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Statistical relations between successive wave heights

Waves
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Waves correlation
Statistics
Theoretical model
Houle
Groupe de vagues
Corrélation inter-vagues
Modèle statistique
Modèle théorique

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ABSTRACT

This paper presents a theoretical and experimental study of the correlation between successive waves. The theoretical model developed here, uses as starting point Rice's results on the autocorrelation function of the envelope of a narrow band random gaussian signal. Recent developments by Hasselmann *et al.* on wave spectra and Cavanie *et al.* on wave heights and periods statistics used as input in Rice's model, lead to a theoretical joint probability for successive wave heights. This model is compared to experimental results computed from 169 North Sea storm wave recordings corresponding to more than 26 000 individual waves.

The model leads to correlation coefficients between successive waves of 0.29 when using "a growing-sea" Jonswap type spectrum, and 0.16 for a "fully arisen sea" Pierson Moskowitz one. This confirms previous experimental observations. The theoretical joint probability function $p(H_i, H_{i+1})$ is very similar to the experimental one. Good agreement between theory and observation is also found for the conditional probability of H_{i+1} given H_i. Both theoretical and experimental probabilities are fitted by a two-parameter Weibull law as a function of H_i. Moreover, it is found theoretically and confirmed by observation, that for heights smaller than $0.75\ H_{1/3}$, waves can be considered as uncorrelated while for high waves the ratio H_i to expectancy of H_{i+1} tends towards 1.9.

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RÉSUMÉ

Relations statistiques entre hauteurs de vagues successives

Cet article présente une étude théorique et expérimentale sur les relations entre vagues successives. Le modèle développé ici utilise comme point de départ les résultats de Rice (1944) sur la fonction d'autocorrélation de l'enveloppe d'un signal gaussien à bande de fréquence étroite. L'application à cette théorie des résultats récents de Hasselman *et al.* (1975) sur la forme des spectres de houle, et de Cavanie *et al.* (1976) sur la distribution composée des amplitudes et des périodes des vagues, a permis de déterminer la loi théorique de distribution composée des hauteurs de deux vagues successives. La densité de probabilité ainsi obtenue est comparée à la distribution expérimentale obtenue à partir d'un ensemble d'enregistrements en Mer du Nord.

Le modèle permet une estimation théorique du coefficient de corrélation entre vagues successives. Un bon accord est observé entre les distributions composées théorique et expérimentale. La distribution liée d'une vague H_{i+1}, étant donnée la vague précédente H_i, est bien représentée par une loi de Weibull dont les paramètres sont fonction de H_i. Une corrélation non nulle entre H_i et H_{i+1} n'apparaît que lorsque H_i est supérieur à 3 fois l'écart type du signal. Pour chiffrer cette corrélation, le rapport de H_i sur l'espérance mathématique de H_{i+1} a été calculé. Ce rapport tend vers une valeur de 1,9 pour les vagues extrêmes, ce qui est confirmé par l'expérience.

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INTRODUCTION

Groups of high waves occuring from time to time are always observed in sea wave recordings. This phenomenon corresponds to a non-zero correlation between successive waves. Information concerning this correlation is of importance in order to determine the behaviour of structures at sea, and to judge the influence of this dependence on the computation of design wave heights and wave groups.

Up to now, the hypothesis of independence between successive waves has been largely accepted and commonly applied; however several publications have previously been devoted to waves groups. Nolte and Hsü (1972) have determined the mean duration of groups of successive waves higher than a given threshold. Starting from this value and assuming independence between successive waves, these authors conclude that group durations are distributed according to a Poisson law. Rye (1974) computed the correlation between successive waves for several Waverider recordings in the North Sea; a mean value of 0.24 was found, but Rye noticed that higher coefficients occured for growing seas and lower ones for fully arisen seas. Assuming a Rayleigh type distribution for heights and independence between successive waves, Rye proposed a theoretical distribution for the length of wave groups.

Other authors have been working on this topic: Wilson and Baird (1972), Goda (1970), Ewing (1973). A comprehensive review of research in this field may be found in Goda's 1976 paper.

In the present work a somewhat different approach has been chosen. The starting point is 1944 Rice's random noise theory. In view of recent developments in wave statistics, it has been possible to apply his results, concerning the autocorrelation function of the envelope of a narrow band signal, to sea waves. Wave energy spectra as expressed by Pierson and Moskowitz (1964), Hasselmann *et al.* (1976) have been used in this work. In order to compare theoretical results with experimental data, the continuous envelope signal had to be converted in terms of individual wave heights. This was done using the joint probability density for heights and periods developped at Cnexo by Cavanie *et al.* (1976).

Testing joint properties of stochastic processes always requires a considerable amount of experimental data; here the data are 169 sea waves storm recordings, from the North Sea, providing more than 26000 individual waves. These records have been lent by the Ukooa (United Kingdom Offshore Operators Association) and partialy processed by the Arae (Association de Recherche Action des Éléments).

This paper begins with a review of Rice's envelope theory and of Hasselmann's latest results concerning wave energy spectra and the choice of their parameters depending on meteorological conditions. Then, starting from these results, an estimation of the correlation coefficient between successive wave heights is computed. These estimates are found to be close to experimental values, and subject, as already mentioned, to variations depending on the spectral shapes for growing or fully arisen seas.

The theoretical joint probability density for successive heights $p(H_i, H_{i+1})$ is then computed. This theoretical density is found, on comparison, to be close to the experimental one. The conditional distribution of H_{i+1}, given H_i, seems to be well fitted by a two-parameter Weibull law. The Weibull parameters, as well as the expected values of the distributions, are determined as function of H_i and compared to experimental results.

BASIC RESULTS FROM PREVIOUS WORK

Rice's envelope theory

Let $I(t)$ be a gaussian noise with a narrow band energy spectrum, f_m being a representative midband frequency. $I(t)$ may be represented as the sum of sinusoidal components of fixed amplitude and random phase

$$I(t) = \sum_{n=1}^{N} c_n \cos(\omega_n t - \varphi_n),$$

where the φ_n are uniformly distributed over the range $[0, 2\pi]$:

$$c_n = [2\Phi(f_n)\,\Delta f]^{1/2}, \qquad \omega_n = 2\pi f_n, \qquad f_n = n\,\Delta f,$$

and $\Phi(f)$ is the spectral energy density.

The random noise may be reformulated as

$$I(t) = I_c \cos \omega_m t - I_s \sin \omega_m t,$$

with $\omega_m = 2\pi f_m$,

$$I_c = \sum_{n=1}^{N} c_n \cos(\omega_n t - \omega_m t - \varphi_n),$$

$$I_s = \sum_{n=1}^{N} c_n \sin(\omega_n t - \omega_m t - \varphi_n).$$

The envelope is defined as

$$R(t) = [I_c^2 + I_s^2]^{1/2}.$$

Rice studied the correlation between $R(t)$ and $R(t+\tau)$. Following his notation, the subscripts 1 and 2 will be writen for the times t and $t+\tau$.

The four random variables $I_{c1}, I_{s1}, I_{c2}, I_{s2}$ have a four dimensional normal distribution determined by the second moments

$$\overline{I_{c1}^2} = \overline{I_{s1}^2} = \overline{I_{c2}^2} = \overline{I_{s2}^2} = m_0,$$

$$\overline{I_{c1} I_{s1}} = \overline{I_{c2} I_{s2}} = 0,$$

$$\overline{I_{c1} I_{c2}} = \overline{I_{s1} I_{s2}} = \int_0^\infty \Phi(f)\,[\cos 2\pi(f-f_m)\,\tau]\,df = \mu_{13},$$

$$\overline{I_{c1} I_{s2}} = \overline{I_{c2} I_{s1}} = \int_0^\infty \Phi(f)\,[\sin 2\pi(f-f_m)\,\tau]\,df = \mu_{14},$$

where $m_0 = \int_0^\infty \Phi(f)\,df.$

Using the transformation

$I_{ci} = R_i \cos \theta_i,$

$I_{si} = R_i \sin \theta_i, \quad i = 1 \text{ or } 2,$

and averaging the resulting probability density over θ_1 and θ_2, Rice obtains

$$p(R_1, R_2) = R_1 R_2 A^{-1} \times I_0 (R_1 R_2 A^{-1} [\mu_{13}^2 + \mu_{14}^2]^{1/2}) \times \exp(-m_0 (2A)^{-1} (R_1^2 + R_2^2)),$$

where $A = m_0^2 - \mu_{13}^2 - \mu_{14}^2$, and I_0 is the Bessel function of the first kind with imaginary argument.

The previously defined envelope is a "crest to mean level" envelope. Since these theoretical results are to be compared to wave height data, R_i is transformed into $R_i/2$, which leads to the equation

$$p(R_1, R_2) = (16A)^{-1} R_1 R_2 \times I_0 (R_1 R_2 (4A)^{-1} [\mu_{13}^2 + \mu_{14}^2]^{1/2}) \times \exp(-m_0 (8A)^{-1} (R_1^2 + R_2^2)), \quad (1)$$

which will be used in the following.

Pierson - Moskowitz and Jonswap Spectra

As can be seen from (1), the shape of the energy spectrum is needed to apply Rice's result. Various spectral shapes for sea-waves have been proposed in the past 20 years. At present, the most commonly used are the Jonswap and the Pierson and Moskowitz spectra, which correspond respectively to growing and fully arisen seas.

From Hasselmann *et al.*, the Jonswap spectral energy density is properly expressed by

$$\Phi(\omega) = \alpha g^2 \omega^{-5} \exp\left[-\frac{5}{4}\left(\frac{\omega}{2\pi f_m}\right)^{-4}\right] \times \gamma^{\exp - [(\omega - 2\pi f_m)^2 / 2(2\pi f_m \sigma)^2]}. \quad (2)$$

In the case of time or fetch-limited sea-states, the Jonswap experiment, conducted in the North Sea in 1969, led to the following mean values for the parameters in this formula:

$\sigma = \sigma_a = 0.07 \quad \text{for} \quad \omega \leq 2\pi f_m,$

$\sigma = \sigma_b = 0.09 \quad \text{for} \quad \omega > 2\pi f_m,$

$\gamma = 3.3,$

$f_m = 3.5 \frac{g}{U_{10}} (\tilde{X})^{-0.33},$

$\alpha = 0.076 (\tilde{X})^{-0.22},$

where $\tilde{X} = X g / U_{10}^2$ is the dimensionless fetch.

The fetch X is expressed in meters, and the wind velocity at 10 m height, U_{10}, in meters per second. g is the acceleration due to gravity.

It must be noticed that different values of these parameters have been found by other authors working in different areas, but since the data used in this paper were collected in the North Sea, the Jonswap values were chosen.

In the case of a fully arisen sea, the γ parameter reduces to unity and the Jonswap formulation becomes equivalent to that of Pierson and Moskowitz (PM) in which the spectral density depends only on the wind velocity:

$$\Phi(\omega) = \alpha g^2 \omega^{-5} \exp\left[-\beta\left(\frac{g}{U_{19.5}\,\omega}\right)^4\right], \quad (3)$$

where $\alpha = 8.1 \times 10^{-3}$ and $\beta = 0.74$ are dimensionless constants.

Correction for the height at which the wind velocity is measured (19.5 m for PM spectrum, 10 m for Jonswap spectrum) will be disregarded in the following. From Hasselmann *et al.*, the portion of the (X, U) plane corresponding to growing seas has been found to be

$X < 0.86\, U^2,$

where X is the fetch in NM and U the wind celerity in meters per second.

THE AUTOCORRELATION FUNCTION OF THE WAVE ENVELOPE

The water surface signal satisfies both conditions required by Rice's model. Except in very shallow water, where sea-waves may show important non-linearities, the distribution of the free surface does not deviate considerably from a normal law. Moreover sea wave spectra in storm conditions can always be considered as narrow spectra, in the sense required by Rice: although one may frequently observe values of the spectral width ε (as defined by Cartwright, Longuet-Higgins, 1956) close to unity, this is due to the ω^{-5} spectral decrease at high frequencies. As an example, cutting off the five per cent of total energy at highest frequencies in a PM spectrum makes the spectral width fall from 1 to 0.49. This shows that, in spite of high values of ε, high frequencies contribute little to the total energy of the spectrum, the main part of which is concentrated near the peak.

Using Rice's results, the autocorrelation function of the wave envelope is computed from the following integral,

$$r(\tau) = \int_0^\infty \int_0^\infty (R(t) - RM)(R(t+\tau) - RM) \times p(R(t), R(t+\tau))\, dR\, dR, \quad (4)$$

where RM is the mean value of $R(t)$. To deal with a dimensionless function $\rho(\tau)$, $r(\tau)$ is divided by the variance of $R(t)$, σ_R^2:

$$\rho(\tau) = \frac{r(\tau)}{\sigma_R^2}. \quad (5)$$

Since the probability density of $R(t)$ is, for a narrow band spectrum, a Rayleigh law:

$$p(R) = \frac{R}{4 m_0} \exp\left[-\frac{R^2}{8 m_0}\right],$$

values of RM and $\sigma_R{}^2$ are given by

$$\mathrm{RM}=(2\pi m_0)^{1/2},$$

$$\sigma_R{}^2=8\, m_0\left(1-\frac{\pi}{4}\right).$$

These expressions and expression (1) are used respectively for RM, $\sigma_R{}^2$ and $p(\mathrm{R}(t), \mathrm{R}(t+\tau))$ in equations (4) and (5), leading to the normalized autocorrelation function. This computation was carried out for both Pierson-Moskowitz and Jonswap spectra, a cut-off frequency of 1 Hz being applied. Results are plotted on Figure 1. TM being the mean zero-up-crossing period deduced from spectral moments, an estimate of the correlation coefficient between successive waves, $\rho_{i,i+1}$, is obtained setting τ equal to TM. In the same way, the correlation coefficients between H_i and H_{i+2}, or H_i and H_{i+3}, are estimated by $\rho(2\,\mathrm{TM})$ or $\rho(3\,\mathrm{TM})$.

All these values are listed in the following table, together with the experimental values computed from North Sea data, and the value given by Rye.

	Theory		Experiment	
	PM spectrum	Jonswap spectrum	North Sea spectrum	From Rye spectrum
$\rho_{i,i+1}$	0.163	0.298	0.297	0.24
$\rho_{i,i+2}$	<0.02	0.113	0.051	-
$\rho_{i,i+3}$	<0.02	0.043	0.036	-

The estimates obtained from the Jonswap spectrum are close to the mean experimental values. If Rye proposes 0.24 as the mean value of $\rho_{i,i+1}$, he also indicates that during sea growth this coefficient tends to be close to 0.30, while during decay it is closer to, or lower than 0.20. These variations explain the relatively low PM estimate of $\rho(i, i+1)$, for it has been seen in paragraph I that the Jonswap spectrum applies to growing seas, while the PM spectrum applies to fully arisen sea states. Larger coefficients during wave growth may appear as a rather surprising result, for growing seas are generally thought to be "confused seas", with much energy at high frequencies. The reason given by Goda to the observed variations is that the spectrum is more sharply peaked during wave growth and this latter phenomenon is preponderant for wave group formation.

All the values reported hereabove for $\rho(i, i+2)$ and $\rho(i, i+3)$ are very weak, from which it must be concluded that H_{i+2} and H_{i+3} can be considered as independent of H_i.

THE JOINT PROBABILITY DENSITY $p(\mathrm{H}_i, \mathrm{H}_{i+1})$

Equation (1), used as in the preceeding paragraph with τ replaced by the mean period TM, furnishes a first order approximation of the joint probability density between

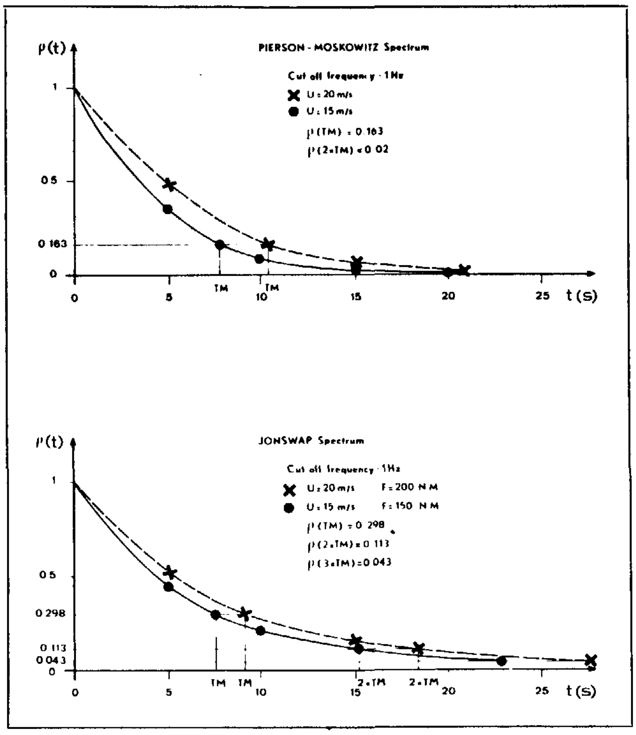


Figure 1

Wave envelope autocorrelation.

successive waves. This approximation which has been used to get a first estimate of the correlation coefficient between successive waves, is rigorously valid for infinitely narrow spectra only. As the scattering of periods around their mean value reflects the importance of the spectral width, a better approximation of $p(\mathrm{H}_i, \mathrm{H}_{i+1})$, for real spectra, has been computed by introducing the period distribution in the model.

The relation (1) gives the probability for the envelope to have values R_1 and R_2 at two points separated by the time interval τ. To underline the importance of this time lag, $p(\mathrm{R}_1, \mathrm{R}_2)$ will be written as $p(\mathrm{R}_1, \mathrm{R}_2\,|\,\tau)$. It appears from this notation that the distribution of lags between successive wave crests, given the heights of these waves, is needed to transform the autocorrelation function on the envelope into the correlation betwen successive waves. This distribution has been estimated from the height-period joint probability density given by Cavanie *et al.* written here in dimensional form:

$$p(\mathrm{H}, \mathrm{T})=\mathrm{A}_1\,\mathrm{H}^2\,\mathrm{T}^{-5}$$
$$\times \exp\left\{\mathrm{A}_2\,\mathrm{H}^2\,\mathrm{T}^{-4}\,[(\mathrm{A}_3\,\mathrm{T}^2-\alpha^2)^2+a^2\,\alpha^2]\right\}, \qquad (6)$$

where

$$\mathrm{A}_1=\frac{\alpha^3\,\varepsilon^{-1}\,(1-\varepsilon^2)^{-1}\,m_0{}^{-3/2}\,\mathrm{TM}^4\,\bar{\tau}^{-4}}{4\,(2\,\pi)^{1/2}},$$

$$\mathrm{A}_2=-\frac{m_0{}^{-1}\,\varepsilon^{-2}\,\mathrm{TM}^4\,\bar{\tau}^{-4}}{8},$$

$$\mathrm{A}_3=\bar{\tau}^2\,\mathrm{TM}^{-2},$$

ε is the spectral width;

$$\alpha = \frac{1+(1-\varepsilon^2)^{1/2}}{2},$$

$$a^2 = \frac{\varepsilon^2}{1-\varepsilon^2}.$$

$\bar{\tau}$ is a dimensionless function of ε which has been inserted in the formulation to set the expected value of periods equal to TM. Because the mean experimental value of ε was 0.86, this value was used in the theoretical model; the corresponding value of $\bar{\tau}$ is 0.94. Although based on the hypothesis of a relatively narrow spectrum, that model proved to fit correctly storm recordings with a spectral width value of about 0.9.

From expression (6) the conditional probability density of periods, given the height H, can be defined as:

$$p(\mathrm{T}\,|\,\mathrm{H}) = \frac{p(\mathrm{H},\mathrm{T})}{\int_0^\infty p(\mathrm{H},\mathrm{T})\,d\mathrm{T}}.$$

The joint probability density to have two successive waves with heights H_i and H_{i+1}, $p(H_i, H_{i+1})$, is then estimated, assuming that the lag between the crests of the two waves is equivalent to the period of a wave with a height $(H_i+H_{i+1})/2$. This simple estimate of the time interval between crests has been retained because it is likely to furnish the same degree of approximation as other hypotheses in the model. It leads to the expression

$$p(H_i, H_{i+1}) = \int_0^\infty p(H_i, H_{i+1}\,|\,\mathrm{T})\; p\left(\mathrm{T}\,\middle|\,\frac{H_i+H_{i+1}}{2}\right) d\mathrm{T}.$$

This probability density has been computed using a Jonswap type spectrum, and plotted on Figure 2 together with the equivalent experimental density deduced from the North Sea data. For reasons of symmetry, only the half-plane corresponding to $H_i \geqq H_{i+1}$ has been reported.

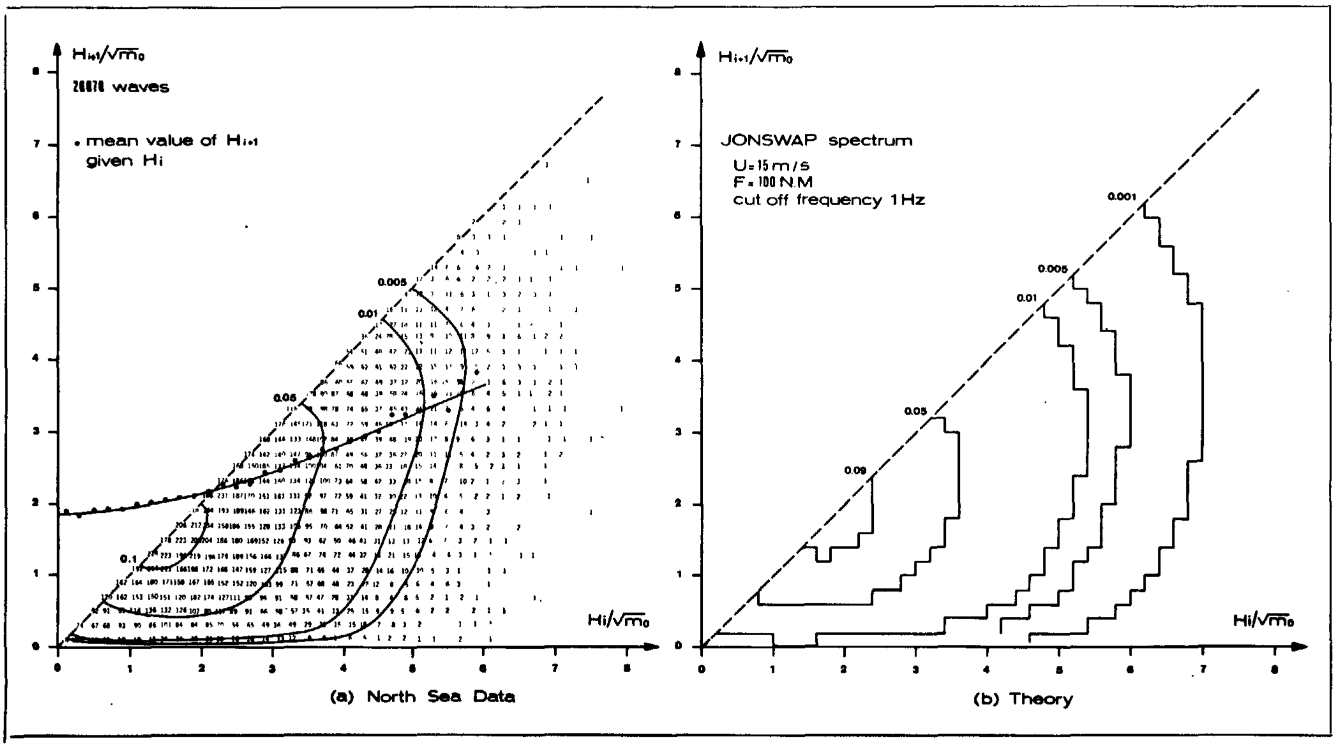
From these drawings it appears that the theoretically computed distribution is very close to the experimental one. This comparison will be extended, in the next paragraph, using the conditional distributions.

The mean values of H_{i+1}, given H_i, have been reported on the experimental plot of Figure 2. The curve defined by these points is a growing function of H_i, which is another indication of a non-zero correlation betwwen H_i and H_{i+1}.

THE CONDITIONAL PROBABILITY DENSITY OF H_{i+1}, GIVEN H_i

The complexity of the expressions involved in the works of Rice, Cavanie *et al.* and Hasselmann *et al.*, is not suited to an analytical formulation of the theoretical probability density $p(H_i, H_{i+1})$. In order to outline the main conclusions obtained by numerical means and to simplify the use of the present model, special attention was given to the conditional distributions of H_{i+1}, given H_i. If H_i and H_{i+1} were uncorrelated, the conditional and individual distributions of H_{i+1} would be identical to Rayleigh laws. For that reason, a two-parameter Weibull law, of which a Rayleigh law is a particular case, was fitted to the conditional distribution of the dimensionless variable $H_{i+1}/\sqrt{m_0}$.

Figure 2
Joint probability density function $p(H_i, H_{i+1})$.



A two-parameter Weibull distribution is expressed as:

$$P(X) = 1 - \exp\left(-\left(\frac{X}{a}\right)^c\right),$$

which can be written

$$\mathrm{Log}[-\mathrm{Log}(1-P(X))] = c\ \mathrm{Log}\,X - c\ \mathrm{Log}\,a.$$

This latter expression furnishes estimates of both c and a since on a Weibull probability paper c and $-c$ Log a are respectively the slope and zero ordinate of the straight line fitted through the data.

Since high waves are of special interest, the conditional distributions of H_{i+1} were determined for $H_i/\sqrt{m_0}$ equal to 2, 3, 4, 5, 6 and 7. These theoretical and experimental distributions plotted on a Weibull paper are presented on Figure 3 with the corresponding fitted Weibull straight lines. Quite good agreement is observed in every case between the theoretical or experimental distributions and the Weibull law, in the range of probability $P(H_{i+1}) < 0.98$. A slight divergence occurs beyond this limit which affects only the highest waves.

Figure 3 also reveals a monotonous variation of the fitted Weibull law parameters as functions of H_i. Values of these parameters are plotted on Figure 4. The deviation, appearing on this figure, between the theoretical and experimental values of c and a, are in every case except one less than 10%. The largest difference, corresponding to $H_i/\sqrt{m_0} = 6$, is 13% and may result from the relatively limited number of waves (70) involved in the determination of the experimental distribution.

The variation of the Weibull parameters as a function of H_i can be summarized as follows, using the mean experimental and theoretical values. When $H_i/\sqrt{m_0}$ increases from 2 to 6, c and a vary in a monotonous and almost linear way, respectively from 2.04 to 2.75 and from 2.5 to 3.95. The increase of c is an indication of the shift of the mode of the distribution towards the high values. This tendency, as well as the increase of a, which is proportional to the standard deviation of the distribution, indicates that high waves generally follow high waves. It is well known that the individual distribution of H_{i+1} is the Rayleigh distribution corresponding to the Weibull parameters $c = 2$

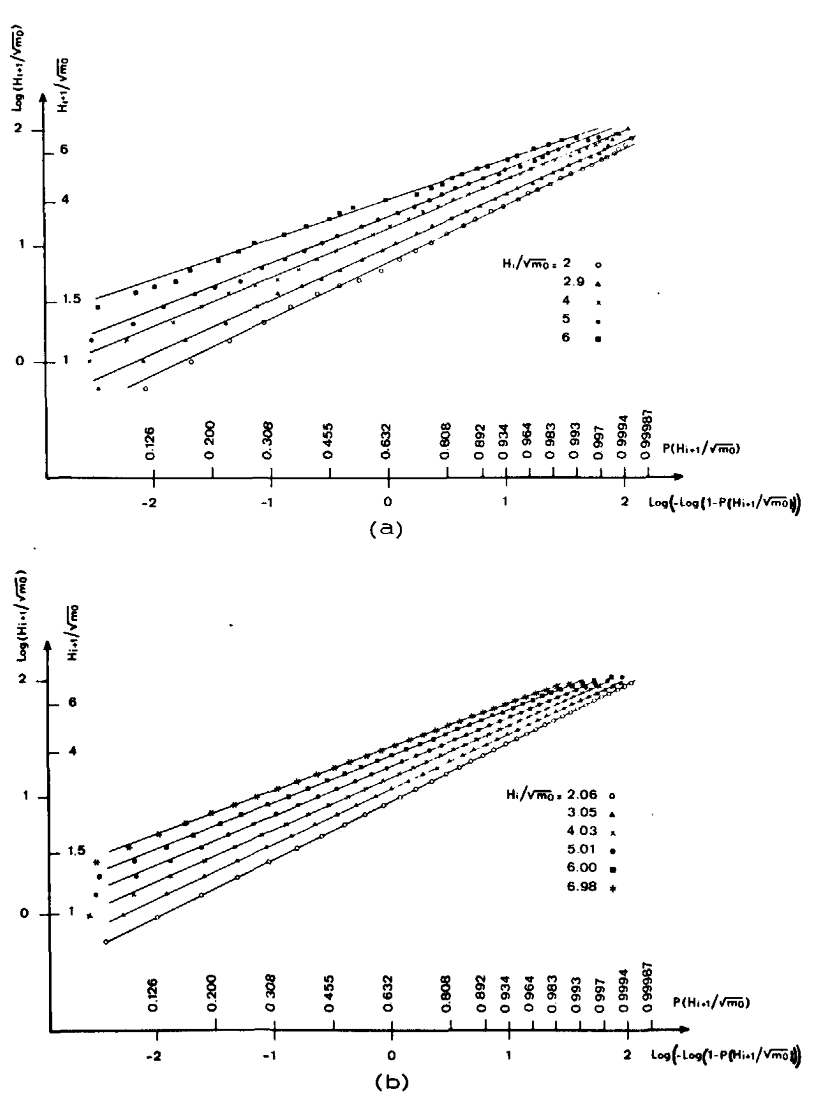


Figure 3
Conditional probability distribution function of H_{i+1}, *given* H_i, *plotted on a Weibull scale. (a) North Sea data; (b) theory.*

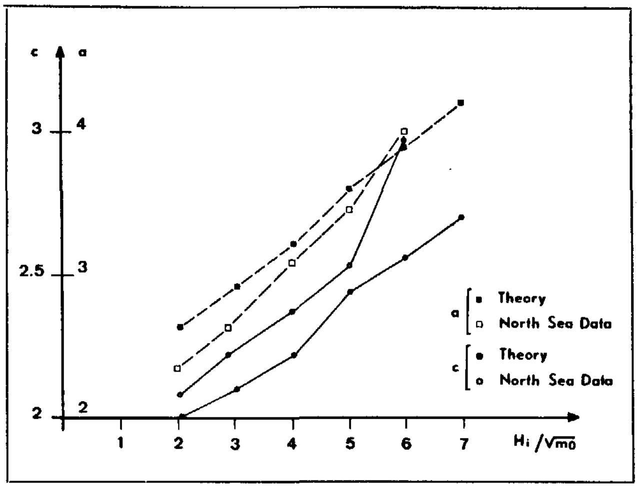


Figure 4
Parameters of a Weibull law fitted to the conditional probability distribution function $P(H_{i+1} | H_i)$ *as a function of* H_i.

and $a = 2\sqrt{2} = 2.83$. The conditional distribution found for $H_i/\sqrt{m_0} = 2$ has the parameter values 2.0 and 2.5. This indicates that the conditional law of H_{i+1}, given a small value of H_i, is very close to the global distribution of heights. In other words, the correlation between H_i and H_{i+1} is very weak for small values of H_i.

An important parameter of the conditional distribution of H_{i+1}, given H_i, is its expected value. The ratio of H_i to this expected value, $H_i/\mathrm{Exp}[H_{i+1}]$ is plotted on the upper drawing of Figure 5. This ratio has been computed for several different cases:

– from the joint probability density presented in paragraph III, using results of Rice, Hasselmann *et al.* and Cavanie *et al.*;

– from results of Rice and Hasselmann *et al.*, assuming that all the waves have the same period, equal to TM;

– from the North Sea experimental data.

For the highest H_i values some scatter appears in the data due to the decreasing number of points used in the computation of the mean value. Nevertheless a comparison of the three curves can be made which shows that the joint probability density for wave heights and periods leads to a better agreement between theory and experiment, especially for the highest H_i values. For $H_i/\sqrt{m_0} = 8$ which can be regarded as an extreme wave, both theoretical and experimental ratios $H_i/\mathrm{Exp}[H_{i+1}]$ tend to 1.9.

If H_i and H_{i+1} were uncorrelated, the expected value of H_{i+1} would be $\sqrt{2\pi} = 2.51$, i.e. the expected value of the global distribution of heights. This hypothesis would lead to the straight line on the drawing for the ratio $H_i/\mathrm{Exp}[H_{i+1}]$. It appears, when comparing this line to the real curve, that the hypothesis of independence can be retained for $H_i/\sqrt{m_0} < 3$, and that beyond this value the dependence increases with H_i.

On the lower drawing of Figure 5 are plotted the ratios $H_i/\mathrm{Exp}[H_{i+2}]$ as a function of $H_i/\sqrt{m_0}$. This ratio has

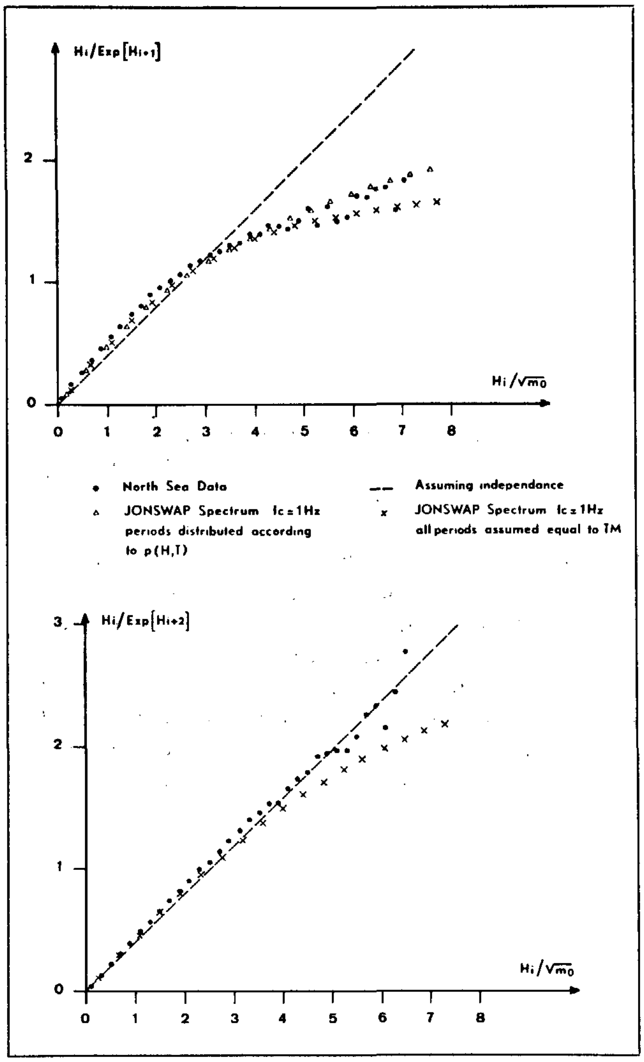


Figure 5
Ratio of H_i *to expected values of* H_{i+1} *and* H_{i+2} *as a function of* H_i.

been computed using the North Sea data, and also from the theoretical model assuming that all periods are equal to the mean one. These curves confirm the very weak correlation between H_i and H_{i+2}, for all values of H_i. The difference observed between the two curves for high values of H_i is probably due to the neglect of the height-period relationship for H_{i+2}.

CONCLUSIONS

Analysis of the important data set obtained from North Sea Storm recordings clearly demonstrates the following properties of successive waves:

- for low wave heights, independence between successive waves is a good approximation;
- for wave heights superior to $0.75\ H_{1/3}$, the height of a given wave depends in practice only on that of the adjacent wave, a Weibull law fitting the experimental distribution of H_{i+1}, given H_i, quite well;
- this dependence increases with wave heights, so that for extreme waves the ratio $H_i/\mathrm{Exp}[H_{i+1}]$ reaches 1.9.

In order to explain this behaviour correctly by a theoretical model, it has proved necessary to develop a three stage model starting from Rice's theory for the envelope, applying either Jonswap or Pierson-Moskowitz spectra, and making use of the joint probability distribution for wave heights and periods developed by Cavanie *et al.* Although this effort proved time consuming it offers a satisfactory concordance with the previously mentioned features of observations at sea. Moreover, the difference in correlation coefficients for wave heights of growing or fully arisen seas, noted by different authors, is found anew by the theory.

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