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A parametrization of the geometry of sea straits

Sea straits
Froude conditions
Paleoclimate
Evaporation
Water balance

Détroits marins
Conditions de Froude
Paléoclimat
Évaporation
Bilan d'eau

D. A. Anati
The Weizmann Institute of Science, Rehovot, Israël.

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ABSTRACT

A parametrization of the bottom topography of sea straits is proposed, which leads to a simple relation between several strait parameters and other properties pertaining to the whole sea. Two different examples of the usefulness of this relation are given: (1) the net water deficit, i. e. evaporation minus precipitation and runoff, from the whole sea is estimated from strait parameters alone; and (2) the salinity difference at the strait of Tiran in the Pleistocene is estimated to have been five times greater than at present.

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RÉSUMÉ

Paramétrisation de la géométrie des détroits marins.

Nous proposons une paramétrisation de la topographie du fond des détroits marins, à partir de laquelle peut être déduite une relation simple liant des paramètres propres d'une part au détroit et d'autre part à l'hydrologie. Nous présentons deux exemples différents de l'application d'une telle relation : (1) le déficit en eau, c'est-à-dire la différence entre l'évaporation et les apports en eau douce, pour l'ensemble du bassin, est estimé à partir des paramètres du seul détroit; et (2) on a pu également avancer que la différence de salinité entre les masses d'eau inférieure et supérieure du détroit de Tiran était, à l'époque du Pléistocène, cinq fois plus grande que de nos jours.

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INTRODUCTION

Sea straits often control the balance of water, salt, enthalpy, and other parameters in semi-closed seas, hence the importance of understanding their control mechanism and estimating quantitatively "how much" a strait impedes the flows of water through it. When an actual sea strait is examined, however, it is soon realized that the three-dimensional bottom topography is in most cases so complex that it cannot possibly be taken into account in all its details. One way to circumvent this inconvenience is to approximate the actual bottom relief to some simpler one, for instance to a straight channel of rectangular, parabolic, or triangular cross-section, and with constant depth.

This method has been used extensively in the past, but its simplicity comes, of course, at the expense of similarity. The present work is an attempt to parametrize a strait's bottom topography. It applies to sea straits with a two-layer regime (Anati *et al.*, 1977, and Stommel, Farmer, 1953) with critical interfacial Froude cônditions

$$U_1^2/g'D_1+U_2^2/g'D_2=1, \qquad (1)$$

or, equivalently

$$Q_1^2/(N^2 n)+Q_2^2/(1-N)^2(1-n)=g'A^2 D. \qquad (2)$$

U_i are the velocities, indices $i=1,2$ being allotted to the upper and lower layers respectively, D_i are the depths, A_i the cross-sectional areas of the layers, Q_i the volume transports and ρ_i the densities. $g'=g(\rho_2-\rho_1)/\rho$ is the reduced gravity, $D=D_1+D_2$ is the total sill depth, $A=A_1+A_2$ is the total cross-sectional area of the strait, $n=D_1/D$ is the relative depth of the interface, and $N=A_1/A$.

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It is seen from (2) that the geometry comes into play through the relation $N=N(n)$, and one strait parameter, say

$$R=(A^2 D)^{-1} \tag{3}$$

of dimension $(\text{length})^{-5}$.

By way of illustration, and since $(Q_1-Q_2)/Q_i \ll 1$, an approximate form of (2) is

$$Q^2 \propto g'/R, \tag{4}$$

which emphasizes the role of g' as the driving force, and the role of R as a resistance to the flow.

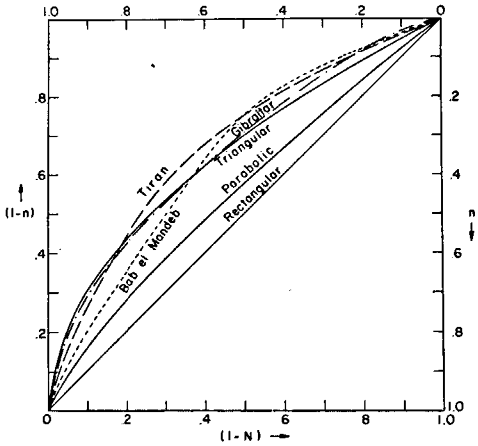
When a strait has a complicated geometry, it is not immediately clear where the control section, or the "dynamic strait" is. In fact, there is no reason to assume it at the narrowest cross-section, or the shallowest, or at the one of minimal cross-sectional area. If anything, the cross-section of maximal R would be the fairest assumption, by equivalence with resistances in other domains, provided this maximum is sharp enough to be well defined.

In each of the three instances examined below, Bab-el-Mandeb, Gibraltar, and Tiran, the value of R changes continuously along the main axis of the strait (in the direction of the flow), and the point of maximal R is unmistakebly conspicuous. This point is not always at the narrowest, or the shallowest, cross-section.

The locations, and the values of the maximal R, are presented in the Table below. The relations $N=N(n)$ are shown in the Figure.

Strait	Cross-section of maximal R	R $(\text{km})^{-5}$
Bab-el-Mandeb	Ras Dumeira, 12°47′N, 43°26′E, Dubabb.	11.8
Gibraltar	Bancos del Fenix, 35°58′N, 5°46′W, Pta Paloma.	0.173
Tiran	27°59.2′N, 34°26.2′E, Gordon Reef, Jackson Reef, 28°00.5′N, 34°29.2′E.	82.0

The basic assumption in the following calculations will be that the value of R to be used in (2) is the maximal value found along the main axis.



Figure

Interdependence of n and N for Tiran, Bab-el-Mandeb, Gibraltar, and three other hypothetical geometries.

WATER DEFICIT

With the above formulated assumption, we can proceed as follows: conservation of salt

$$Q_1 S_1 = Q_2 S_2, \tag{5}$$

combined with (2) gives

$$[S_2^2/(N^2 n)+S_1^2/(1-N)^2(1-n)](Q_1-Q_2)^2 = (S_2-S_1)^2 g'/R, \tag{6}$$

which can be solved for the net water deficit (Q_1-Q_2), provided that S_1, S_2, and g' at the strait are known, and the relative depth of the interface, n, is measured or is taken as the one which minimizes g' (Assaf, Hecht, 1974). For Bab-el-Mandeb, the values $n=0.343$, $N=0.601$ are found (graphically, from the Figure, since $N(n)$ is not analytic) to minimize g'. Using $S_1=36.6\,^0/_{00}$, $S_2=39.7\,^0/_{00}$ and $g'=3.2\times10^{-3}g$ (Siedler, 1969) as inputs to (6), the value

$$Q_1-Q_2=31.5\times10^3\ \text{m}^3.\text{sec}^{-1} \tag{7a}$$

is obtained, which is equivalent to an average evaporation (precipitation and runoff in the Red Sea, including the Gulf of Aqaba, are negligible) from the Red Sea

$$E=2.2\ \text{m/year}. \tag{8a}$$

This value of E is slightly higher than Privett's (1959) estimate of 1.83 m/year, but reasonably close to it.

Note that only information from the strait itself was used (S_1, S_2, g', $N(n)$, R), the Red Sea itself being taken just as a "black box".

The same method, when applied to the strait of Tiran, with the inputs to (6) taken from Paldor and Anati (1979), yields $n=0.309$, $N=0.609$, and

$$Q_1-Q_2=330\ \text{m}^3.\text{sec}^{-1}, \tag{7b}$$

which is equivalent to an average evaporation from the Gulf of Aqaba:

$$E=4.2\ \text{m/year}, \tag{8b}$$

again slightly higher than a previous estimate of 3.65 m/year (Assaf, Kessler, 1976).

For the strait of Gibraltar, we find $n=0.355$, and $N=0.581$. Using inputs to (6) from direct measurements taken in May-June 1961 (Lacombe, 1971), with $g'=1.7\times10^{-3}$ g, equation (6) yelds

$$Q_1-Q_2=124\times10^3\ \text{m}^3.\text{s}^{-1}, \tag{7c}$$

which lies near to the value measured in September 1960 and that measured in May-June 1961 (120×10^3 and $60\times10^3\ \text{m}^3.\text{sec}^{-1}$ respectively). Seasonal variations in Q_1-Q_2 in the strait of Gibraltar is rather high (much higher than in the other two straits considered), and therefore (7c) does not necessarily represent the annual average. With an area of 3×10^6 km^2, (7 *c*) would give

$$E = 1.3 \text{ m/year}, \tag{8c}$$

where E is water deficit in the case of the Mediterranean Sea, where precipitation is not negligible and whereas the annual average is estimated between 0.6 and 0.8 m/year (Lacombe, 1971).

PALEOCLIMATE

There is evidence that at the peak of glaciation in the Pleistocene, the sea level of the world's oceans was about 130 m below the present mean sea level (Fairbridge, 1961) and therefore some of the sea straits known today were dry, others considerably shallower. The strait of Tiran is today 253 m deep and, if no marked morphologic changes took place in the last few thousands of years, was at that time about 120 m deep. Its shape in the deepest 120 m is nearly parabolic, so that Figure 1 can be used to find the N (n) of the cross-section of maximal R.

The values which would minimize the g' in equation (2) are $n = 0.45$, $N = 0.53$, and

$$R_{\text{Paleo}} \sim 2\,900 \text{ (km)}^{-5}. \tag{9}$$

The surface area of the gulf was about 2000 km^2 (vs. 2400 km^2 today), so that, if the evaporation, precipitation and runoff rates did not change much,

$$(Q_1 - Q_2)_{\text{Paleo}} \sim 270 \text{ m}^3.\text{sec}^{-1}. \tag{10}$$

Isotopic considerations (^{18}O with oceanic $d\delta/ds$) seem to indicate that salinities in the Red Sea were about $10°/_{oo}$ higher than today (Deuser, Degens, 1969; Reiss *et al.*, 1980):

$$S_{1,\text{Paleo}} \sim 50°/_{oo}. \tag{11}$$

Assume now that the salinity difference was large enough, so that

$$g' = g\beta(S_2 - S_1), \tag{12}$$

where $\beta = (1/\rho)(\partial\rho/\partial S)$, and (6) is rewritten as

$$\left[\frac{(S_1 + (S_2 - S_1))^2}{N^2 n} + \frac{S_1^2}{(1-N)^2(1-n)}\right](Q_1 - Q_2) = \frac{g\beta}{R}(S_2 - S_1)^3, \tag{13}$$

which can be solved for $(S_2 - S_1)$, using (9), (10), and (11), yielding

$$(S_2 - S_1)_{\text{Paleo}} \sim 1°/_{oo}, \tag{14}$$

or about five times greater than at present.

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