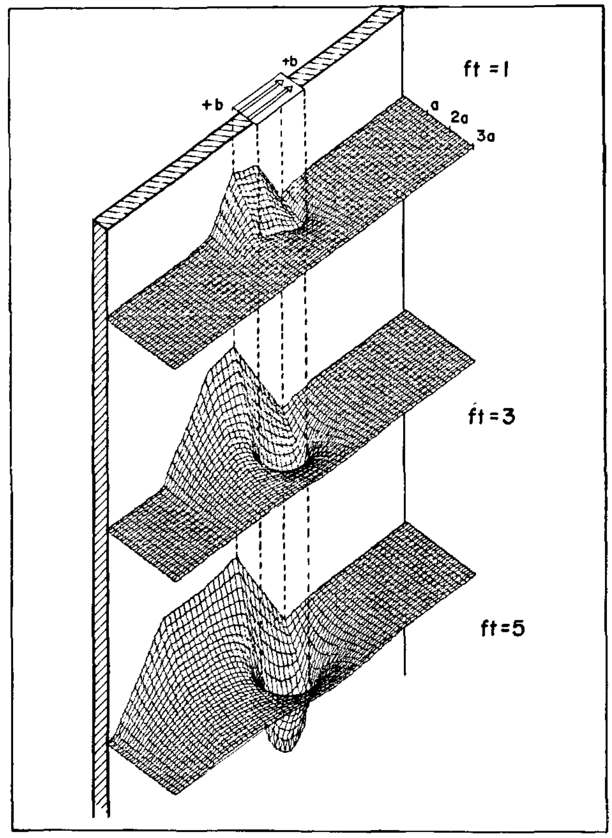

Figure 5

Wind blowing over a patch adjacent to the coast. Interface elevation at three different times ($b=2r$, $a=r$).

Vent soufflant sur une aire rectangulaire au voisinage de la côte. Élevation de l'interface à trois instants différents.

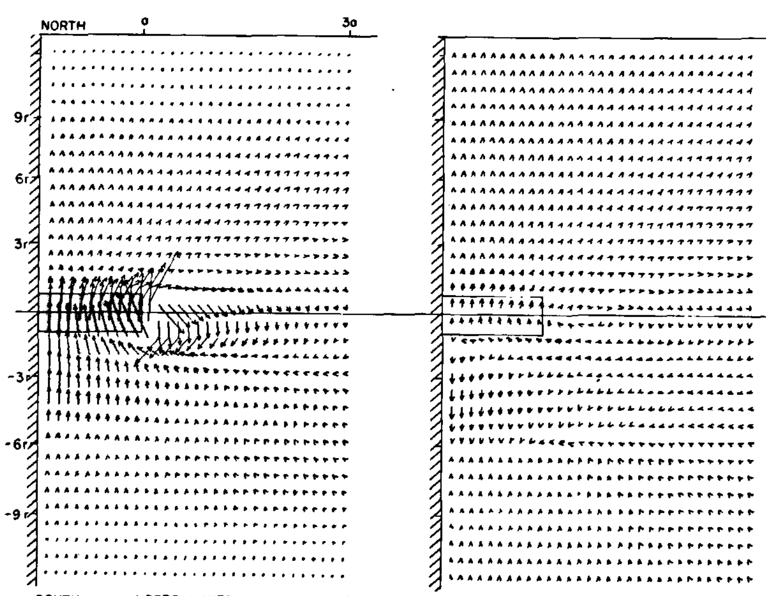

Figure 6

Wind blowing over a patch adjacent to the coast. Currents in the upper and lower layers at $ft=5$ ($b=r_2$, $a=r_2$). A different scale is used on Ox and Oy.

Vent soufflant sur une aire rectangulaire au voisinage de la côte. Champ de courant dans la couche supérieure et dans la couche inférieure à l'instant $ft=5$ (les échelles sont différentes sur Ox et Oy.)

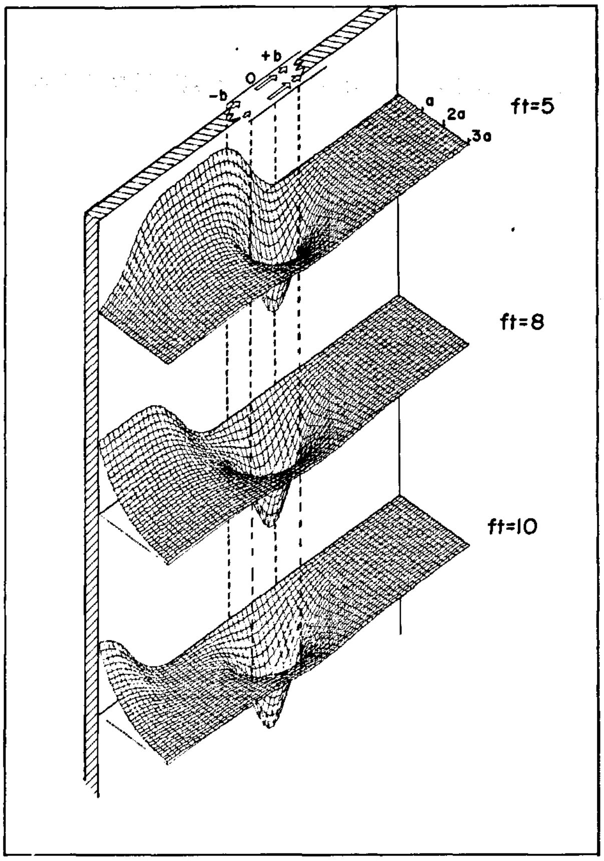

Figure 7

Wind blowing over a patch adjacent to the coast. The modulus of the wind is a gaussian function with respect to y. Evolution of the interface elevation after the wind stops blowing at $ft=5$.

Vent soufflant sur une aire rectangulaire proche de la côte. Le module du vent est une fonction gaussienne de y. Evolution de l'élévation de l'interface après que le vent ait cessé de souffler à l'instant $ft=5$.

Should the wind stop blowing at time $ft=T$, then the coastal zone returns to its original state, but the offshore downwelling at $x=a$, and the currents there, persist. Figure 7 shows the evolution of the interface after $ft=5$ (when the wind stops blowing) for a gaussian wind with respect to y.

THE β DISPERSION EFFECT

By using a similar procedure to that of Gill and Anderson (1975), it is found that the β plane approximation to (2.3) is

$$\frac{\partial}{\partial t}(r^2\nabla^2\zeta-\zeta)+\beta r^2\frac{\partial\zeta}{\partial x}=\frac{1}{\rho f}\operatorname{rot}\tau. \qquad (5.1)$$

In order to obtain a first physical insight of the dispersion introduced by the β effect in the previous problem, attention is only focused on the x-dependence of parameters. According to the results of section 4, this is a reasonable approximation to study variation of parameters along the x axis. In the subsequent, it is assumed that the parameters do not depend on y.

A solution of (5.1) may be then obtained by using a Fourier transformation in x. Equation (5.1) yields:

$$\frac{\partial}{\partial t}(4\pi^2k^2r^2+1)Z-2\pi ik\beta r^2 Z=-\frac{2\pi ik}{\rho f}T, \qquad (5.2)$$

where Z and T are the Fourier transforms of ζ and τ^y with respect to x.

First, let us investigate the response of an unbounded ocean to a windstress shear of the form

$$\tau^y(x)=\tau_0\,Y(x)\,Y(t). \qquad (5.3)$$

This yields

$$T(k)=\frac{\tau_0}{2}\left[\frac{1}{i\pi k}+\delta(k)\right]Y(t). \qquad (5.4)$$

Equation (5.2) becomes

$$\frac{\partial}{\partial t}((4\pi^2k^2r^2+1)Z)-2\pi ik\beta r^2 Z=-\frac{1}{\rho f}\tau_0\,Y(t). \qquad (5.5)$$

Assuming that the fluid starts from rest at $t=0$ ($\zeta=0$ at $t=0$), a solution of (5.5) is

$$Z=\frac{\tau_0}{2\pi ik\beta r^2\rho f}\times\left[1-e^{2\pi ik\beta r^2 t/(4\pi^2k^2r^2+1)}\right]Y(t). \qquad (5.6)$$

Applying the Fourier inversion theorem yields

$$\zeta=\frac{\tau_0}{2\rho f\beta r^2}\left[\operatorname{sgn}(x)-\int_{-\infty}^{+\infty}\frac{1}{i\pi k}\times e^{(2\pi ik\beta r^2 t/(4\pi^2k^2r^2+1)+2i\pi kx)}\,dk\right]Y(t). \qquad (5.7)$$

The integral in (5.7) is rewritten in the form

$$\int_{-\infty}^{+\infty}\frac{1}{i\pi k}\left[e^{[2\pi ik\beta r^2 t/(4\pi^2k^2r^2+1)+2i\pi kx]}-e^{[i2\pi kx+i2\pi k\beta r^2 t]}\right]dk+\int_{0}^{+\infty}\frac{1}{i\pi k}\exp(i2\pi kx+i2\pi k\beta r^2 t)\,dk. \qquad (5.8)$$

Hence (5.7) becomes

$$\zeta=\frac{\tau_0}{2\rho f\beta r^2}Y(t)\Bigg[[\operatorname{sgn}(x)-\operatorname{sgn}(x+\beta r^2 t)]-\int_{-\infty}^{+\infty}\frac{1}{i\pi k}\exp\left[i\left(2\pi k\xi+\frac{2\pi k\beta r^2}{4\pi^2k^2r^2+1}\right)t\right]\left[1-\exp\left[i2\pi k\beta r^2 t\left(1-\frac{1}{4\pi^2k^2r^2+1}\right)\right]\right]\Bigg]dk, \qquad (5.9)$$

where $\xi=x/t$.

It proves useful to let $\xi=x/t$ very small (Cane, Sarachik, 1975). This arises when considering long-term behavior at a fixed point in space.

The integral in (5.9) being definite for all k, its large t response can be calculated by applying the method of stationary phase. One obtains

$$\left[\frac{1}{\pi r \beta t}\right]^{1/2} 8\cos(x/r + 3/4\,\beta rt - \pi/4) \times \sin(\beta rt/4). \qquad (5.10)$$

Hence, at long time, a first estimate of (5.9) is

$$\zeta = \frac{\tau_0}{2\rho cf}\,\frac{2f^2}{\beta c}\,[\mathrm{Y}(x) - \mathrm{Y}(x + \beta r^2 t)]. \qquad (5.11)$$

Equation (5.11) can be interpreted in the following manner. At small time, the motion is controlled by a f plane dynamics. The elevation is given by (4.3). It increases linearily in time and is confined in a region of length scale of order r around the wind discontinuity. As the time evolves, β dispersion becomes important. The elevation becomes constant and spreads out from the origin to West at the velocity $c = -\beta r^2$ which is the velocity of long Rossby waves (Fig. 8). This steady value corresponds to the value reached by (4.3) after a time equal to $t = 2f/\beta c$. It is the adjustment time found by Anderson and Gill (1975) for the Rossby waves dispersion becoming effective in an Eastern coastal jet. By 30° latitude, it is equal to 60 days if $c = 2\ \mathrm{ms}^{-1}$.

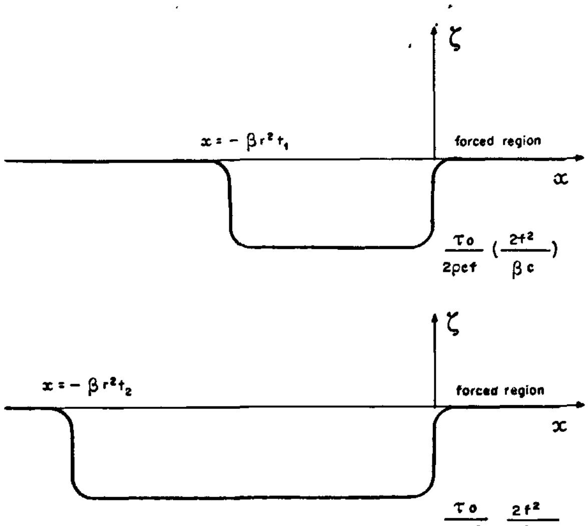

Figure 8

β-plane solution in an unbounded ocean: interface elevation at two different times, the wind blowing continuously ($\tau = y\,\tau_0\,\mathrm{Y}(x)\,\mathrm{Y}(t)$).

Solution dans le plan β dans le cas d'un océan infini : élévation de l'interface à deux instants différents, le vent soufflant d'une manière continue.

Equation (5.11) is a rough estimate of the elevation. A more accurate one is given in Cane and Sarachik (1976). In fact, the propagating front is not abrupt, but is a continuous curve, the slope of which steepens as $t^{2/3}$ and its width narrows as $t^{-2/3}$. The wake consists of the steady response plus wiggles which decay as $t^{-1/2}$. At $x = 0$, the steady front located at the wind discontinuity matches an elevation of the form $\mathrm{J}_0(2\sqrt{\beta xt})$ which represents the contribution of short Rossby waves which propagate eastwards.

Integration of (4.3) and (5.11) with respect to x at a given time yields the same value. Hence the volume of water displaced in both cases (f-plane and β-plane) are equal.

If the wind stops, after having blown during a time t_0, the f-plane solution becomes steady. The β-plane solution is obtained by taking the difference of (5.11) with its translated of t_0 in time. It is found that the steady front occurring at the origin in (5.11) moves to West at the celerity c (Fig. 9). The elevation forms a bump propagating to West the width of which is equal to $\beta r^2 t_0$. At a fixed point and after the passage of the bump, the elevation relaxes to zero.

In conclusion, it can be said that a transient upwelling or downwelling which is at small time confined in a region of order of the radius of deformation and increases with respect to time, becomes steady at time $t_1 = 2f/\beta c$ and spreads out to West extending over a large distance. If the upwelling (downwelling) gets steady at time $t \ll t_1$ (for instance because the wind stops), the interface elevation at a fixed point becomes equal to zero at large time (Fig. 9).

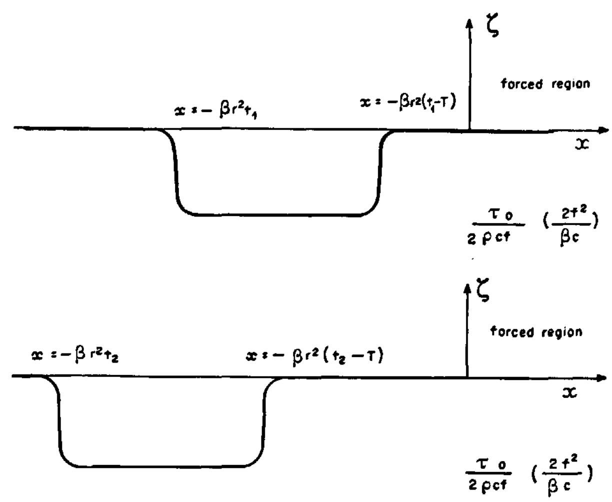

Figure 9

β-plane solution in an unbounded ocean: interface elevation at two different times, the wind blowing during a time t_0 ($\tau = y\,\tau_0\,\mathrm{Y}(x)\,\mathrm{Y}(t_0 - t)$).

Solution dans le plan β dans le cas d'un océan infini : élévation de l'interface à deux instants différents, le vent soufflant pendant un laps de temps t_0.

DISCUSSION

According to the results derived here, the atmospheric jet along the coast of Peru will drive an equatorward coastal surface jet and will cause upwelling at the coast; offshore, in the shear zone where the amplitude of the winds attenuate, there should be downwelling and a southward flowing current. This southward flowing water, which could advect warm water from the neighborhood of the equator, is likely to attain a great intensity. It is conceivable that this offshore current

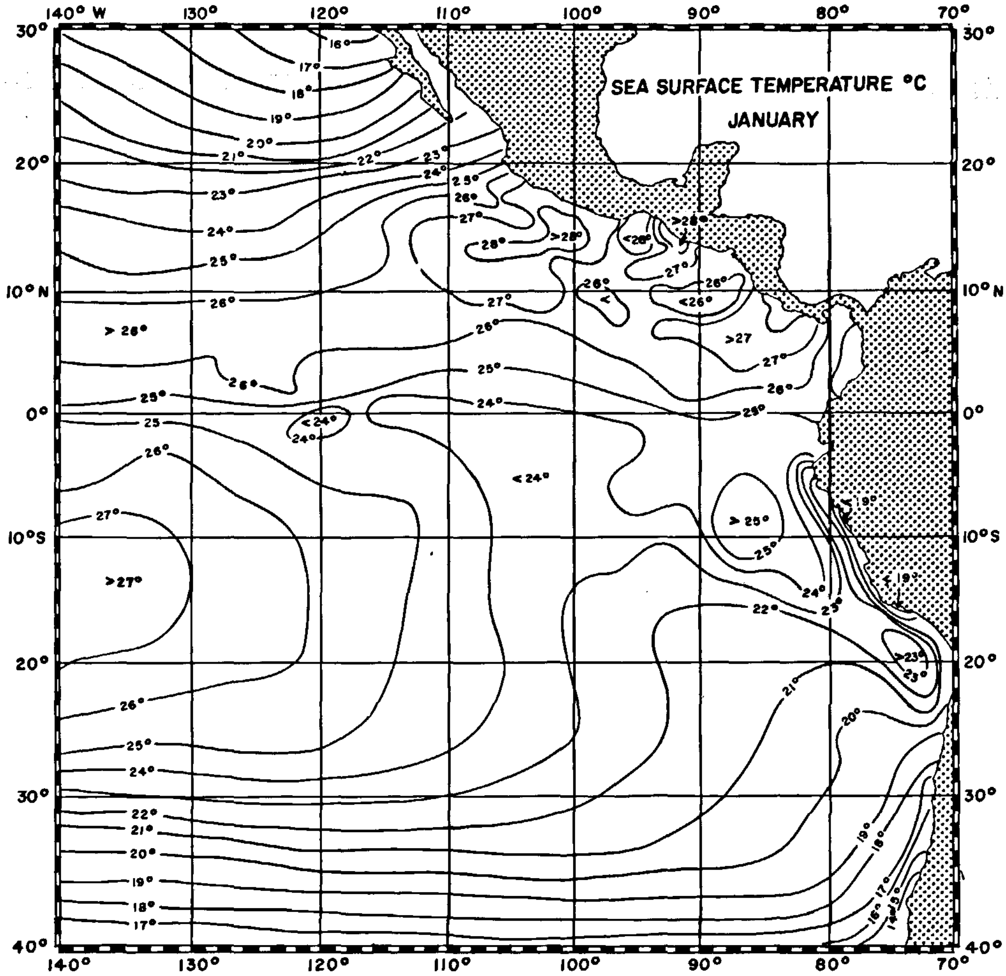

Figure 10
Sea surface temperature in Pacific Ocean in January (from Wyrtki, 1964).
Température de surface de l'Océan Pacifique en janvier (d'après Wyrtki, 1964).

contributes to the tongue of warm water that appears off the coast of Peru in February when the wind shear is a maximum (Fig. 10). The β effect enhances the downwelling due to the wind-stress curl. Because no motion is propagated eastwards, the results of section 5 can also apply in a presence of an Eastern Ocean boundary. The analytical model shows that, as long as the wind blows, the downwelling increases with time for small time, then spreads out to the West, keeping a constant value. For a wind variation of $4\,\mathrm{ms}^{-1}$, the interface drops by 60 m at 20° of latitude. On the contrary, the coastal upwelling becomes steady because arrested by coastal Kelvin waves, after a time $t=2b/c_2 \# $ 10 days. The upwelling is then propagated away as a small bump. At long time, only the downwelling remains, extending from the wind discontinuity to West over a distance $\beta r^2 t$. In fact, the β dispersion becomes effective at relatively short time scales, as it can be seen on numerical model (Philander, Yoon, 1982). Further calculations are necessary to determine the importance of this mechanism because the model used here neglects a number of important processes. For example, the acceleration of the offshore current will rapidly cause nonlinearities to become important. Finally, there is a lack of information about the shear of the wind. The intensity of the offshore current depends on the magnitude of the windshear. All that is known, however, is that the atmospheric jet extends approximately 100 kms offshore. More accurate wind data are needed.

Acknowledgements

This work was supported by the French Centre National de la Recherche Scientifique (LA 175, PNEDC) and the Centre National pour l'Exploitation des Océans (CNEXO).

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