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# An inverse estimate of the dynamic topography of the ocean

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## Abstract:

The mean dynamic topography of the surface of the ocean is estimated using a finite difference inverse model of the ocean circulation constrained by a prior TOPEX/Poseidon-EGM96 estimate and an ocean density climatology. The EGM96 geoid uncertainties are represented by their full covariance matrix. The model finds a solution that yields a new 1°-resolution dynamic topography estimate with a 5 cm precision. The existence of this solution suggests that the uncertainties in EGM96, despite being much smaller than in previous geopotential models, are nonetheless realistic. The geodetic information constrains the inverse estimate of the ocean circulation, particularly at high latitudes.

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# An inverse estimate of the dynamic topography of the ocean

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[1] The mean dynamic topography of the surface of the ocean is estimated using a finite difference inverse model of the ocean circulation constrained by a prior TOPEX/Poseidon-EGM96 estimate and an ocean density climatology. The EGM96 geoid uncertainties are represented by their full covariance matrix. The model finds a solution that yields a new 1°-resolution dynamic topography estimate with a 5 cm precision. The existence of this solution suggests that the uncertainties in EGM96, despite being much smaller than in previous geopotential models, are nonetheless realistic. The geodetic information constrains the inverse estimate of the ocean circulation, particularly at high latitudes. *INDEX TERMS:* 1227 Geodesy and Gravity: Planetary geodesy and gravity (5420, 5714, 6019); 4512 Oceanography: Physical: Currents; 1243 Geodesy and Gravity: Space geodetic surveys; 4536 Oceanography: Physical: Hydrography. **Citation:** LeGrand, P., E. J. O. Schrama, and J. Tournadre, An inverse estimate of the dynamic topography of the ocean, *Geophys. Res. Lett.*, *30*(0), XXXX, doi:10.1029/2002GL014917, 2003.

## 1. Introduction

[2] The limiting factor in determining the steady component of the large-scale ocean circulation from altimetric data is the low precision of currently available geoid height models. Indeed, in the absence of a precise geoid model, the contribution of the quasi-permanent ocean circulation cannot be separated from the contribution of geoid undulations. The GOCE mission (Gravity Field and Steady-State Ocean Circulation Explorer, www.esa.int/export/esaLP/goce.html), scheduled for launch by the European Space Agency in 2006, will yield much improved estimates of the geoid height, but in the mean time one must rely on existing data sets. The on-going CHAMP mission (CHAllenging Minisatellite Payload, http://op.gfz-potsdam.de/champ), managed by the GeoForschungsZentrum in Postdam, and the GRACE mission (Gravity Recovery And Climate Experiment, http://essp.gsfc.nasa.gov/grace), directed by the National Aeronautics and Space Administration are not intended to resolve the short spatial scales (~100 km) of the gravity field. They will thus need to be completed with oceanographic information if the very energetic oceanic currents associated with these spatial scales are to be determined. Furthermore, the application of GOCE data to the estimation of the ocean circulation will also require the combination of geodetic information with oceanographic information in order to constrain the deep layers of the ocean. Such a combination is carried out here, with the objectives of 1) producing an estimate of the mean dynamic topography that can be used to correct existing models of the marine geoid for oceanographic applications and 2) preparing future analyses of satellite gravity observations in terms of ocean circulation.

[3] Parallel modeling efforts that combine geodetic and oceanographic information are underway in the geodetic community (http://bowie.gsfc.nasa.gov/926/PGM2000A). The geodetic approach combines a prior estimate of dynamic topography derived from an ocean general circulation model with satellite tracking data, surface gravity data, and altimeter data. The resulting geopotential solution is consistent with the geodetic constraints but there is no guarantee that it is consistent with ocean dynamical constraints since these constraints are not explicitly imposed. In contrast, in the approach implemented here a prior geodetic estimate of dynamic topography is calculated using altimeter data and a geoid height model derived from a geopotential model. This prior estimate is updated to make it consistent with in situ oceanic observations and the dynamical constraints relevant to the large-scale circulation of the ocean. In the present approach, the consistency of the solution with ocean dynamics is explicitly imposed and can be verified *a posteriori*. The consistency of the solution with the normal equations from satellite tracking data cannot be verified, however, since these equations are not explicitly used. Thus, the oceanographic and geodetic approaches use similar information but, in practice, proceed in a different order and may yield different results.

[4] The ocean inverse modeling approach first proposed by *Roemmich and Wunsch* [1982] is used here with the finite difference implementation of *LeGrand et al.* [1998]. The global ocean is covered, except the Arctic Ocean, semi-enclosed seas like the Mediterranean Sea, and continental shelf areas. The spatial resolution is set to 1°, corresponding to a spherical harmonic expansion up to degree and order 180. This resolution ensures the presence of at least 2 gridpoints across strong oceanic density fronts such as the one associated with the Kurushio. Note that the problem of

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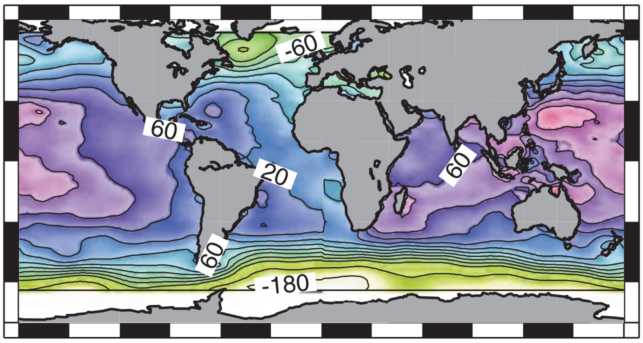

**Figure 1.** Prior estimate of mean dynamic topography (color scale indicated in Figure 2).

omission errors is circumvented here because the inverse model and the satellite prior constraint on dynamic topography are implemented at the same resolution and omit the same spatial scales.

[5] The next section briefly describes the Laboratoire de Physique des Océans (LPO) inverse model used in this work and the treatment of observational and modeling uncertainties. The third section presents the inverse estimate of the mean ocean dynamic topography. Finally, the last section discusses the impact of the geoid height model on the inverse solution.

## 2. The Finite Difference Inverse Model

[6] The LPO inverse model has been extensively described in previous papers [*Mercier et al.*, 1993; *LeGrand et al.*, 1998], thus only its main features are presented here. This model searches for an ocean circulation that best fits, in a least-squares sense, dynamical and observational constraints. This best-fit is found by minimizing a cost function that is the sum of a dynamical imbalance term (mass imbalances in pre-defined volume elements, for instance), weighted by the inverse of the covariance of the model errors, and a data misfit term, weighted by the inverse of the covariance of the observational uncertainties [*Mercier et al.*, 1993].

[7] The dynamical constraints consist of mass conservation, the thermal wind balance in the interior of the ocean, and the geostrophic balance at the surface. These constraints are applicable to the large-scale ocean circulation to a very good approximation [*Pond and Pickard*, 1983]. The mass conservation constraints are implemented in each volume element defined horizontally by the 1° finite difference grid of the inverse model and vertically by the whole water column. Large-scale mass conservation is also imposed in boxes such as the one defined by the South Atlantic basin delimited by the Drake Passage, the South African "choke point" in the Circumpolar Current, and the zonal section linking Africa to South America at 2°S. Non-geostrophic Ekman transports, which are caused by frictional stress exerted by the wind in the upper layers of the ocean, are calculated using the ERS wind stress products (www.ifremer.fr/cersat). These transports are taken into account in the mass conservation equations as given information; they are not adjusted in the inverse calculation.

[8] The observational constraint on dynamic topography is provided by the satellite estimate derived by Chambers using the EGM96 model [*Lemoine et al.*, 1998] and the mean of 5 years of TOPEX/Poseidon (T/P) sea surface height data (ftp://ftp.csr.utexas.edu/pub/sst/sstgrid.tp_5yr_avg.egm96). The geodetic information contained in the EGM96 model is therefore implicitly taken into account in the inverse calculation. The covariance of this prior topography estimate is derived from the sum of the full covariance matrix of the errors in the EGM96 geoid height (spherical harmonic expansion to degree 180) and the variance of the mean sea surface height. The covariance of EGM96 dominates the uncertainty in the prior topography estimate with standard deviations ranging from 5 cm at high latitudes to 25 cm near the equator. The sea surface height variance is globally set to $(5\ \text{cm})^2$ to account for measurement errors and more importantly for interpolation errors from T/P tracks to model grid. Figure 1 shows that the altimetric/geoid prior estimate of dynamic topography is too smooth to be fully consistent with the 1° resolution of the inverse model. However, it is assumed here that the difference between the present estimate and an estimate actually containing signal down to 1° would be within the uncertainties assigned to the present estimate. Because of the high precision of the low spherical harmonic coefficients of EGM96, the prior estimate of dynamic topography is quite precise at large spatial scales. This information is passed on to the inverse model through the EGM96 covariance matrix, which describes geoid errors at all spatial scales.

[9] Hydrographic observations provide the other observational constraint. These observations consist of a gridded estimate of the mean density field calculated from the temperature and salinity fields derived by Gouretski and Jancke from a large compilation of historical and recent hydrographic data (http://www.dkrz.de/~u241046/SACserver/SACHome.htm). Uncertainties in the climatological density field are assumed to be horizontally homogeneous but to vary vertically according to an *ad-hoc* profile decreasing from 1 $\text{kg/m}^3$ at the surface of the ocean down to 0.02 $\text{kg/m}^3$ at the bottom. This profile was derived from the variance of vertical profiles of density in 1° bins using the dense North Atlantic historical database. Accounting for horizontal variations of the uncertainties would be more rigorous [*LeGrand et al.*, 1998] but this information is not available (only mapping errors are provided to the users of the climatology).

## 3. Estimated Mean Dynamic Topography

[10] An ocean circulation, and the corresponding mean dynamic topography (Figure 2), that is consistent with the observational constraints and the dynamical constraints is found by the inverse model. In the North Atlantic, many known features such as the Gulf Stream and the associated recirculation, the North Atlantic Current, and the Azores Front are apparent. In the South Atlantic, the Brazil Current and the Falkland Current, the Benguela Current west of South Africa, and the broad return flow that closes the surface circulation of the subtropical gyre in the South Atlantic are resolved. The Antarctic Circumpolar Current (ACC) is evidenced by an area of strong meridional gradients in the map of mean dynamic topography. The Kurushio Current off the coast of Japan partly intersects the coastline, but this problem is attributable to the isolines of

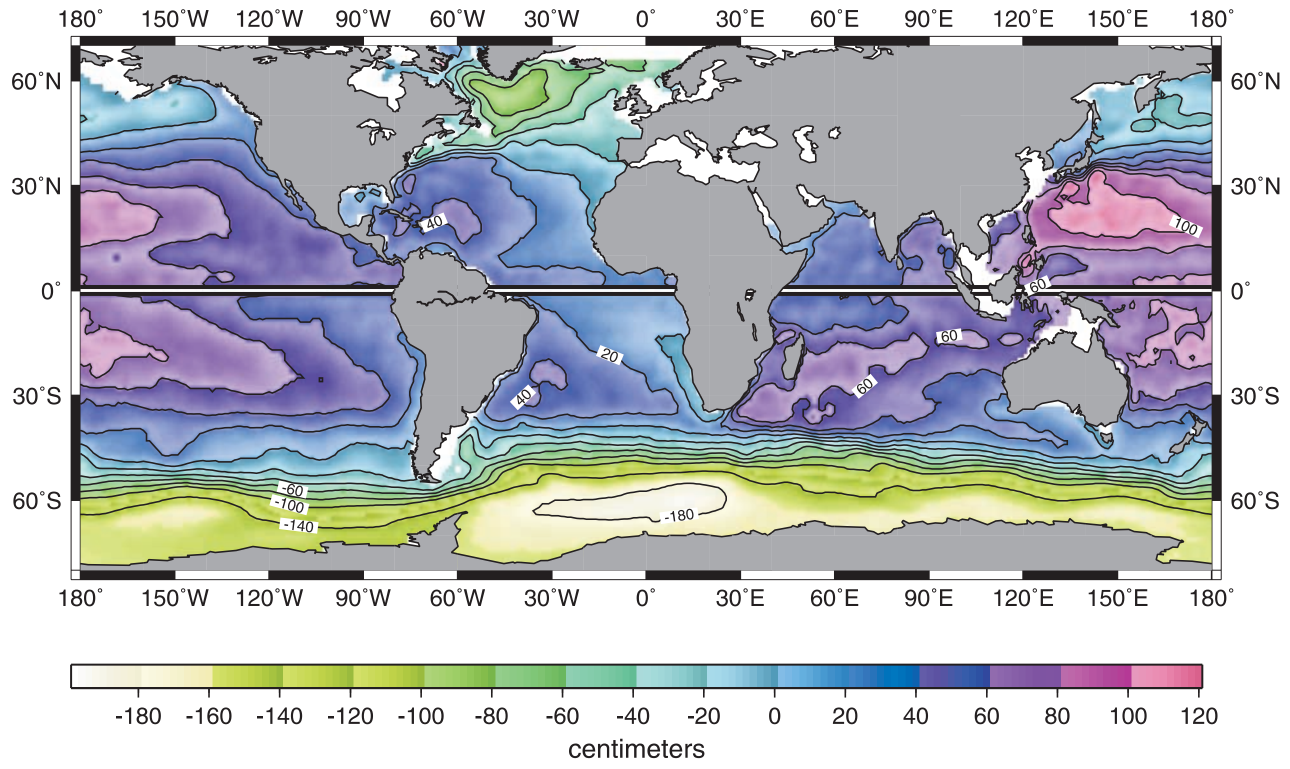

**Figure 2.** Mean dynamic topography estimated over the 1° × 1° grid of the inverse model.

the climatological density field, which are too zonal in this region. Because the geostrophic and thermal wind balances break down near the equator, no attempt is made to estimate dynamic topography between ±1° (the southern and northern hemispheres are treated separately in the inverse calculation). In summary, except for relatively minor and local problems, the inverse estimate of dynamic topography appears to be qualitatively consistent with previous oceanographic knowledge. It is beyond the scope of this paper to fully assess the quantitative details of the inverse solution, but the assessment of a similar estimate in the Atlantic Ocean [*LeGrand*, 2001] was quite conclusive. For instance, the transport of the Gulf Stream at 60°W is on the order of $60 \times 10^6 \mathrm{m}^3/\mathrm{s}$ above 1 km depth, which is consistent with climatological values published in the literature [*Schmitz and McCartney*, 1993].

[11] The constraints imposed in the inverse calculation are satisfied within the error bars that were specified prior to inversion (error bars interpreted as standard deviations), except for the geostrophic constraint on surface velocities in a small region near Indonesia. However, the effect of the complex bathymetry of this region on the circulation cannot be represented with the simplified dynamics of the inverse model, and moreover this region is too close to the equator for the discrepancy to be considered as a serious problem.

[12] The mean dynamic topography estimated by the inverse calculation is a robust quantity. It varies little when assumptions about reference velocities in the deep ocean or model errors are modified. This robustness is explained by the fact that dynamic topography is determined from the altimetric/geodetic constraints at large spatial scales and from the integral of the density field over a large depth range at small spatial scales [*LeGrand et al.*, 1998]. In particular, the integration of density from the reference level to the surface of the ocean tends to filter out errors present at individual depth levels. Unlike prognostic general circulation model estimates, the inverse estimate is little affected by systematic model errors since it is mainly constrained by the observations.

[13] The uncertainties can be calculated as part of the least-squares inverse procedure using the prescribed model errors and observational errors. Model errors are difficult to quantify, but they do not seem critical. Indeed, multiplying or dividing these errors by 2 changes the uncertainty in mean dynamic topography by 20% only. Observational errors are estimated directly from the data, and despite some crude approximations are accurate in magnitude. After inversion, the geoid height uncertainty is on the order of 5 cm, the range of values being 3 cm to 6 cm in the northern hemisphere. Because the geographical variations of the uncertainties in the climatology of the density field are not available to us, the error bars on dynamic topography provided here can only be interpreted as amplitude estimates, and could in reality vary over a slightly larger range.

[14] Because the strongest observational constraint in the inverse calculation is the climatological estimate of the density field, which is an average of a large number of hydrographic data collected since the 1930's, the present estimate of dynamic topography is best understood as an estimate of the mean over most of the 20th century. That strong oceanic signals can still be seen despite the averaging illustrates the permanence of some oceanic signals. One should not interpret, however, this permanent signal as actual currents. Indeed, the signal in the Gulf Stream region, for instance, corresponds to the envelope within which the

current meanders, rather than the intensity of the actual current at any one time. The mean dynamic topography estimate obtained here is made available on ftp://hocus.geo.tudelft.nl/pub/ejo/dyntop. This estimate can easily be converted into a correction of the EGM96 geoid by subtracting it from the Chambers prior EGM96-T/P estimate.

## 4. Impact of the Prior Estimate of Dynamic Topography on the Inverse Calculation

[15] The inverse calculation indicates that there is quantitative consistency, within the given errors, between oceanographic knowledge and the altimetric/geodetic constraint derived from the EGM96 geopotential model. This consistency is obtained despite the fact that the uncertainties provided with this model are much smaller than the uncertainties in other geoid models, by a factor of 2 to 3 for spherical harmonic degrees above degree 8 when compared to the JGM-3 uncertainties for instance [*Lemoine et al.*, 1998]. Therefore, we conclude that the uncertainties provided with EGM96 are realistic over the ocean and can be used as such in oceanographic studies.

[16] As noted by *LeGrand et al.* [1998], the inverse estimate of mean dynamic topography exhibits a proper separation of the Gulf Stream at Cape Hatteras (Figure 2), which is an improvement over the altimetry/geoid prior estimate (Figure 1). In other areas, it is the prior satellite estimate of dynamic topography that improves the circulation obtained from hydrographic constraints only, especially at high latitudes where the uncertainties in the EGM96 geoid height model are relatively small. Between Antarctica and South Africa, for instance, the prior T/P-EGM96 estimate implies a larger south-north dynamic topography difference across the ACC than the estimate derived from hydrography only. This, in turn, implies larger surface velocities, and thus results in an increased inverse estimate of the transport across the section. The integrated geostrophic transport is also increased across the Drake Passage, to about $110 \times 10^6 m^3/s$, as a consequence of large-scale volume conservation. This increase corrects the unrealistically small value mentioned by *LeGrand* [2001] for a calculation based on geoid height errors that were unrealistically large at high latitudes (the geoid errors were assumed to be homogeneous over the ocean).

[17] The results of the present study illustrate how the combination of geodetic and oceanographic information can improve our knowledge of the ocean circulation. This combined approach will reach its full potential when highly precise observations provided by forthcoming space gravity missions become available.

[18] **Acknowledgments.** N. Pavlis provided valuable assistance in validating the calculation of the EGM96 geoid uncertainties. This study was partially funded by the Earth Oriented Research program of the Technical University of Delft (DEOS). PLG thanks R. Klees for having organized his stay in the Department of Geodesy at TU Delft, stay during which this work was completed. The inverse calculations were carried out on the NEC SX5 of the French IDRIS center for high performance computing. M. Guyon parallelized the inverse model, which greatly speeded up the calculations.

## References

LeGrand, P., H. Mercier, and T. Reynaud, Combining T/P altimetric data with hydrographic data to estimate the mean dynamic topography of the North Atlantic and improve the geoid, *Ann. Geophysicae*, *16*, 638–650, 1998.

LeGrand, P., Impact of the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) mission on ocean circulation estimates. Volume fluxes in a climatological inverse model of the Atlantic, *J. Geophys. Res.*, *106*, 19,597–19,610, 2001.

Lemoine, F. G., et al., The development of the joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) geopotential model EGM96, *NASA Technical Publication-1998-206861*, NASA Center for Aerospace Information, Hanover, MD, 1998.

Mercier, H., M. Ollitrault, and P.-Y. Le Traon, An inverse model of the North Atlantic general circulation using Lagrangian float data, *J. Phys. Oc.*, *23*, 689–715, 1993.

Pond, S., and G. L. Pickard, *Introductory dynamical oceanography*, 329 pp., Pergamon Press, Oxford, 1983.

Roemmich, D., and C. Wunsch, On combining satellite altimetry with hydrographic data, *J. Mar. Res.*, *40*, 605–619, 1982.

Schmitz, W. J., Jr., and M. McCartney, On the North Atlantic circulation, *Review of Geophysics*, *31*, 29–49, 1993.

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