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A model for prediction of tidal elevations over the English Channel

Tides
Model of prediction
Harmonic prediction
Satellite altimetry
English Channel
Marées
Modèle de prédiction
Prédiction harmonique
Altimétrie par satellite
Manche

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ABSTRACT

A model for prediction of tidal elevations in the English Channel is presented. It is based on the classical harmonic description of tides deduced from the harmonic development of the tide generating potential. The spatial distribution of the amplitudes and the phases of the twenty nine harmonic constituents included in the procedure of prediction is deduced from a previous study: spline functions determined on the basis of these data allow to compute these harmonic parameters at every point of the English Channel.
Two examples of application of the model are presented: firstly, a tidal prediction is realized in the harbour of Le Havre over a period of half a year and compared with *in situ* observation; secondly, the model is used to analyse altimetric measurements of the sea level along subsatellite flights across the English Channel, supplied by the oceanographic satellite Seasat: the predicted tides, computed by the proposed procedure, are removed from the altimetric measurements; this allows us to demonstrate that the precision of the altitude of the satellite over the Channel, processed during these flights, is not better than several meters; nevertheless, these results give an estimate of the slope of the geoïd between England and France, along the subsatellite track.

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RÉSUMÉ

Un modèle pour la prédiction des dénivellations de la marée dans la Manche

Un modèle de prédiction des variations de la surface de la mer dues à la marée dans la Manche, est présenté. Il est fondé sur la description harmonique classique des marées, déduite du développement harmonique de leur potentiel générateur. La distribution spatiale des amplitudes et des phases des vingt-neuf composantes harmoniques introduites dans la prédiction est déduite d'une étude antérieure : des fonctions spline déterminées sur la base de ces données permettent de calculer ces paramètres harmoniques en tout point de la Manche.
Deux exemples d'application du modèle sont présentés : premièrement, une prédiction de marée est réalisée pour le Port du Havre, sur une période d'une demi-année, et comparée avec l'observation des niveaux *in situ;* deuxièmement, le modèle est utilisé pour analyser les mesures altimétriques du niveau de la surface libre de la mer sous la trajectoire d'un satellite survolant la Manche, réalisées par le satellite océanographique Seasat : les prédictions de marée, calculées suivant la procédure proposée, sont retranchées des mesures altimétriques; on peut ainsi démontrer que la précision sur l'altitude du satellite au-dessus de la Manche déterminée durant ces vols n'est que de plusieurs mètres; cependant, ces résultats donnent une estimation de la pente du géoïde entre l'Angleterre et la France, le long de la trace du satellite.

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INTRODUCTION

In the European seas: North East Atlantic, English Channel, North Sea, the variations of the sea surface elevations are predominantly influenced by tides, which induce a large variability in space and time. On rather small areas, tidal elevations can differ in a ratio of several unity; in the Channel for instance, along the 2°W meridian, tidal heights on the English coast are divided by four, if compared to the nearest French coast, distant by only a hundred kilometers and by nine if referred to the Mont Saint-Michel Bay, where the ranges are of the order of 14 m. In the past, tidal predictions have been intensively developed in some places, along the coasts, especially in harbours, where such data were necessary for navigation; but correct predictions need long time series of previous observations, which are difficult to realize *in situ* (Godin, 1972); consequently, the number of such places is rather limited. With the increase of the size of ships, the development of offshore engineering, the detailed investigation of the sea bed structure... it becomes now necessary to establish such predictions over extended marine areas. Several approaches are possible. As the routine computations used in harbours are based on *in situ* observation, a first way can be to realize offshore records using special tidal gages moored near the bottom of the sea (Cartwright *et al.*, 1980). But, as long series of several months of observation are needed, and offshore measurements have often to be done in areas of intense traffic, such a procedure is not an easy thing; it demands experience, time and money; it can be applied only for restricted areas, where the characteristics of the tides are not varying too much. For larger domains, interpolations between observed points are often difficult and uncertain.

A second way to obtain prediction of tides over a given area is to build and use a numerical model; corresponding to the degree of precision needed, the complexity of the physical processes, and the extent of the studied area, various classes of two or even three dimensional models can be used. Based on the classical hydrodynamic equations of fluid motion under the shallow water hypothesis, they are established by using finite difference approximations of these partial differential equations, for finite difference models (FD), or finite element formulation, when using finite element technics (FE). Usually, along the coasts, it is sufficient to state as boundary conditions (BC) that the velocity is parallel to the shore; but along the open boundaries, external data are needed: tidal elevations and, sometimes also, velocity field; here is an important difficulty: generally, these BC are estimated from some rough knowledge of the phenomenon already published in the literature, and from what can be deduced from coastal observations, but this approach is not easy, and often subject to controversy. In the particular case where *in situ* observations along open boundaries are available, or possible to obtain by adapted offshore cruises, better use of these FD or FE models can be made. But there remains an important constraint coming from the tidal variability in time: the tide is not a strictly periodic phenomenon, and its complete spectrum is rather complex. Consequently, for tidal predictions, it is necessary to define correct BC throughout the period of interest, and to run the model over that period: it results in new difficulties, and very high computer costs.

The purpose of this paper is to present an other way to predict offshore tidal elevations, which is in fact a synthesis of the two first methods. It is based on the harmonic method for computing tides, extensively developed since the beginning of this century for classical harbour predictions; the basic parameters necessary for the computations are deduced from a complete study of the different constituents of the tide, realized previously by using FD or FE modelling technics. Initially, it demands an important and adapted effort to obtain the precise description of all the significant constituents of the tidal spectrum, all over the studied area, but any future prediction is easy to realize everywhere, at any time, with a very small computing cost. In this paper, after a brief summary of the harmonic method of prediction, we present an illustration of the proposed method for the English Channel, with the justification of the basic elements of the procedure: choice of the different tidal constituents, definition of their spatial distributions, and the comparison of some typical predictions with *in situ* observation; as a practical application of this model, we present in the last paragraph the use of this prediction for the filtering of the tidal contribution from the variations of the sea surface measured by satellite altimetry along some tracks of the oceanographic satellite Seasat A over the English Channel.

MATERIALS AND METHODS

Method of prediction: the harmonic theory of tides

One of the most classical methods used for tidal predictions is the harmonic method, based on the very typical form of the tidal spectrum whose frequencies can be deduced from those of the tidal potential. Darwin (1883) was the first to establish a development of the tide generating potential under the sum of sinusoïdal functions:

$$V = G \sum_{i=1}^{N_p} \lambda_i f_i c_i \cos(\omega_i t + v_{0i} + u_i), \qquad (1)$$

where:

$$G = \frac{3}{4} g \frac{M_L}{M_T} \frac{a^4 \rho^2}{c_L^2},$$

for the lunar terms and:

$$G = \frac{3}{4} g \frac{M_S}{M_T} \frac{a^4 \rho^2}{c_S^3},$$

for the solar terms with the notations:

g: earth gravity;
M_T, M_L and M_S: mass of the earth, the moon and the sun;
c_L and c_S: mean distance to the center of the earth of the moon and the sun;

a: mean radius of the earth;
ρ: distance from the center of the earth of the point where V is computed.
The other coefficients of (1) are:
λ_i: latitude factor corresponding to:
$(1-3\sin^2\varphi)/2$ for the long period constituents,
$\sin 2\varphi$ for the diurnal period constituents,
$\cos^2\varphi$ for the semi-diurnal period constituents,
with φ: latitude of the considered point on the earth;
f_i: nodal correcting factor, slowly varying with a basic period of 18.61 years;
c_i: amplitude of the constituent of index i;
ω_i: pulsation of the constituent of index i;
v_{0i}: phase of this constituent for $t=0$;
u_{0i}: nodal phase correction, slowly varying over 18.61 years.

Notice that (1) is only quasi harmonic: the nodal correcting coefficients f_i and u_{0i} are functions of the slow displacement of the ascending node of the lunar orbit on the equator.

This development contains 32 lunar terms; a more complete computation presented by Shureman (1958) gives 63 lunar terms, and 59 solar terms. The corresponding frequencies are not distributed all over the spectrum: they are concentrated in species: long period, diurnal, semi-diurnal and third diurnal, and very near each others. On Figure 1 are presented the main constituents in each species, with their name given by Darwin. Following the dynamic theory of tides, it can be assumed that to each constituent of this spectrum corresponds a wave in the ocean; consequently, the variation of the sea surface elevation in the ocean can be written as:

$$H(x, y, t) = H_0(x, j) + \sum_{i=1}^{N_p} f_i A_i(x, y) \times \cos[\omega_i t + (v_0+u)_i - g_i(x, y)], \quad (2)$$

with $H_0(x, y)$: mean sea level at point (x, y):
$A_i(x, y)$: amplitude of the component i at point (x, y);
$g_i(x, y)$: phase lag of this component, related to the phase of the corresponding term in the tidal potential, the reference being taken on the Greenwich meridian.

In shallow water areas, the development (2) can still be used, but new frequencies must be introduced in the spectrum, because of non linear effects governing the dynamic of propagation of the different astronomical tidal waves coming from the ocean. The frequencies of these non linear components are easy to deduce from the frequencies of their generating waves: they correspond to harmonics or interactions and are distributed in new species (quarter diurnal, sixth diurnal,...) or superimposed to already existing astronomical constituents: for example 2 MN_2 (interaction between the generating constituents M_2 and N_2, of frequency $\nu_{2MN_2} = 2\nu_{M_2} - \nu_{N_2}$) coincides with the minor lunar elliptic component L_2; notice that the nodal corrections for the astronomical wave L_2 and the non linear wave 2 MN_2 have not the same value, the later being deduced from those of the generating components M_2 and N_2, following some non linear theory explaining their interaction (*cf.* Le Provost, 1976).

Thus, the harmonic theory of tides assumes that the variation of the sea surface elevation can be developed under a form similar to (2), with a number of constituents N_R bigger than N_p: the parameters $A_i(x, y)$ and $g_i(x, y)$ are typical parameters of the tidal spectrum at the point (x, y), constant in time, and completely defining the time variable quantity $H(x, y, t)$. Classically, these parameters $A_i(x, y)$ and $g_i(x, y)$ are deduced from harmonic analysis of time series of *in situ* observations (notice that such analysis need long time series, because of the complexity of the tidal spectrum and the closeness of some frequencies). And their definition allows then the prediction of $H(x, y, t)$; the phases v_{0i} and the nodal corrections f_i and u_{0i} are known from the harmonic development of the tidal potential, and can be either directly computed from astronomical data of the earth's and the moon's position relative to the sun, or deduced from tables established by Shureman (1958), or more recently by Horn (1967), and giving the values of these parameters as functions of time, over the period 1850-1999.

Using formula (2), it is thus easy to predict tides at any place of a domain $\mathscr{D}$ if the amplitude and the phase of the significant tidal constituents are defined everywhere over the studied area. The English Channel is a typical area where tides have been intensively studied and for which an atlas of these constituents has been published (Chabert-d'Hières, Le Provost, 1979). We shall present in the following the application of the proposed method for all the English Channel.

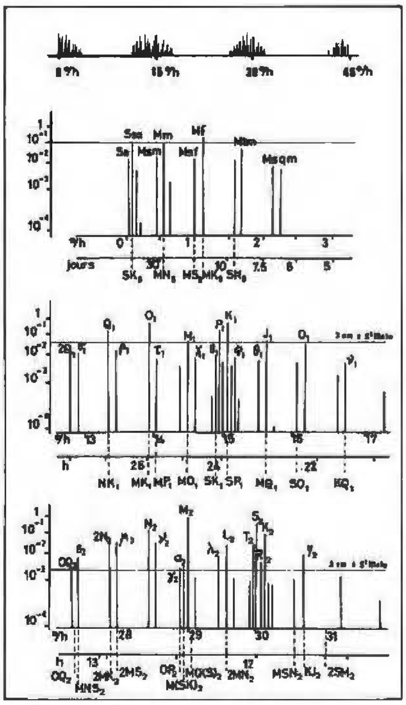


Figure 1
Spectrum of the tidal potential following the development of Darwin (1883).
Spectre du potentiel générateur des marées, selon le développement de Darwin (1883).

Materials: the atlas of harmonic tidal constituents over the Channel

Although the propagation of tides in shallow water areas is a strongly non linear phenomenon, we have realized some years ago a complete study of the significant constituents of the tide all over the English Channel. A preliminar theoretical analysis of the problem has been carried on the basis of a perturbation method where the astronomical waves coming from the ocean are considered of first order, and the harmonic and interaction waves appear at higher orders, generated by the non linear effects accompanying the propagation of the astronomical waves (*cf.* Le Provost, 1976). Practical investigations on the English Channel have been realized with the help of a hydraulic reduced model of that sea (Chabert d'Hières, Le Provost, 1976). All the significant components of the tide have been reproduced and checked by reference to the collection of harmonic constants available along the coasts, at the places where tidal elevations have been observed and analyzed (*cf.* Publication of the IHB, 1966; Desnoës, Simon, 1975), 26 constituents have been determined, the list of their symbols and frequencies is presented in Table 1: we find 3 diurnal (K_1: luni-solar declinational, O_1 major lunar, and P_1 major solar), 10 astronomical semi-diurnal (M_2: mean lunar, S_2 mean solar, N_2: major lunar elliptic, K_2 luni-solar declinational, L_2 minor elliptic lunar, μ_2 variational, 2 N_2 second order elliptic, T_2 major solar elliptic, ν_2 major evectional and λ_2 minor evectional), 7 non linear semi-diurnal (2 MS_2, 2 MN_2, 2 NK_2, MNS_2, MSN_2, 2 SM_2, 3 MSN_2), 3 non linear quarter diurnal (M_4, MS_4, MN_4) and 3 non linear six diurnal (M_6, 2 MS_6, 2 MN_6). For each constituent, the amplitude A_i and the phase g_i, as defined in formula (2), have been computed at 125 points over the studied area (the geographic location of these points is presented on Figure 2) and visualized on maps. We reproduce the co-amplitude and co-phase nets for the astronomical semi-diurnal M_2 (Fig. 3 *a*), the non linear semi-diurnal 2 MS_2 (Fig. 3 *b*), the quarter diurnal M_4 (Fig. 3 *c*) and the six diurnal 2 MS_6 (Fig. 3 *d*) which are typical examples of each class of constituents; notice the very different aspect of these cotidal maps, not only between the different species (1/2, 1/4, 1/6 diurnal), but also inside the semi-diurnal group between the astronomical and the non linear constituents.

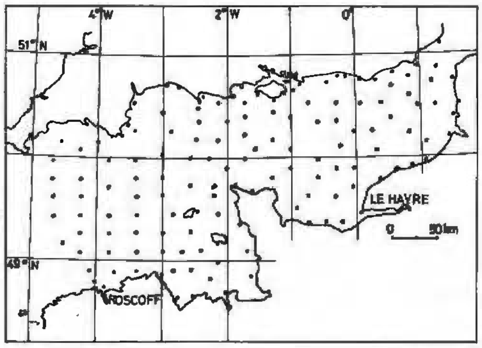


Figure 2

Location of the points where numerical values of the principal tidal constituents are known from Chabert d'Hières and Le Provost (1976).

Situation géographique des points où les valeurs numériques des principales composantes de la marée sont connues, suivant Chabert d'Hières et Le Provost (1976).

Before introducing this material in an automatic predicting procedure, one important problem to consider is the number of tidal constituents to use. Our aim is here to predict variations of water level offshore, i. e. in areas where non linear effects are generally less important than in estuaries (let us remember that Rossiter and Lennon (1968) took into account 114 components to reproduce tides in the Thames estuary!). Over the Channel, we have to focus our attention on three regions: Gulf of Saint-Malo, Bay of Seine, and Bay of Somme; the second area is the most complicated, because of the presence of noticeable high harmonics, six diurnal for example. A recent study presented by Desnoës and Simon (1975)

Table 1

List of constituents included in the prediction; (*) *constituents not published in Chabert d'Hières and Le Provost* (1979).

Liste des composantes introduites dans la prédiction; (*) composantes non publiées par Chabert d'Hières et Le Provost (1979).

Constituent	Angular speed (°/h)		
O_1	13.943 035 6	T_2	29.958 933 3
P_1	14.958 931 4	S_2	30.000 000 0
K_1	15.041 068 6	K_2	30.082 137 3
ε_2	27.423 833 7	MSN_2	30.544 374 7
2 MK_2	27.886 071 2	2 SM_2	31.015 895 8
2 N_2	27.895 354 8	MN_4	57.423 833 7
μ_2	27.968 208 4	M_4	57.968 208 4
2 MS_2	27.968 208 4	MS_4	58.984 104 2
N_2	28.439 729 5	MK_4 (*)	59.066 241 5
ν_2	28.512 583 1	2 MN_6	86.407 938 0
3 MSN_2	28.512 583 1	M_6	86.952 312 7
M_2	28.984 104 2	MSN_6 (*)	87.423 833 7
λ_2	29.455 625 3	2 MS_6	87.968 208 4
L_2	29.528 478 9	2 MK_6 (*)	88.050 345 7
2 MN_2	29.528 478 9		

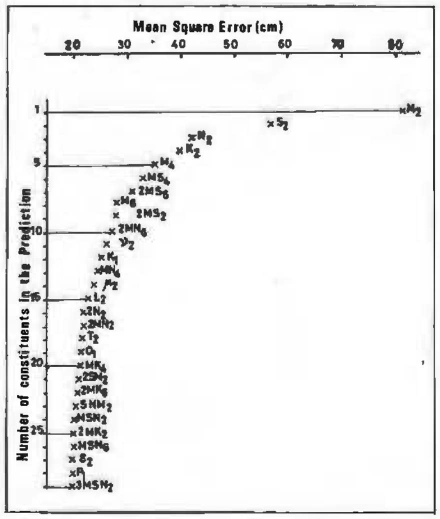


Figure 4

Mean square error between prediction and observation in Le Havre, as a function of the number of constituents included in the prediction. The order of introduction of the constituents follows a classification according to their amplitude.

Erreur quadratique moyenne entre prédiction et observation au Havre, en fonction du nombre de composantes introduites dans la prédiction. L'ordre d'introduction des composantes suit une classification basée sur leur amplitude.

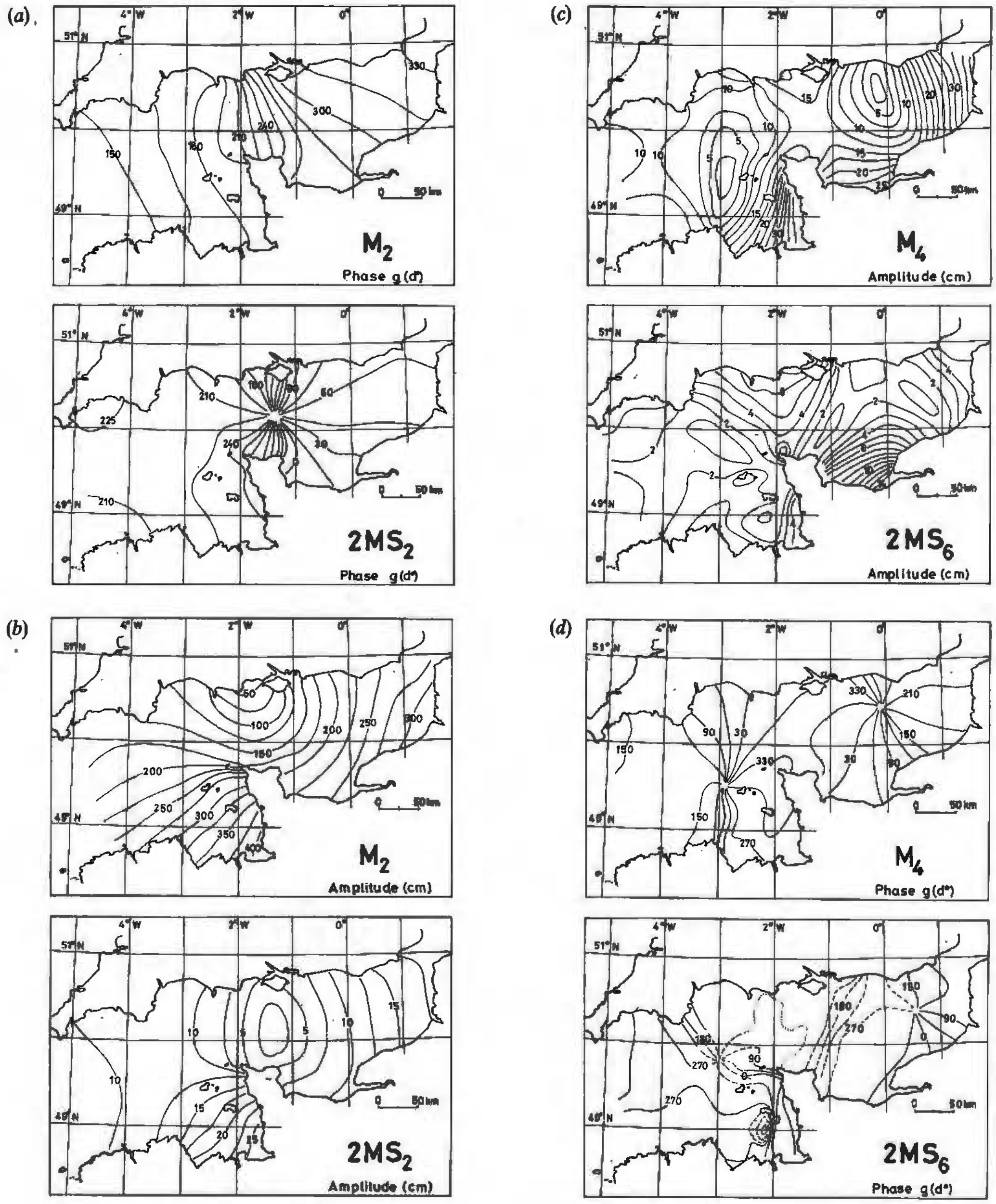


Figure 3
Typical cotidal maps over the Channel, from Chabert d'Hières and Le Provost (1979).

Cartes cotidales typiques pour la Manche, selon Chabert d'Hières et Le Provost (1979).

shows that a very high precision can be obtained for predictions in the harbour of Le Havre, by using 68 constituents: the mean square error between observation and prediction over one year 1938 being of only 11 cm. It would be possible to imagine a model taking into account such a big number of components, we shall give some ideas how to do it in the following but as a first approach, we have preferred to limit the method to a smaller number. On Figure 4 is visualized the mean square error (MSE) obtained between observation and prediction at Le Havre by using an increasing number of constituents introduced in the computations, in decreasing order of their amplitudes; the test period covers the second half of 1977. The order of magnitude of the sea surface variation is of 7 meters; as the number of constituents in the prediction increases, the MSE is reduced, of course, but the gain is less and less important as smaller and smaller amplitudes are taken into account: with ten constituents (5 semi-diurnal, 2 quarter diurnal, 3 six diurnal) the MSE is 27 cm, with ten additional components (2 D, 6 S−D, 2 1/4 D), the MSE is reduced to 22 cm and, using 29 constituents, 20 cm. To this level, the amplitude of the waves considered is less than 3 cm, and we have decided to limit our model to these 29 components. This limitation is arbitrary and it is motivated only because of practical facilities; we shall see later, in the discussion, how smaller constituents can be introduced without too much difficulty.

Characteristics of the model

The prediction model is based on formula (2). As noticed before, our previous study of the tides in the English Channel provides us the numerical values of A_i and g_i for the 29 considered constituents at the 125 points located on Figure 2. An interpolating procedure is necessary to deduce the values of these parameters everywhere between these points: this is realized by using spline functions. But these interpolations are not easy to obtain for the phases g_i when real amphidromic points exist, as shown on Figures 3*b*, 3*c* and 3*d*; we prefer to use (2) under the form:

$$H(x, y, t) = H_0(x, y) + \sum_{i=1}^{N_R} \{ f_i\, a_i(x, y) \times \cos[\omega_i t + (v_0 + u)_i] + f_i\, b_i(x, y) \sin[\omega_i t + (v_0 + u)_i] \}, \quad (3)$$

with:

$$\begin{cases} a_i(x, y) = A_i(x, y) \cos g_i(x, y), \\ b_i(x, y) = A_i(x, y) \sin g_i(x, y). \end{cases}$$

As a matter of fact, the distribution of these parameters a_i and b_i over the Channel are very regular, without singular areas like amphidromic points. Thus, for each of the 29 constituents introduced in the predictions, 2 spline functions are computed on the basis of the 125 values of a_i and b_i (Montes, Diaz, 1978).

The phase situations v_{0i}, function of the time origin choosen for the prediction, and the time variable correction coefficients f_i and u_i are tabulated for the first of January of each year, from 1970 to 1990 (*cf.* Horn, 1967), and their values at the specified time origin used for a particular prediction are deduced by interpolation.

The model offers two typical ways of prediction:

— at a special place, specified by its (λ, φ) coordinates in longitude and latitude, over a given period limited by the dates t_1 and t_2 giving the year, month, day, hour, minute and second of beginning and end of prediction. The parameters $a_i(\lambda, \varphi)$ and $b_i(\lambda, \varphi)$ are computed for the 29 components at the point $(x_0(\lambda, \varphi), y_0(\lambda, \varphi))$ by using the corresponding spline functions. The tidal heights $H(x_0, y_0, t)$ are computed by using (3) at time $t = t_1 + k\,\Delta t$, from t_1 to t_2, with a time interval Δt specified by the user;

— at a special time t_s (giving the year, month, day, hour, minute and second), for a group of points (x_m, y_m) specified by their coordinates (λ_m, φ_m).

The model has been adapted on a small computer: less than 10 K real words, and a few number of elementary operations are necessary. The computing time cost is very low.

RESULTS AND DISCUSSIONS

We shall present two typical applications of the model, in order to illustrate its possibilities: a prediction of water level, over a long period, in a particular location where tidal elevations have been observed *in situ*, and the use of a prediction of tidal heights, along a given line across the channel, to interpret sea level measurements obtained from satellite altimetry.

Example of prediction in Le Havre

The model has been used to compute tidal predictions in Le Havre where *in situ* observations are available. We have realized a prediction over 180 days, during the second half of 1977. As an example, the results for the beginning of this period are presented on Figure 5: on the left, prediction and observation are superposed on the same graph (given the scale of time abscissa, and the good correlation between the two signals, it is nearly impossible to distinguish one from the other); on the right side of the figure, the deviation between these two signals is presented. Some remarks can be made from looking at these results:

— the two signals are well in phase; and this is true over all 180 days considered (not presented on Fig. 5);

— most of the time, the deviations are rather small: within 30 cm, often less than 15 cm, while the studied signals are of the order of (+3 m, −4 m);

— the mean value of these residues is slowly varying with time. On Figure 5, it is of the order of +15 cm from the

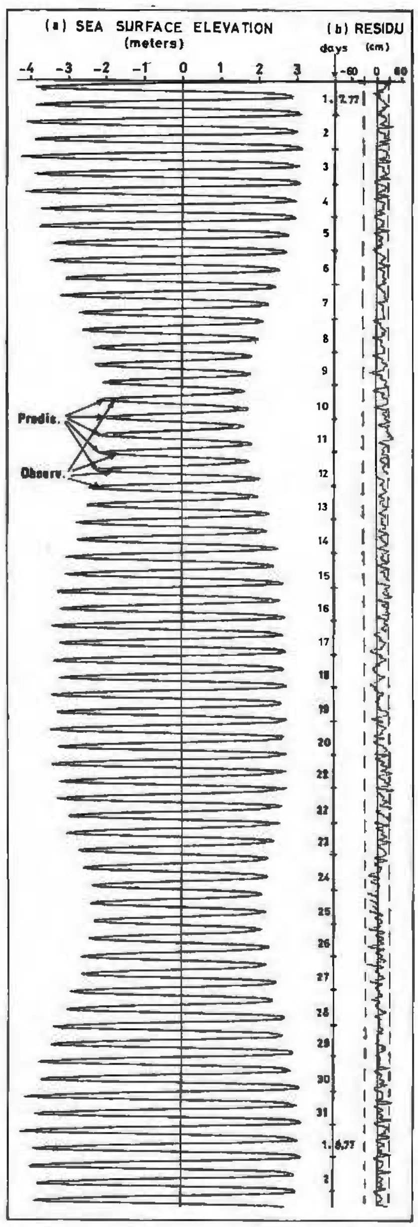


Figure 5

Example of tidal prediction in Le Havre: (a) prediction and observation; (b) deviation between prediction and observation.

Exemple de prédiction de marée au Havre : (*a*) prédiction et observation; (*b*) écart entre la prédiction et l'observation.

first to the 23 of July, except the 10 and 11 during which it goes up to +25 cm, and the 17, 18 and 19 when it goes down to zero. A detailed examination of the residues over the 180 days shows that, on the contrary, this mean deviation reaches sometimes negative values of the same order (it must be noticed that, the observed signal has been referred to its mean level computed over the 180 days, in order to be compared with the predicted signal). These slow variations are partly due to long period tidal constituents not included in the predicting model, and partly to meteorological effects;

– within each day, the residues oscillate with apparent semi-diurnal, and quarter diurnal periods and varying amplitudes from day to day. These deviations can be imputed to the small constituents of the real spectrum not included in the prediction, to the not strictly exact values of the harmonic constants prescribed by the spline functions of the model, and also to the fact that meteorological effects slightly modify these harmonic values by non linear interactions with tides.

Nevertheless, this example shows that, even in the area of Le Havre, where non linear deformations of the tides are very important, and greatly complicate the tidal spectrum, the present model allows us to predict sea surface variations quite correctly. It gives an idea of the standard deviation which can be expected, by reference to the real value. Other tests of the same character have been realized for other places in the Channel; it is not possible to present them in this paper, but we can say that similar results have been obtained: thus the present example is representative of the standard deviation which can be expected almost everywhere over the modelled area.

Tidal prediction and satellite altimetry

In September and the beginning of October 1978, the oceanographic satellite Seasat passed over the Channel along the same track every 3 days. On board this satellite, a radar altimeter measured the distance ρ from the vessel to the sea surface with an expected precision less than 10 cm (cf. Tapley *et al.*, 1979) (see Fig. 6); as the instantaneous geocentric radial position of the orbit r, and the radial geocentric position R_s to the reference ellipsoïd ($a = 6\,378\,140$ m, $e = 1/298.157$) are deduced

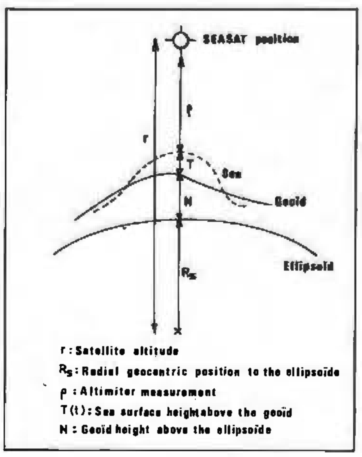


Figure 6
Definition of the satellite altimetric parameters.
Définition des paramètres caractéristiques de l'altimétrie par satellite.

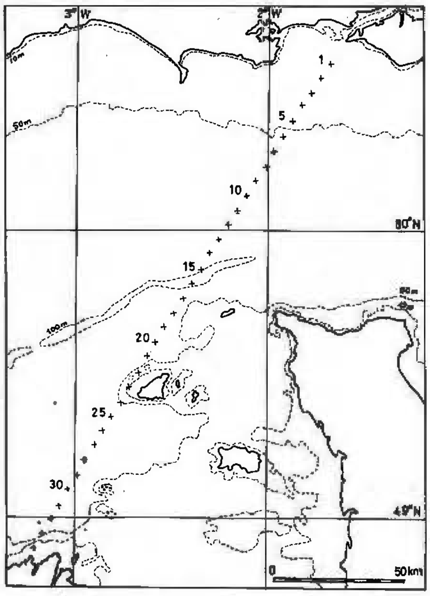


Figure 7
Seasat subsatellite track analysed, with the conventional abscissa used.
Trace au sol des vols Seasat analysés, et abscisse conventionnelle utilisée.

from satellite tracking, the altitude of the sea surface referred to this ellipsoïd is given by the relation:

$$N + T = r - R_s - \rho + \Delta r, \qquad (4)$$

N is the position of the geoïd relative to the ellipsoïd and T(t) the instantaneous sea-surface altitude relative to the geoïd, including the solid earth tide; Δr is the radial orbit error and is principally due to gravity model error propagated into the orbit determination.

The quantity T(t), variable in time, is a function of tides h_t, wind and atmospheric pressure effects, h_w and h_p, earth tide h_e, loading effects h_1, density h_d and mean circulation effects h_c. h_d is negligible; h_c, h_1 and h_e reach only some centimeters; $h_w + h_p$ are sometimes important, and can go up to 1 m; but the most important contribution is h_t which varies in the limits of several meters. Thus, it is interesting to use our model, in order to remove this quantity h_t from the altimetric signal.

We have selected three passages of the satellite over the same track visualized on Figure 7: points number 1 to 34 are the location of the successive altimetric measurements, processed every second, along the first flight over the channel; the data base has been communicated by the Jet Propulsion Laboratory, NASA (Brossier, 1979; Scott, 1980). The sea surface altitude referred to the ellipsoïd is presented on Figure 8 *a*. We can notice the important variations of this value from the English coast (near abscissa 1), to the French coast (point 35), and from one passage to the other: the 19.9, the altitude is quite constant, the 25.9, it rises from 40 to 44 m, and the 1.10, it is flat from the English coast to the middle of the Channel, near 43 m, and then rises up to 45 m; without anymore information these results are impossible to interpret. Let us compute tides along that line, as the satellite was passing, by using our model. The results are presented on Figure 8 *b*: the phases of the tides are completely different from one flight to the other, distributed from −4 m to 1 m. When we subtract the

Figure 8

Analysis of Seasat altimeter measurements over the English Channel: (a) measured sea surface altitude by reference to the ellipsoid: a = 6 378 140 m, c = 1/298 257; (b) tidal heights predicted along the subsatellite track; (c) altimeter residuals when tidal heights are removed.

Analyse des mesures de l'altimètre de Seasat sur la Manche: (*a*) altitude de la surface libre mesurée par référence à l'ellipsoïde: $a = 6\,378\,140$ m, $c = 1/298\,257$; (*b*) hauteurs de la marée prédites le long de la trace du satellite; (*c*) résidus altimétriques après élimination de la contribution de la marée.

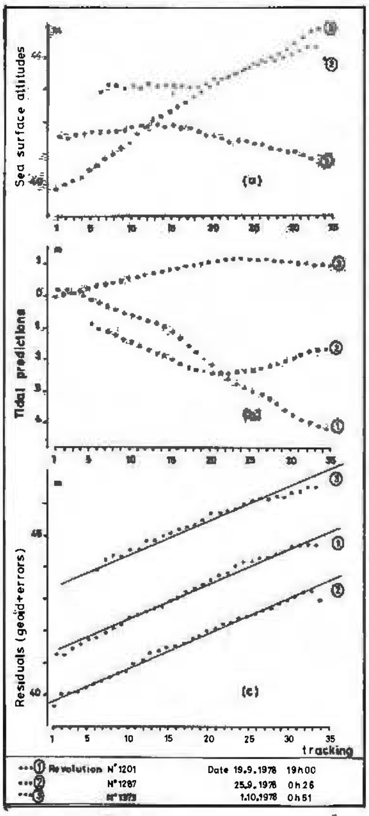
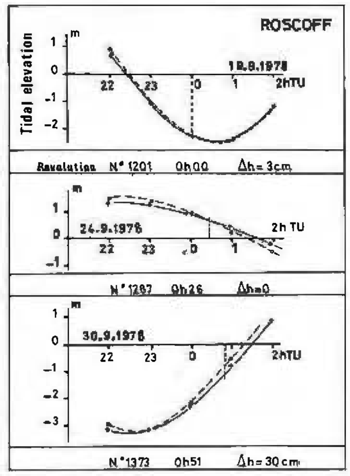


Figure 9

Deviation between prediction (----) and observation (———) at Roscoff during the satellite overflights.

Écart entre prédiction (---) et observation (———) à Roscoff pendant les survols du satellite.



tidal height predictions to the altimetric measurement, we obtain the curves presented on Figure 8 *c*: they are quite parallel, which can be surprising, at first sight. In fact, if we consider *a priori* that all the contributions to the sea surface variation $T(t)$ other than tides are negligible, these altimetric residuals correspond to the geoïd altitude N plus the radial orbit error Δr; the quantity N is constant in time, and Δr can be considered as a constant value for each passage, because of the small distance between points 1 and 35; consequently, Figure 8 *c* can be interpreted as a visualization of the slope of the geoïd: along the line 1-35, the geoïd altitude referred to the ellipsoïd of characteristics ($a = 6\,378\,140$ m, $e = 1/298.257$) is rising of about 3.5 m from England to France; moreover, if Δr was nul, the three curves would coincide: this is not the case, and we can conclude that the radial orbit errors are of the order of several meters, over the Channel. New tracking computations are actually carried out by the JPL in order to reduce these errors.

On Figure 8 *c*, we have drawn a mean straight line, with the same slope for each curve, and we can notice that the deviation is within 35 cm: of course, the geoïd is surely not strictly flat along this line, but it may be considered as a first approximation, and thus, we deduce that the deviations of the true altimetric residuals, corrected from the geoïd slope and the radial orbit error, are less than 35 cm. Within these residuals are the meteorological contributions, and the proper errors of our predicting procedure. In order to have an idea of these deviations, we present on Figure 9 a comparison between our predictions and observations in the harbour of Roscoff (not too far from point 35 on the French coast), in the neighbouring of the three studied overflights: it can be seen that the present errors Δh are of the same order, as the deviations noticed in the preceding application.

Possible refinement of the model

The present model is based on 29 constituents deduced from the atlas of reference, and we have already noticed that better predictions could be expected by introducing other components of smaller but yet significant amplitude, in areas of complex tidal spectra, like Gulf of Saint-Malo, Bay of Seine and Bay of Somme. But how to take into account these constituents, which are negligible almost everywhere, and thus difficult to characterize over all the domain? Rather than defining new spline functions for these new waves, we suggest to deduce their characteristic parameters a_k and b_k (or A_k and g_k) from the main 29 constituents already in the computer memory. This is possible because of the similarities existing between tidal waves of neighbouring frequencies and same origin: astronomical and radiational diurnal and semi-diurnal, non linear semi-diurnal, quarter diurnal, six diurnal... (cf. Chabert d'Hières, Le Provost, 1977). Indeed, over restricted areas, the amplitude and the phase of some unknown constituent can be deduced from the cotidal maps of the nearest known in the spectrum; within some distorsion (very small if the considered area is limited and the waves close in the spectrum), the ratio between their amplitude and their difference of phase can be considered as constant. As a first approximation, these ratios and these differences of phase can be deduced from the development of the tide generating potential (astronomical and radiational) for the oceanic waves, and from some non linear theory of tidal interactions, for the non linear constituents (cf. Le Provost, 1974); a better improvement can be obtained if a detailed analysis of some *in situ* observation is available.

As an illustration of this procedure, let us consider the example of the English Channel. Our reference will be the tidal spectrum in Le Havre where excellent analysis of *in situ* observations are available (cf. Desnoës, Simon, 1975) and where non linear waves have important amplitudes. On Table 2, are listed all the constituents in Le Havre which are smaller than 3 MSN_2 (the smaller of the 29 constituents actually in the model), and bigger than 2 cm: we have 5 quarter diurnal, 6 six diurnal, and 1 height diurnal. Each of them can be connected to a constituent already in the computer memory:

$2SM_6$ (angular speed: $88.984\,104\,2°/h$) to $2MS_6$ ($87.968\,208\,4°/h$);

Table 2

List of new constituents of significant amplitude in Le Havre which can be included in the prediction, with their constituents of reference, and the ratio and the difference of phase used to deduce their characteristics from those of the constituents of reference. Comparison between observed and estimated values (following the proposed procedure) in Antifer *(49°39N, 0°09E), near Le Havre, and Boulogne (50°44N, 1°36E), far from Le Havre.*

Liste de nouvelles composantes d'amplitude significative au Havre, qui peuvent être introduites dans la prédiction, avec leurs composantes de référence, et les rapports d'amplitude et les décalages de phase utilisés pour déduire leurs caractéristiques de celles des composantes de référence. Comparaison entre les valeurs observées et estimées (suivant la procédure proposée) à Antifer (49°39N, 0°09E), près du Havre, et Boulogne (50°44N, 1°36E), loin du Havre.

	New constituent	$2SM_6$	$2NM_6$	$3MSN_6$	$3MN_4$	$2MV_6$	MV_4	$3MS_4$	SN_4	$3MS_8$	MSK_6	$2MSN_4$	$2ML_6$
	Constituent of reference	$2MS_6$	$2MN_6$	$2MS_6$	MS_4	$2MN_6$	MN_4	MN_4	M_4		$2MS_6$	MS_4	$2MS_6$
	Le Havre												
New constituents	Amplitude (cm)	3.52	3.07	2.84	2.71	2.64	2.48	2.33	2.33	2.33	2.20	2.12	2.12
	g (°)	22	224	175	262	272	43	140	148	169	20	311	326
Constituents of reference	Amplitude (cm)	15.51	8.32		16.74		8.81		24.66				
	d (°)	334	266		131		54		77				
	C_{NR}	0.227	0.369	0.183	0.162	0.317	0.282	0.264	0.094		0.142	0.127	0.138
	Δg_{NR} (°)	48	318	201	131	6	349	86	71		46	180	352
	Antifer												
Estimated value	Amplitude (cm)	2.3	2.1	1.8	2.2	1.8	2.0	1.9	1.9		1.4	1.7	1.4
	g (°)	112	313	265	324	1	109	206	212		110	13	56
Observed value	Amplitude (cm)	2.5	2.0	2.0	2.4	1.6	1.8	1.8	1.8	2.0	0.9	1.7	1.8
	g (°)	112	325	260	322	358	104	206	203	294	125	13	28
Absolute error	ε amplitude (cm)	0.2	0.1	0.2	0.2	0.2	0.2	0.1	0.1		0.5	0	0.4
	ε phase (°)	0	12	5	2	3	5	0	9		15	0	28
	Boulogne												
Estimated value	Amplitude (cm)	1.5	1.3	1.2	3.5	1.1	3.1	2.9	3.1		1.0	2.8	0.9
	g (°)	184	28	335	47	76	188	285	294		180	96	126
Observed value	Amplitude (cm)	1.6	1.2	1.4	4.6	1.2	3.4	3.3	2.3	3.3	0.9	2.9	1.3
	g (°)	198	64	322	33	32	166	292	294	34	183	102	62
Absolute error	ε amplitude (cm)	0.1	0.1	0.2	1.1	0.1	0.3	0.4	0.8		0.1	0.4	0.1
	ε phase (°)	14	36	13	14	44	22	7	0		3	6	64

$2\,NM_6$ (angular speed: 85.863 563 3°/h) to $2\,MN_6$ (86.407 938 0°/h);

$3\,MN_4$ (angular speed: 58.512 583 1°/h) to MS_4 (58.984 104 2°/h)...

Except for one wave: $3\,MS_8$, in the heigh diurnal species, for which no cotidal map is known. Using the detailed analysis of *in situ* observations available in Le Havre, the ratio C_{NR} between the amplitudes of each new constituent H_N and its constituent of reference H_R, and their difference of phase g_{NR} are computed, following the relations:

$$C_{NR}=\frac{H_N\text{ (Le Havre)}}{H_R\text{ (Le Havre)}},$$

$$\Delta g_{NR}=g_N\text{ (Le Havre)}-g_R\text{ (Le Havre)}.$$

They are listed on Table 2. As an illustration of the degree of approximation obtained by using these coefficients C_{NR} and Δg_{NR} to estimate the amplitude $H_N(x, y)$ and the phase $g_N(x, y)$ of the new waves, from the known characteristics of the constituents of reference $H_R(x, y)$ and $g_N(x, y)$, following the formula:

$$H_N(x, y)=C_{NR}\,.\,H_R(x, y),$$

$$g_N(x, y)=g_R(x, y)+\Delta g_{NR},$$

we have compared the values thus obtained with the real ones deduced from *in situ* observation in two places: Antifer, situated near Le Havre, and Boulogne, which is far from the Bay of Seine. The obtained values are listed on Table 2: we notice that, for Antifer, the agreement is excellent; and that, for Boulogne, the results are still correct, except the amplitude of $3\,MN_4$ which is too small (3.5/4.6 cm) and the phase of $2\,ML_6$ (126/62°). On the basis of these relations between the 29 constituents already in the predicting procedure, and the smaller but yet significant constituents existing in some areas of the modelled areas, it is thus possible to include the later in the prediction for the English Channel. This can be done without difficulty for the constituents situated in the diurnal, semi-diurnal, quarter diurnal, and the six diurnal species; a problem remains for other species: long period, third diurnal, height diurnal... which have not been investigated, till now, all over the Channel.

CONCLUSIONS

The model presented allows prediction of tidal height variations of the sea level over all the English Channel, at any time, with a very low computer cost. The procedure is based on the harmonic method of tidal prediction and needs a detailed description of the different harmonic constituents of the tide over the whole studied area; the present application is a little peculiar because it uses an atlas of these constituents established with the help of a reduced hydraulic model of the Channel, but as a general rule, a numerical modelling of the different astronomical and non linear tidal constituents can be realized over any particular area, by using for example the spectral approach defined recently by Le Provost *et al.* (1980) who have built a finite element code allowing to compute quasi-automatically the main components of the tide over any given domain on condition that bathymetry and open boundary conditions are known.

The precision of these predictions is of course limited by the meteorological effects, not included in the forecasting, and by the limited number of constituents introduced in the computations. In the preceding discussion, we have suggested a simplified way to improve the second point; the prediction of meteorological effects is much more difficult to introduce, and is not in the scope of this study.

Nevertheless, the two examples presented in this paper show that the actual model, including only 29 constituents, gives interesting results and can be used in practical applications, such as satellite altimetry, elimination of tides in bathymetric sounding, computation of depths along shipping routes,...

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