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Is a management framework based on spawning-stock biomass indicators sustainable? A viability approach

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Abstract:

Fisheries management agencies have to drive resources on sustainable paths, i.e. within defined boundaries for an indefinite time. The viable-control approach is proposed as a relevant method to deal with sustainability. We analyse the ICES precautionary approach (PA) by means of the notion of viability domain, and provide a mathematical test for sustainability. It is found that the PA based on spawning-stock biomass (SSB) and fishing mortality (*F*) indicators is sustainable only when recruits make a significant contribution to SSB. In this case, advice based upon SSB, with an appropriate reference point, is sufficient to ensure sustainability. In all other cases, SSB is not a sufficient metric of stock productivity and must be complemented with other management indicators to ensure sustainability. The approach is illustrated with numerical applications to the northern hake and Bay of Biscay anchovy.

Keywords: ICES precautionary approach, indicators, sustainable management, viability

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Introduction

Sustainability is a major goal of international agreements and guidelines to fisheries management (FAO, 1999; ICES, 2004). However, the meaning and operational content of sustainability is not always well defined. We define sustainability as the ability to maintain a system within the limits of given objectives for an indefinite time.

Sustainability of management framework based on SSB

Indicators and their associated reference points are key elements of current management advice, as well as of the developing ecosystem approach. One or several indicators are used to monitor the progress towards management objectives, be they implicit or explicit. Reference points are selected as benchmarks for the indicator values (Deriso *et al.*, 1998; FAO, 1999; ICES, 2004). In the ICES precautionary approach (PA), the objectives are to maintain spawning-stock biomass (SSB) above a limit reference point B_{lim}, while keeping fishing mortality (F) below a limit reference point F_{lim} (ICES, 2004).

We claim that the indicators in the PA play a confusing double role of implicit sustainability objectives and explicit management tools. The issues at stake are the following. Can the objectives defined by SSB and F be achieved by operational advice based only on SSB and F used as indicators? Assuming the management would follow the advice and be complied with, would it be sufficient to keep an indicator above a reference point to be able to keep it there again after subsequent time steps? The answer is far from being obvious, because exploited stocks are not at equilibrium and their dynamics bear a shared inertia.

Optimum-control theory has been extensively used to define fisheries management strategies (Hilborn and Walters, 1992; Quinn and Deriso, 1999). Although it offers a dynamic perspective, undesirable outcomes of optimality approaches undermine its applicability in a sustainability perspective. First, it could be optimal to exhaust the resource. Second, optimum solutions depend largely on the selected discount rate, which measures relative preference for future payoffs with respect to present ones. Moreover, optimum control is not easy to apply when multiple objectives are pursued.

The viability approach put forward here does not strive to determine optimum paths for the co-dynamics of resources and exploitation, but paths belonging to acceptable corridors. These corridors are defined by the management objectives, and sustainability is then defined as the ability to maintain the system within these corridors for an indefinite time. The viable-control approach (Aubin, 1991), or weak-invariance approach (Clarke *et al.*, 1995), enables us to define the corridor borders formally, and allows us to provide advice for decision-making, given a set of objectives, by computing the conditions that allow these objectives to be fulfilled at any time, in the present and in the future. This approach has been applied to renewable-resource management (Béné *et al.*, 2001; Doyen and Béné, 2003; Eisenack *et al.*, 2006; Rapaport *et al.*, 2006), and has been suggested to be useful potentially in integrating ecosystem considerations (Cury *et al.*, 2005). When system dimension increases and/or relationships become non-linear, technical and numerical difficulties in applying viability concepts generally arise. However, in a companion paper (De Lara *et al.*, 2006), we show how viability tools allow us to take advantage of some monotonicity properties of age-structured population models.

Here, we use the viable-control approach to make the relationships between management objectives, the PA and the stock dynamics explicit. We show how the age-structured dynamics can be taken into account to test whether SSB and F are appropriate indicators for keeping SSB above a reference point. First, the age-structured population dynamics model is presented, together with viability tools. Then, sustainability of the PA is examined. The approach is illustrated with applications to the northern hake and Bay of Biscay anchovy.

Material and methods

Age-structured stock model

We describe the dynamics of the exploited resource by a so-called controlled dynamic system in discrete time, where the time step is one year. At each discrete time $t = t_0,\ t_0+1,\ \ldots,$ let us consider $N_a(t)$, the abundance of the stock at age $a \in \{1,\ldots,A\}$, and $\lambda(t)$ the fishing mortality multiplier (control), supposed to be taken at the beginning of period $[t, t+1]$. Introducing the state vector $N(t) = (N_1(t),\ldots,N_A(t))$ (in short: stock), belonging to the state space $\mathbb{R}_+^A$ ($\mathbb{R}_+$ the set of non-negative real numbers), the following dynamic system is considered (Quinn and Deriso, 1999):

$$N(t+1) = g(N(t),\lambda(t)), \quad t = t_0, t_0+1,\ldots, \qquad N(t_0) \text{ given}, \tag{1}$$

where the vector function $g = (g_a)_{a=1,\ldots,A}$ is defined for any $N \in \mathbb{R}_+^A$ and $\lambda \in \mathbb{R}_+$ by

$$\left\{\begin{array}{lcl} g_1(N,\lambda) & = & \varphi(\mathrm{SSB}(N))\,, \\ g_a(N,\lambda) & = & e^{-(M_{a-1}+\lambda F_{a-1})}N_{a-1}, \quad a = 2,\ldots,A-1\,, \\ g_A(N,\lambda) & = & e^{-(M_{A-1}+\lambda F_{A-1})}N_{A-1} + \pi \times e^{-(M_A+\lambda F_A)}N_A\,. \end{array}\right. \tag{2}$$

The function φ describes a stock-recruitment (S-R) relationship. The SSB is defined by

$$\mathrm{SSB}(N) = \sum_{a=1}^{A} \gamma_a w_a N_a\,, \tag{3}$$

with γ_a the proportion of mature individuals-at-age and w_a the weight-at-age. The parameter $\pi \in \{0,1\}$ is related to the existence of a plus group to describe the population dynamics. If we neglect the survivors after age A, then $\pi = 0$, else $\pi = 1$ and the last age class is a plus group.

Sustainability of management framework based on SSB

Indicators and reference points

Two indicators are used in the PA, with associated limit reference points. Let us stress here that using limit reference points implies defining a boundary between unacceptable and acceptable states, whereas desirable states would rather be defined by target reference points. The first indicator, denoted by SSB in (3), is associated with the reference point $B_{\mathsf{lim}} > 0$. For management advice, an additional precautionary reference point $B_{\mathsf{pa}} > B_{\mathsf{lim}}$ is used, intended to incorporate uncertainty about stock state.

The second indicator, denoted by F, is the mean fishing mortality over a pre-determined age range from a_r to A_r, i.e.

$$F(\lambda) := \frac{\lambda}{A_r - a_r + 1} \sum_{a=a_r}^{a=A_r} F_a \,. \tag{4}$$

The associated limit reference point is F_{lim} and the precautionary approach reference point is $F_{\mathsf{pa}} < F_{\mathsf{lim}}$. Acceptable controls λ, according to this reference point, are those for which $F(\lambda) \leq F_{\mathsf{lim}}$, because higher F rates might drive SSB below its limit reference point.

Acceptable configurations

To define sustainability, we now assume that the decision-maker can describe "acceptable configurations of the system", i.e. acceptable couples (N, λ) of states and controls, which form a set $\mathbb{D} \subset \mathbb{R}_+^A \times \mathbb{R}_+$, the acceptable set. Let us insist upon the fact that $\mathbb{D}$ includes both system states and controls. In practice, the set $\mathbb{D}$ may capture ecological, economic and/or sociological requirements.

Considering sustainable management within the PA, involving SSB and F indicators, we introduce the following PA configuration set

$$\mathbb{D}_{\mathsf{lim}} := \{(N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid \mathrm{SSB}(N) \geq B_{\mathsf{lim}} \text{ and } F(\lambda) \leq F_{\mathsf{lim}}\}. \tag{5}$$

B_{lim} and F_{lim} are used in this definition, assuming that the uncertainty will be accounted for in the assessment and advice process, relying on B_{pa} and F_{pa}. Actually, any reference point could be used here, because the focus of the study is on indicators, not on reference points.

Viability domains and viable controls

A subset $\mathbb{V} \subset \mathbb{R}_+^A$ of the state space $\mathbb{R}_+^A$ is said to be a viability domain for dynamics g in the acceptable set $\mathbb{D}$ if

$$\forall N \in \mathbb{V}\,, \quad \exists \lambda \in \mathbb{R}_+\,, \quad (N, \lambda) \in \mathbb{D} \text{ and } g(N, \lambda) \in \mathbb{V}. \tag{6}$$

In other words, if one starts from a stock in $\mathbb{V}$, there exists an appropriate fishing mortality multiplier such that the system is in an acceptable configuration and the next time step state is also in $\mathbb{V}$. For example, acceptable equilibria ($(\bar{N}, \bar{\lambda}) \in \mathbb{D}$ and $g(\bar{N}, \bar{\lambda}) = \bar{N}$) are viability domains.

Given a viability domain $\mathbb{V}$, the viable controls associated with any state $N \in \mathbb{V}$ are those controls that let the state within the viability domain at the next time step, i.e. that belong to the following (non-empty) set

$$\Lambda_{\mathbb{V}}(N) := \{\lambda \in \mathbb{R}_+ \mid (N, \lambda) \in \mathbb{D} \text{ and } g(N, \lambda) \in \mathbb{V}\}\,. \tag{7}$$

Interpreting PA in the light of viability

The PA can be sketched as follows: an estimate of the stock vector N is made; the condition $\mathrm{SSB}(N) \geq B_{\mathsf{lim}}$ is checked; if valid, the following usual advice is given:

$$\lambda_{UA}(N) = \max\{\lambda \in \mathbb{R}_+ \mid \mathrm{SSB}(g(N, \lambda)) \geq B_{\mathsf{lim}} \text{ and } F(\lambda) \leq F_{\mathsf{lim}}\}\,.$$

However, the existence of a fishing mortality multiplier for any stock vector N such that $\mathrm{SSB}(N) \geq B_{\mathsf{lim}}$ is tantamount to non-emptiness of a set of viable controls. This justifies the following definitions. Let us define the PA state set

$$\mathbb{V}_{\mathsf{lim}} := \{N \in \mathbb{R}_+^A \mid \mathrm{SSB}(N) \geq B_{\mathsf{lim}}\}\,. \tag{8}$$

We shall say that the PA is sustainable if the PA state set $\mathbb{V}_{\mathsf{lim}}$ given by (8) is a viability domain for dynamics g in the acceptable set $\mathbb{D}_{\mathsf{lim}}$ given by (5).

Whenever the PA is sustainable, the set (7) of viable controls $\Lambda_{\mathbb{V}_{\mathsf{lim}}}(N)$ is not empty. This implies the existence of a viable fishing mortality multiplier $\lambda \in \Lambda_{\mathbb{V}_{\mathsf{lim}}}(N)$ that allows the SSB of the population to remain above B_{lim} at any time. When $\mathbb{V}_{\mathsf{lim}}$ is not a viability domain for dynamics g in the acceptable set $\mathbb{D}_{\mathsf{lim}}$, maintaining the SSB above B_{lim} from year to year will not be sufficient to ensure the existence of controls ensuring status quo. For example, in a stock with high abundance in the oldest age class and low abundances in the other age classes, SSB would be above B_{lim} but would be at high risk of falling below B_{lim} the subsequent year, whatever the fishing mortality, if recruitment is low.

Results

The PA is sustainable if, and only if,

$$\min_{B \in [B_{\mathsf{lim}}, +\infty[} \left[\min\left(\min_{a=1,\dots,A-1,\; \gamma_a w_a \neq 0} \left[\frac{\gamma_{a+1} w_{a+1}}{\gamma_a w_a} e^{-M_a} \right], \pi e^{-M_A} \right) B + \gamma_1 w_1 \varphi(B) \right] \geq B_{\mathsf{lim}} \tag{9}$$

i.e. if, and only if, the lowest possible sum of survivors (weighted by growth and maturation) and newly recruited spawning biomass is above B_{lim} (proof to be found in the Appendix).

Notice that, when $\gamma_1 = 0$ (the recruits do not reproduce) condition (9) is never satisfied (because $\pi e^{-M_A} < 1$) and the PA is not sustainable, whatever the value of B_{lim}. In other words, to keep SSB above B_{lim} for an indefinite time, it is not enough to keep it there from year to year. Other conditions based upon more indicators have to be checked.

We stress that condition (9) involves biological characteristics of the population (see Table 1) and the S-R relationship φ, as well as the threshold B_{lim}. However, it is important to note that condition (9) does not depend on the S-R relationship φ between 0 and B_{lim}. It does not depend on F_{lim} either.

If we suppose that the natural mortality is independent of age, i.e. $M_a = M$, and that the proportion γ_a of mature fish and weight w_a are increasing with age a, condition (9) becomes

$$\min_{B \in [B_{\mathsf{lim}}, +\infty[} \left[\pi e^{-M} B + \gamma_1 w_1 \varphi(B) \right] \geq B_{\mathsf{lim}} \, . \tag{10}$$

When, in addition, constant recruitment R is used, the PA is sustainable if, and only if, we have $\pi e^{-M} B_{\mathsf{lim}} + \gamma_1 w_1 R \geq B_{\mathsf{lim}}$, i.e. if, and only if,

$$R \geq \underline{R} \quad \text{where} \quad \underline{R} := \frac{1 - \pi e^{-M}}{\gamma_1 w_1} B_{\mathsf{lim}} \, , \tag{11}$$

making $\underline{R}$ the minimum recruitment required to preserve B_{lim}.

The previous condition is easy to understand when there is no plus group $\pi = 0$. Assuming a constant recruitment R and no plus group, the PA is sustainable if, and only if,

$$\gamma_1 w_1 R \geq B_{\mathsf{lim}}. \tag{12}$$

This means that, in the worst case where the whole population would spawn and die in a single time step, the resulting recruits would be able to restore the spawning biomass to the required level. This does not mean that longer-lived species that do not reproduce as recruits cannot be fished sustainably, but that SSB is not an indicator to monitor them safely and to ensure they will be maintained for more than one year.

Case studies

Our results are applied to two stocks with contrasting life histories (parameters given in Table 1), Bay of Biscay anchovy (*Engraulis encrasicolus*), a short-lived small pelagic fish, and northern hake (*Merluccius merluccius*), a longer-lived top predator. Both stocks are currently assessed by ICES as being at risk of reduced reproductive capacity (ICES, 2005b; ICES, 2005c). We examine here if this can be ascribed partly to the way management advice has been designed.

Bay of Biscay anchovy

Because the first age class of anchovy accounts for ca. 80% of SSB, the sustainability of the PA will depend on the relationship between the biomass reference point and the stock dynamics, mainly determined by the S-R relationship because there is no plus group. Assuming various S-R relationships, and taking $\pi = 0$ (as no plus group is present), we determine whether the PA based on the current value of B_{lim} is sustainable. The answer is given in the final column of Table 2. The second column contains an expression whose value is given in the third column, and has to be compared, according to condition (10), with the threshold in the fourth column.

Constant recruitment

Assuming a constant R, as is usual in stock projections, the PA is sustainable with R_{mean} (average over 1987-2004) or even the geometric mean R_{gm} of low R years (ICES, 2004). Actually, any other R above $\underline{R} \approx 1\ 312 \times 10^6$ fish defined in (11) will be sustainable. In 2004, however, there was a minimum historical R-value R_{min} of 696×10^6 fish, for which the PA is no longer sustainable.

Linear S-R *relationship*

Assuming a linear S-R relationship $\varphi(B) = rB$ with $r = f_b n_b s_r S_0$ as e.g. in a Leslie matrix model (ICES, 2005c), where batch fecundity $f_b = 500$ g^{-1}, number of batches per female per year $n_b = 21$, sex ratio $s_r = \frac{1}{2}$, egg survival to age 1 $S_0 = 10^{-5}$ (Methot, 1989; Motos, 1996), condition (10) becomes $\gamma_1 w_1 f_b n_b s_r S_0 \geq 1$. Egg survival S_0 to age 1 is highly variable. Assuming all other parameters to be known and constant, condition (10) would be satisfied if, and only if, $S_0 \geq 1.2 \times 10^{-5}$, which is not the case with the average value (over 20 y) $S_0 = 10^{-5}$ (Methot, 1989). For the given set of parameters, there is no B_{lim} value large enough for the PA to be sustainable, because the asymptotic growth rate of the population is less than 1. With $S_0 \geq 1.2 \times 10^{-5}$, the population growth rate would be ≥ 1 and the PA advice would be sustainable with any B_{lim}.

Ricker S-R *relationship*

Assuming a Ricker S-R relationship $\varphi(B) = aBe^{-bB}$, where B is measured in tonnes and with parameters $a = 0.79 \times 10^6$ and $b = 1.8 \times 10^{-5}$ (De Oliveira *et al.*, 2005), the PA is, strictly speaking, not sustainable. This is because the Ricker S-R relationship decreases towards zero when SSB is large: under this assumption, there is a risk in letting the stock grow too large.

This counter-intuitive result stems from the model allowing for unrealistically large numbers. Assuming a very large SSB accumulated in the oldest age class, recruitment would be close to zero, and the lowest possible sum of survivors could decrease below B_{lim} within a single year without any fishing mortality; this is unrealistic, but mathematically possible.

Northern hake

For hake, the PA is never sustainable, because $\gamma_1 = 0$ (see the discussion above).

Discussion

Because it is based on a single constraining indicator SSB, the PA appears to be sustainable only when the contribution of recruits to the spawning-stock is substantial. This is understandable because the PA advice relies upon a short-term perspective, which projects stock dynamics over the next year and does not consider longer-term dynamics. For stocks with a large contribution of recruits to spawning, sustainability of the PA then depends both on stock dynamics, mainly the assumed S-R relationship, and on the biomass reference point, without any need to constrain F below a reference point. The PA for Bay of Biscay anchovy is sustainable for constant recruitment as long as the value used is not too low. We stress that, in accordance with the ICES precautionary approach, we make no assumptions about stock dynamics below B_{lim}. However, the minimum observed recruitment R_{min} is always provisional. In 2002, R_{min} was $3\,964 \times 10^6$, which was sustainable. In 2004, recruitment decreased to 696×10^6, and the PA was no longer sustainable.

For stocks with a low contribution of recruits to spawning, the SSB-based PA is not sustainable. No SSB reference point would be high enough to prevent stock collapse in future years, because SSB as the sole indicator is not sufficient to manage the stock. This is because, as established by many empirical studies over the past decade (Marshall *et al.*, 1998; Murawski *et al.*, 2001; Marteinsdottir and Begg, 2002), SSB is not a sufficient index of the renewal capacity of a stock. In that case, the PA is not appropriate to ensure sustainable management within the acceptable set $\mathbb{D}_{\mathsf{lim}}$ given by (5). The viability approach can be used to examine how additional indicators could be included into the PA to make it sustainable.

We have demonstrated how the viability approach can be used to check the sustainability of a simple objective in relation to a simple single-stock dynamic model. This could be extended to more complex models and/or to different and mutliple objectives. For more complex models, the generalized results of the present study will hold (De Lara *et al.*, 2006) as long as the monotonicity assumption is verified (the more fishing mortality, the less stock at the next time step). For example, mixed fisheries models in which the dynamics of several stocks are linked by the joint pressure exerted by fishing fleets generally satisfy monotonicity properties. The monotonicity assumption will generally not be fulfilled in, for instance, multispecies models including trophic relationships, and adapted methods will have to be developed.

The viability approach provides a convenient tool to reconcile apparently contradictory objectives. It is reasonable for fisheries managers to require not only to preserve resources, but in addition, to be able to allow harvesting at any time. Another reasonable objective could be to provide a minimum yield every year. Our first computations indicate viability domains depending upon SSB among other indicators, partly justifying the PA, but with a more restricted set of options. This may be useful to develop policies aiming at restoring or maintaining stock at the maximum sustainable yield, as required by the Johannesburg summit (UN, 2002). Such a tool will also be useful in the development of an ecosystem approach to fisheries management where, inevitably, contradictory objectives will have to be considered (FAO, 2003; ICES, 2005a).

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References

Aubin, J-P. 1991. Viability Theory. Birkhäuser, Boston. 542 pp.

Béné, C., Doyen, L., and Gabay, D. 2001. A viability analysis for a bio-economic model. Ecological Economics, 36: 385–396.

Clarke, F. H., Ledayev, Y. S., Stern, R. J., and Wolenski, P. R. 1995. Qualitative properties of trajectories of control systems: a survey. Journal of Dynamical Control Systems, 1: 1–48.

Cury, P., Mullon, C., Garcia, S., and Shannon, L. J. 2005. Viability theory for an ecosystem approach to fisheries. ICES Journal of Marine Science, 62: 577–584.

De Lara, M., Doyen, L., Guilbaud, T., and Rochet, M-J. 2006. Monotonicity properties for the viable control of discrete time systems. Systems and Control Letters. In press.

De Oliveira, J. A., Uriarte, A., and Roel, B. A. 2005. Potential improvements in the management of Bay of Biscay anchovy by incorporating environmental indices as recruitment predictors. Fisheries Research, 75: 2–14.

Deriso, R., Quinn, T., Collie, J., Hilborn, R., Jones, C., Lindsay, B., Parma, A., Saila, S., Shapiro, L., Smith, S. J., and Walters, C. 1998. Improving fish stock assessments. 177 pp.

Doyen, L., and Béné, C. 2003. Sustainability of fisheries through marine reserves: a robust modeling analysis. Journal of Environmental Management, 69: 1–13.

Eisenack, K., Sheffran, J., and Kropp, J. 2006. The viability analysis of management frameworks for fisheries. Environmental Modelling and Assessment, 11: 69–79.

FAO. 1999. Indicators for sustainable development of marine capture fisheries. FAO Technical Guidelines for Responsible Fisheries, 8. 68 pp.

FAO. 2003. The ecosystem approach to fisheries. FAO Technical Guidelines for Responsible Fisheries, 4 (Suppl. 2). 112 pp.

Hilborn, R., and Walters, C. F. 1992. Quantitative Fisheries Stock Assessment. Choice, Dynamics and Uncertainty. Chapman and Hall, New York. 570 pp.

ICES. 2004. Report of the ICES Advisory Committee on Fishery Management and Advisory Committee on Ecosystems, 2004. ICES Advice, 1. 1544 pp.

ICES. 2005a. Ecosystem effects of fishing: impacts, metrics, and management strategies. ICES Cooperative Research Report, 272. 177 pp.

ICES. 2005b. Report of the Working Group on the Assessment of Hake, Monk and Megrim (WGHMM). ICES Document CM 2006/ACFM: 01. 875 pp.

ICES. 2005c. Report of the Working Group on the Assessment of Mackerel, Horse Mackerel, Sardine and Anchovy (WGMHSA). ICES Document CM 2006/ACFM: 08. 615 pp.

Marshall, C. T., Kjesbu, O. S., Yaragina, N. A., Solemdal, P., and Ulltang, Ø. 1998. Is spawner biomass a sensitive measure of the reproductive and recruitment potential of Northeast Arctic cod? Canadian Journal of Fisheries and Aquatic Sciences, 55: 1766–1783.

Marteinsdottir, G., and Begg, G. A. 2002. Essential relationships incorporating the influence of age, size and condition on variables required for estimation of reproductive potential in Atlantic cod *Gadus morhua*. Marine Ecology Progress Series, 235: 235–256.

Methot, R. 1989. Synthetic estimates of historical abundance and mortality of northern anchovy. American Fisheries Society Symposium, 6: 66–82.

Motos, L. 1996. Reproductive biology and fecundity of the Bay of Biscay anchovy population (*Engraulis encrasicolus, L.*). Scientia Marina, 60: 195–207.

Murawski, S. A., Rago, P. J., and Trippel, E. A. 2001. Impacts of demographic variation in spawning characteristics on reference points for fishery management. ICES Journal of Marine Science, 58: 1002–1014.

Quinn, T. J., and Deriso, R. B. 1999. Quantitative Fish Dynamics. Biological Resource Management Series. Oxford University Press, New York. 542 pp.

Rapaport, A., Terreaux, J-P., and Doyen, L. 2006. Sustainable management of renewable resource: a viability approach. Mathematics and Computer Modeling, 43: 466–484.

UN. 2002. Report of the World Summit on Sustainable Development Johannesburg, South Africa, 26 August-4 September 2002. Technical report A/CONF.199/20*. 167 pp.

Table 1. Parameter definitions and values for two case studies (ICES, 2005b; ICES, 2005c). RP denotes reference point.

Definition	Notation	Anchovy	Hake
Maximum age	A	3	8
Mean weight-at-age (kg)	$(w_a)_a$	$(16,\ 28,\ 36) \times 10^{-3}$	(0.126, 0.2, 0.319, 0.583, 0.986, 1.366, 1.748, 2.42)
Maturity ogive	$(\gamma_a)_a$	(1, 1, 1)	(0, 0, 0.23, 0.60, 0.90, 1, 1, 1)
Natural mortality	M	1.2	0.2
F-at-age	$(F_a)_a$	(0.4, 0.4, 0.4)	(0, 0, 0.1, 0.25, 0.22, 0.27, 0.42, 0.5, 0.5)
Presence of plus group	π	0	1
F precautionary RP	F_{pa}	$1 - 1.2$	0.25
SSB precautionary RP (t)	B_{pa}	33 000	140 000
F limit RP	F_{lim}	/	0.35
SSB limit RP (t)	B_{lim}	21 000	100 000

Table 2. Bay of Biscay anchovy: sustainability of advice based on the SSB indicator for various S-R relationships. The answer is given in the final column of the table.

S-R relationship	Condition	Parameter values	Threshold	Sustainable?
Constant (mean)	$R_{\mathsf{mean}} \geq \underline{R}$	$14\ 016\ \times 10^6$	$1\ 312\ \times 10^6$	Yes
Constant (geometric mean)	$R_{\mathsf{gm}} \geq \underline{R}$	$7\ 109\ \times 10^6$	$1\ 312\ \times 10^6$	Yes
Constant (2002)	$R_{2002} \geq \underline{R}$	$3\ 964\ \times 10^6$	$1\ 312\ \times 10^6$	Yes
Constant (2004)	$R_{2004} \geq \underline{R}$	$696\ \times 10^6$	$1\ 312\ \times 10^6$	No
Linear	$\gamma_1 w_1 r \geq 1$	0.84	1	No
Ricker	$\min_{B \geq B_{\mathsf{lim}}} [\cdots] \geq B_{\mathsf{lim}}$	0	21 000	No

Appendix: proof

Proof is given here for a slightly more general problem with minimum-F objectives. To avoid having closing the fishery at any time, a lower bound $F_{\mathsf{min}} \geq 0$ could be set for F in addition to the upper bound, and the corresponding acceptable set is

$$\mathbb{D}_{\text{effort},\mathsf{lim}} = \{(N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid \text{SSB}(N) \geq B_{\mathsf{lim}} , \quad F_{\mathsf{min}} \leq F(\lambda) \leq F_{\mathsf{lim}}\}. \tag{13}$$

We assume that $F_{\mathsf{min}} \leq F_{\mathsf{lim}}$, so that $\mathbb{D}_{\text{effort},\mathsf{lim}}$ is not empty. F_{min} corresponds, by (4), to a fishing mortality multiplier $\lambda_{\mathsf{min}} \geq 0$, such that $F(\lambda_{\mathsf{min}}) = F_{\mathsf{min}}$. The constraint $F(\lambda) \leq F_{\mathsf{lim}}$ is thus replaced by $\lambda \leq \lambda_{\mathsf{min}}$ in the sequel.

We shall coin sustainability of the PA in the minimum-F sense as the property that the set $\mathbb{V}_{\mathsf{lim}}$ provides a viability domain associated with dynamics g in the desirable set $\mathbb{D}_{\mathsf{effort,lim}}$. If this viability domain is not empty, F-multipliers larger than the minimum required λ_{min} can keep biomass above B_{lim} at any time, meaning that the fishery should never have to be closed.

Define the minimal survival coefficient as

$$\Theta_{\mathsf{min}}(\lambda) := \min\left(\min_{a=1,\dots,A-1,\ \gamma_a w_a \neq 0} \left[\frac{\gamma_{a+1} w_{a+1}}{\gamma_a w_a} e^{-M_a - \lambda F_a} \right], \quad \pi e^{-M_A - \lambda F_A} \right). \tag{14}$$

The PA is sustainable in the minimum-F sense if, and only if,

$$\min_{x \in [B_{\mathsf{lim}}, +\infty[} \left[\Theta_{\mathsf{min}}(\lambda_{\min}) B + \gamma_1 w_1 \varphi(B) \right] \geq B_{\mathsf{lim}}. \tag{15}$$

Again, this is the lowest expected spawning biomass at next time step.

We prove (15), which includes condition (9) as a particular case for $\lambda_{\mathsf{min}} = 0$.

Step 1. Recall that the PA is said to be sustainable in the minimum-F sense if the set $\mathbb{V}_{\mathsf{lim}}$ is a viability domain associated with dynamics g in the desirable set $\mathbb{D}_{\mathsf{effort,lim}}$, i.e. if, and only if,

$$\mathrm{SSB}(N) \geq B_{\mathsf{lim}} \Rightarrow \exists \lambda \in \mathbb{R}_+, \quad F(\lambda) \leq F_{\mathsf{lim}} \text{ and } \lambda_{\mathsf{min}} \leq \lambda \text{ and } \mathrm{SSB}(g(N,\lambda)) \geq B_{\mathsf{lim}}. \tag{16}$$

As the dynamics g in (2) decrease with λ, we only have to check the previous condition for the lowest $\lambda = \lambda_{\min}$ (recall that $F(\lambda_{\mathsf{min}}) \leq F_{\mathsf{lim}}$). Therefore, (16) is equivalent to

$$\mathrm{SSB}(N) \geq B_{\mathsf{lim}} \Rightarrow \mathrm{SSB}(g(N, \lambda_{\mathsf{min}})) \geq B_{\mathsf{lim}}. \tag{17}$$

Defining

$$v_{\mathsf{min}}(B_{\mathsf{lim}}, \lambda_{\mathsf{min}}) := \min \left\{ \mathrm{SSB}(g(N, \lambda_{\mathsf{min}})) \,\middle|\, \begin{array}{l} N \in \mathbb{R}_+^A \\ \mathrm{SSB}(N) \geq B_{\mathsf{lim}} \end{array} \right\}, \tag{18}$$

condition (17) is equivalent to $v_{\mathsf{min}}(B_{\mathsf{lim}}, \lambda_{\mathsf{min}}) \geq B_{\mathsf{lim}}$.

Step 2. For any $\lambda \geq 0$, we denote by $T(\lambda)$, the square matrix that defines the linear part of the dynamics g in (2), i.e.

$$T(\lambda) := \begin{pmatrix} 0 & 0 & \dots & 0 & 0 & 0 \\ e^{-M_1 - \lambda F_1} & 0 & \dots & 0 & 0 & 0 \\ 0 & e^{-M_2 - \lambda F_2} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & e^{-M_{A-2} - \lambda F_{A-2}} & 0 & 0 \\ 0 & 0 & \dots & 0 & e^{-M_{A-1} - \lambda F_{A-1}} & \pi e^{-M_A - \lambda F_A} \end{pmatrix}$$

such that

$$\begin{pmatrix} g_1(N,\lambda) \\ g_2(N,\lambda) \\ \vdots \\ g_A(N,\lambda) \end{pmatrix} = T(\lambda)N + \begin{pmatrix} \varphi(\text{SSB}(N)) \\ 0 \\ \vdots \\ 0 \end{pmatrix} . \tag{19}$$

Let us introduce the vectors

$$\beta := \begin{pmatrix} \gamma_1 w_1 \\ \vdots \\ \gamma_A w_A \end{pmatrix} \quad \text{and} \quad \alpha := T(\lambda_{\mathsf{min}})'\beta = \begin{pmatrix} e^{-M_1-\lambda_{\mathsf{min}}F_1}\gamma_2 w_2 \\ \vdots \\ e^{-M_{A-1}-\lambda_{\mathsf{min}}F_{A-1}}\gamma_A w_A \\ \pi e^{-M_A-\lambda_{\mathsf{min}}F_A}\gamma_A w_A \end{pmatrix} , \tag{20}$$

where $T(\lambda_{\mathsf{min}})'$ is the transpose matrix of $T(\lambda_{\mathsf{min}})$. For all $N \in \mathbb{R}_+^A$, (19) and (3) give

$$\text{SSB}(g(N,\lambda_{\mathsf{min}})) = \langle T(\lambda_{\mathsf{min}})'\beta, N\rangle + \gamma_1 w_1 \varphi(\text{SSB}(N)) = \langle \alpha, N\rangle + \beta_1\varphi(\text{SSB}(N)) , \tag{21}$$

where $\langle \alpha, N\rangle$ is the scalar product between vectors α and N. By splitting the optimization problem (18) into two parts, we obtain

$$\begin{aligned}
v_{\mathsf{min}}(B_{\mathsf{lim}},\lambda_{\mathsf{min}}) &:= \min\left\{ \text{SSB}(g(N,\lambda_{\mathsf{min}})) \,\middle|\, \begin{array}{l} N \in \mathbb{R}_+^A \\ \text{SSB}(N) \geq B_{\mathsf{lim}} \end{array} \right\} \\
&= \min\left\{ \text{SSB}(g(N,\lambda_{\mathsf{min}})) \,\middle|\, \begin{array}{l} N \in \mathbb{R}_+^A \\ \text{SSB}(N) = B,\ B \geq B_{\mathsf{lim}} \end{array} \right\} \\
&= \min_{B \geq B_{\mathsf{lim}}} \left\{ \text{SSB}(g(N,\lambda_{\mathsf{min}})) \,\middle|\, \begin{array}{l} N \in \mathbb{R}_+^A \\ \text{SSB}(N) = B \end{array} \right\} \\
&= \min_{B \geq B_{\mathsf{lim}}} \left[\min\left\{ \langle \alpha, N\rangle \,\middle|\, \begin{array}{l} N \in \mathbb{R}_+^A \\ \text{SSB}(N) = B \end{array} \right\} + \beta_1\varphi(B) \right] \quad \text{by (21)} \\
&= \min_{B \geq B_{\mathsf{lim}}} \left[\omega_{\alpha,\beta}(B) + \beta_1\varphi(B) \right] ,
\end{aligned}$$

where $\omega_{\alpha,\beta}(B)$ is defined in (22) in Step 3. Setting

$$\Theta_{\mathsf{min}}(\lambda_{\mathsf{min}}) = \min_{a,\ \beta_a \neq 0} \frac{\alpha_a}{\beta_a} = \min\left(\min_{a=1,\ldots,A-1,\ \beta_a\neq 0} \left[\frac{\beta_{a+1}}{\beta_a} e^{-M_a-\lambda_{\mathsf{min}}F_a} \right], \quad \pi e^{-M_A-\lambda_{\mathsf{min}}F_A} \right) ,$$

we use Step 3 to obtain

$$v_{\mathsf{min}}(B_{\mathsf{lim}},\lambda_{\mathsf{min}}) = \min_{B \geq B_{\mathsf{lim}}} \left[\Theta_{\mathsf{min}}(\lambda_{\mathsf{min}})B + \beta_1\varphi(B) \right] .$$

Recalling that β is given by (20), this gives condition (15).

Step 3. We prove that, for any $\alpha \in \mathbb{R}_+^A$, $\beta \in \mathbb{R}_+^A \backslash \{0\}$ and $s \in \mathbb{R}_+$, we have

$$\omega_{\alpha,\beta}(s) := \min_{\langle \beta, N \rangle = s,\ N \in \mathbb{R}_+^A} \langle \alpha, N \rangle = \left(\min_{a,\ \beta_a \neq 0} \frac{\alpha_a}{\beta_a} \right) s. \tag{22}$$

Let $N \in \mathbb{R}_+^A$ be such that $\langle \beta, N \rangle = s$. We have

$$\begin{aligned}
\langle \alpha, N \rangle &= \sum_{a=1}^{A} \alpha_a N_a \\
&\geq \sum_{a, \beta \neq 0} \frac{\alpha_a}{\beta_a} \beta_a N_a \ \text{ since } \ \alpha_a N_a \geq 0 \\
&\geq \left(\min_{a,\ \beta_a \neq 0} \frac{\alpha_a}{\beta_a} \right) \sum_{a,\ \beta_a \neq 0} \beta_a N_a \\
&= \left(\min_{a,\ \beta_a \neq 0} \frac{\alpha_a}{\beta_a} \right) \sum_{a=1}^{A} \beta_a N_a \\
&= \left(\min_{a,\ \beta_a \neq 0} \frac{\alpha_a}{\beta_a} \right) \langle \beta, N \rangle \\
&= \left(\min_{a,\ \beta_a \neq 0} \frac{\alpha_a}{\beta_a} \right) s\,.
\end{aligned}$$

Let $a^\sharp \in \{1, \ldots, A\}$ be such that

$$\min_{a,\ \beta_a \neq 0} \frac{\alpha_a}{\beta_a} = \frac{\alpha_{a^\sharp}}{\beta_{a^\sharp}}\,.$$

The vector $N^\sharp$, defined by

$$N^\sharp_{a^\sharp} = \frac{s}{\beta_{a^\sharp}} \qquad \text{and} \qquad N^\sharp_a = 0 \ \text{ if } \ a \neq a^\sharp\,,$$

is such that $\langle \beta, N^\sharp \rangle = s$ and $\langle \alpha, N^\sharp \rangle = \frac{\alpha_{a^\sharp}}{\beta_{a^\sharp}} s$. Hence, $N^\sharp$ achieves the lower bound for $\langle \alpha, N \rangle$ established above. Therefore, (22) holds true.