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Wind-driven flow in a shallow estuary

Estuary
Currents
Wind
Dynamics
Potomac

Estuaire
Courant
Vent
Dynamique
Potomac

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ABSTRACT

The effect of wind forcing on the circulation in the Potomac estuary has been investigated using lowpass records of near-surface and near-bottom current, wind stress and sea level. The response was analysed by integrating the linear equation of motion through upper and lower layers and then estimating each term in the resulting equations. In each layer the acceleration term, $h\,\partial u/\partial t$, was an order of magnitude smaller than the dominant terms. In the upper layer the surface wind stress was balanced by a combination of the effects of side friction and surface slope; in the lower layer the bottom stress was balanced directly by the pressure gradient due to the surface slope. The acceleration term, $h\,\partial u/\partial t$, was small because the time scale of the wind forcing was long and because the water was shallow.

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RÉSUMÉ

Circulation induite par le vent
dans un estuaire de faible profondeur

Les effets de la tension du vent sur l'écoulement des eaux dans l'estuaire du Potomac ont été étudiés à partir du signal basse fréquence des enregistrements du vent, du niveau de la mer et des courants près de la surface et du fond. La réponse a été analysée par intégration de l'équation linéarisée du mouvement d'un système à deux couches en calculant chaque terme dans les équations résultantes. Dans chaque couche, le terme d'accélération $h\,\partial u/\partial t$ est d'un ordre inférieur aux termes dominants. Dans la couche supérieure, la tension du vent en surface est équilibrée par une combinaison des effets du frottement latéral et de la pente de la surface; dans la couche profonde, le frottement sur le fond est compensé directement par le gradient de pression dû à la pente de la surface. Le terme d'accélération $h\,\partial u/\partial t$ est faible à cause des grandes échelles de temps de l'action du vent et de la faible profondeur de l'eau dans l'estuaire.

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INTRODUCTION

During a one-year period from July, 1974, until July, 1975, a current meter mooring was maintained in the Potomac River, a tributary estuary of the Chesapeake Bay (Fig. 1). The mooring was located at approximately 30 km upstream from the mouth of the estuary, where the water depth was around 15.25 m (50 ft). Three current meters were attached to the mooring at depths of 3.05 m (10 ft), 7.60 m (25 ft) and 12.20 m (40 ft), and wind and sea level data were obtained from established recording stations. For an initial analysis the data were lowpass filtered with a rectangular filter that spanned 25 hours of data and were then averaged in 24 hour blocks to obtain mean values centred on 1 200 hours for each day of the year-long experiment (Elliott, 1978). These daily averages were used to investigate the coupling between the currents and the meteorological forcing and two separate forcing mechanisms were isolated: local forcing, which could account for about 55 % of the variance in the records, and non-local forcing caused by interaction with the Chesapeake Bay. The purpose of this note is to look in more detail at the local forcing, the non-local response

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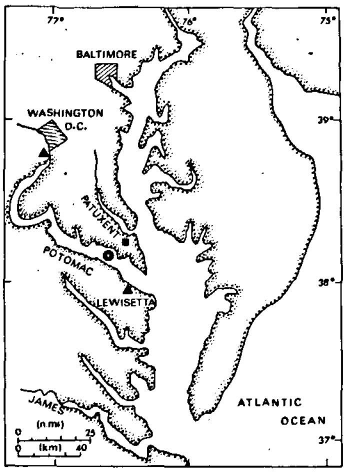

Figure 1
The Chesapeake Bay and Potomac estuary showing the positions of the current meter mooring, the sea level stations and the wind tower.

has been discussed by Wang and Elliott (1978), Elliott *et al.* (1978) and Wang (1979).

THE DATA

Continuous current records were available for the near-surface flow (U10) and the near-bottom flow (U40) between April 13 and June 30, 1975. During these 75 days continuous wind data were available from the Patuxent Naval Station, and sea level data were available for Washington, DC, and Lewisetta (Fig. 1). In addition, the current meter at mid-depth had provided data during the initial 30 days from April 13 to May 12. The data series were filtered to remove the tidal and other high frequency signals and then resampled at 6 hour intervals; the resulting time series had zero amplitude at 1 cycle/day (cpd), half amplitude at 0.7 cpd and 95% amplitude at 0.5 cpd. The wind data were converted to wind stress by a quadratic law with a drag coefficient of 2.5×10^{-3}. The lowpass time series of wind stress, current and sea level are shown in Figure 2; the current at mid-depth was similar to the near-bottom current (the coherence squared exceeded 0.90 at the 5-day time scale) and therefore is not shown.

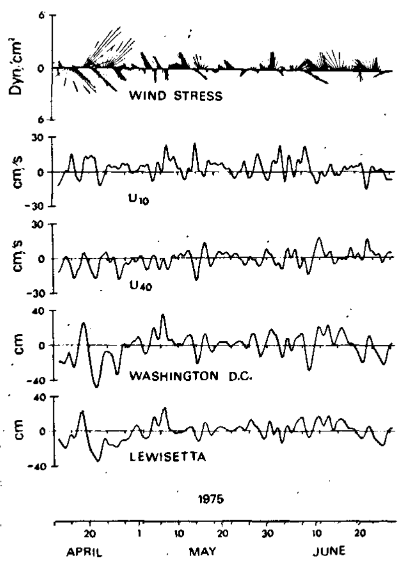

Figure 2
Time series of wind stress, current and elevation. The wind vectors are shown in coordinates that have north directed up the page; the currents are downstream components of the flow in the Potomac (positive to the SE).

RESULTS

The dynamic balance

Examination of U10 and U40 in Figure 2 suggests that the flow had a strong two-layered character during the early summer, and consequently a two-layered analysis should provide a good approximation to the flow structure. The linear two-dimensional equation of motion is:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\left(K\frac{\partial u}{\partial z}\right),$$

where x is the downstream coordinate and z is the vertical coordinate. If this is combined with the hydrostatic equation and then integrated vertically through upper and lower layers of thickness h_1 and h_2, respectively, it gives:

$$h_1\frac{\partial u_1}{\partial t} = \frac{\tau_x}{\rho} - gh_1\frac{\partial \eta}{\partial x} - a(u_1-u_2) - b\gamma u_1, \quad (1)$$

and:

$$h_2\frac{\partial u_2}{\partial t} = -gh_2\frac{\partial \eta}{\partial x} - a(u_2-u_1) - bu_2, \quad (2)$$

where u_1 and u_2 are the mean horizontal velocities within the layers, a and b are linear coefficients for interfacial and bottom drag, τ_x is the downstream wind stress and η is the surface elevation (note that the horizontal pressure gradient due to the horizontal variations in salinity has been omitted. Consequently, the analysis will only be concerned with the wind-driven flow and will not consider the internal density-driven circulation). The bottom drag term in Equation (1) has been multiplied by a factor γ $(0 \leqq \gamma \leqq 1)$ to take account of friction acting directly on the surface layer due to the shallow water near the banks. A linear drag law is appropriate since the tidal motion has been removed by filtering (Hunter, 1975; Heaps, 1978). The choice of values for a, b and γ will be discussed in the following section, the most appropriate values being 0.01, 0.20 and 0.40, respectively in cgs units. Figure 3 shows the calculated time series of the individual terms in Equations (1) and (2) and the corresponding rms values are summarised in Table 1. The acceleration term $h\,\partial u/\partial t$ was negligible in both layers, being of the same order as the interfacial stress term. In the bottom layer the dominant balance was between the bottom stress and the pressure gradient due to the tilt of the free surface. The balance between these two terms was most

Table 1
Rms values of the terms in the equation of motion (cm^2/sec^2).

Upper layer				
$h_1(\partial u_1/\partial t)$	τ_x/ρ	$-gh_1(\partial\eta/\partial x)$	$-b\gamma u_1$	$-a(u_1-u_2)$
0.07	0.89	0.66	0.59	0.12
Lower layer				
$h_2(\partial u_2/\partial t)$		$-gh_2(\partial\eta/\partial x)$	$-bu_2$	$-a(u_2-u_1)$
0.12		1.32	1.34	0.12

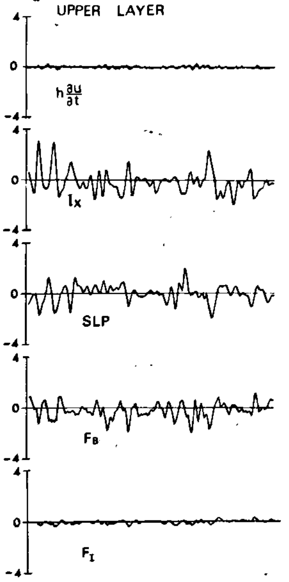

Figure 3*a*

Time series of the terms in the equation of motion of the upper layer. The layer thickness was 5 m, FB and FI represent the bottom and interfacial stresses.

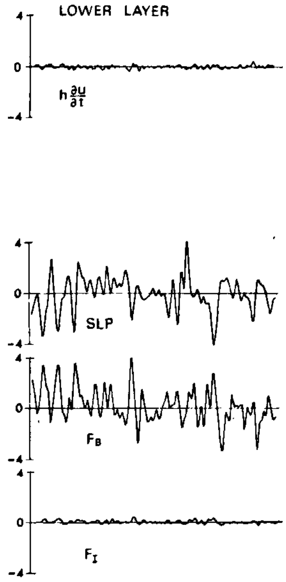

Figure 3*b*

Time series of the terms in the equation of motion of the lower layer (layer thickness was 10 m).

pronounced during the first month of the measurements when there was a series of very regular bottom current fluctuations (Fig. 2 and 3*b*). In the surface layer the direct action of the wind stress was balanced by a combination of side friction and surface slope.

In hindsight it is not suprising that the acceleration term should be so small. Typically, the current speed changed by about 20 cm/sec. over an interval of 2-3 days and with a layer thickness of 10 m this gives a value for $h\,\partial u/\partial t$ of about 0.1. In contrast, the wind stress had a typical value of about 1.0 (Table 1), and this value would be exceeded by the bottom stress term for velocities in excess of 5-10 cm/sec. Consequently the acceleration term was small because the water was shallow and the important time scale was long. The acceleration would be of the same order as the other terms for velocity changes that occur on a time scale comparable to that of the semi-diurnal tide, but for the longer period wind-driven events the acceleration of the fluid can be neglected.

Predicting the two-layered flow

If the time derivatives are neglected then Equations (1) and (2) can be solved for the horizontal velocities giving:

$$u_2 = \frac{agh_1(\partial\eta/\partial x) - a\tau_x/\rho + (a+b\gamma)gh_2(\partial\eta/\partial x)}{a^2-(a+b)(a+b\gamma)}, \qquad (3)$$

and:

$$u_1 = \frac{\tau_x/\rho - gh_1(\partial\eta/\partial x) + au_2}{(a+b\gamma)}. \qquad (4)$$

If $\gamma=0$ then bottom (side) friction is neglected in the upper layer which then feels friction only through the interface with the lower layer. The coefficient of interfacial drag, a, determines the shear character of the flow. For small a the two layers are essentially uncoupled and the flow shows a pronounced shear, while for large a the two layers become locked together and the response is barotropic. When the side friction was neglected from the upper layer (i.e. γ was set to zero) then the only way to obtain realistic upper layer flow was to use a high value for the interfacial friction, but this resulted in a flow that lacked the observed shear. Consequently, side friction must be included in the dynamic balance for the surface layer. The best values were found by first adjusting the interfacial drag coefficient, a, until the flow showed realistic shear. Then the bottom drag coefficient, b, and the surface layer drag factor, γ, were adjusted to give the best fit to the amplitude of the current fluctuations. The best values for a, b and γ were 0.01, 0.20 and 0.40, respectively (cgs). These drag coefficients are in general agreement with other observed values. Pollard and Millard (1970) used a value in the range 0.005-0.010 to represent the interfacial drag acting across the bottom of a surface mixed layer, while Winant and Beardsley (1979) compared several sets of shallow water data and estimated the linear bottom drag coefficient to lie in the range of 0.030-0.200. Theoretical estimates of the linear drag coefficient for low frequency motions (Hunter, 1975; Heaps, 1978) suggest a value of around 0.100. Since the Potomac currents were measured in the interior of the layers the adjusted drag coefficients also include factors that convert the observed velocities into mean layer values, and consequently they agree reasonably well with other published values.

The observed and calculated currents are shown in Figure 4. At times there was an apparent shift in origin between the observed and calculated flow; examples can be found in the lower layer flow during May 1-10 and in the upper layer flow between June 10-20. It is likely that these shifts are due to variations in the density-driven component of the flow. The rms values of the fluctuations in the flow were adequately reproduced as shown by Table 2, and therefore the calculated currents may be suitable for predicting flushing and dispersion within the estuary, processes that are likely to be strongly affected by wind forcing. There was not always good agreement between the peaks in the observed and calculated flow, this was probably due to the flow being more complicated than the assumed two-layered structure. The poor

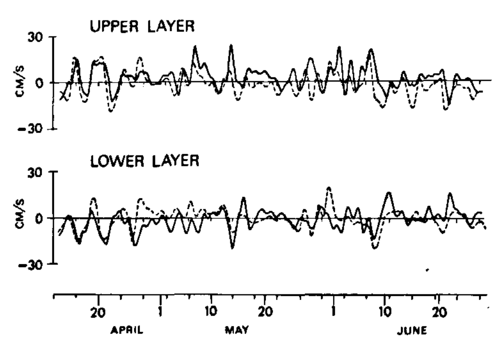

Figure 4

Comparison between the observed currents (solid curve) and the currents calculated by neglecting the acceleration terms.

prediction of the flow during May 28-31 is thought to be due to an error in the sea level data for that period.

Table 2

Comparison between the observed and calculated rms velocities (cm/sec.).

	Upper layer	Lower layer
Observed	7.4	6.7
Calculated	7.8	6.4

Acknowledgements

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REFERENCES

Elliott A. J., 1978. Observations of the meteorologically induced circulation in the Potomac estuary, *Estuarine Coastal Mar. Sci.*, **6**, 285-299.

Elliott A. J., Wang D.-P., Pritchard D. W., 1978. The circulation near the head of Chesapeake Bay, *J. Mar. Res.*, **36**, 643-655.

Heaps N. S., 1978. Linearised vertically-integrated equations for residual circulation in coastal seas, *Dtsch. Hydrogr. Z.*, **31**, 147-169.

Hunter J. R., 1975. A note on quadratic friction in the presence of tides, *Estuarine Coastal Mar. Sci.*, **3**, 473-475.

Pollard R. T., Millard R. C., 1970. Comparison between observed and simulated wind-generated inertial oscillations, *Deep-Sea Res.*, **17**, 813-821.

Wang D.-P., 1979. Wind-driven circulation in the Chesapeake Bay, winter 1975, *J. Phys. Oceanogr.*, **9**, 564-572.

Wang D.-P., Elliott A. J., 1978. Non-tidal variability in the Chesapeake Bay and Potomac River: evidence for non-local forcing, *J. Phys. Oceanogr.*, **8**, 225-232.

Winant C. D., Beardsley R. C., 1979. A comparison of some shallow wind-driven currents, *J. Phys. Oceanogr.*, **9**, 218-220.