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Statistical Analysis of Low Frequency Motions of Floating Bodies in Shallow Water

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ABSTRACT

In this study we discuss various methods to assess the statistics of extreme response of a floating structure subjected to random waves. This issue is of great importance in the case of low frequency motion of floating bodies in shallow water. Those methods are based on the evaluation of the level crossing rate, which is easily related to the extreme value statistics for stationary random processes and large levels. Four main methods are considered, namely the Rice formula with either a Gaussian assumption or an approximation of the joint distribution of the response and its first derivative by a projection method; the Breitung's formula; the Hagberg's approaches using either an asymptotic expansion or a Monte-Carlo integration and finally the time domain simulation which serves as the reference solution. All these methods are applied to the surge motion of a floating body subjected to unidirectional waves. In that case, the Rice formula under the Gaussian assumption strongly underestimates the level crossing rate. The other methods are more or less accurate compared to the time domain simulation, but Hagberg's Monte-Carlo integration is shown to give the best approximation.

KEY WORDS: extreme response; crossing rate; second-order wave loading; shallow water; low frequency motion.

INTRODUCTION

In the context of mooring design and analysis of offshore operations in shallow water, an accurate prediction of the motions of the floating bodies is of great importance. For that purpose a dynamic structural analysis is necessary to compute the wave-induced structural response. The traditional approach to perform dynamic analysis of a structure is to work in the time domain and solve the motion's equations. However, in the offshore environment where the waves have random properties, that solution scheme becomes limited since only a single instance of the motion is generated. Of course the time-domain solution scheme can be repeated for various random instances of the loading and the statistics of the response can be estimated. Unfortunately, such an approach is often very time consuming and thus impractical. It is therefore interesting to look at various alternative methods to assess the statistical distribution of the structural response to the random waves.

In this paper we are interested in the low frequency loadings, which generate the largest amplitudes of the structural resonant behaviour. Low frequency loading is recognized to originate from the second order approximation of the wave forces (Pinkster, 1960). In this study, the sea state condition is represented by a stationary Gaussian random process. Then, the second-order approximation of the structural response is extracted using a quadratic filter. The resulting structural motion is a stochastic process, the properties of which are to be determined, namely the probability density function of the extreme values.

In practice, the assessment of the extreme value distribution is based on the evaluation of the level crossing rate which is simply related to the extreme value statistics for stationary random process and large level:

$$\Pr(M_T > \beta) \cong T\mu^+(\beta) \tag{1}$$

where T is the time duration, $\mu^+(\beta)$ is the up-crossing rate of level β and $M_T = \sup_{0\le t\le T}\nu(t)$ ($\nu(t)$ denotes a stationary random process). Moreover, the up-crossing rate $\mu^+(\beta)$ is half $\mu(\beta)$, the crossing rate of the level β by the stationarity of the response process. Therefore, the distribution of the extreme response can be easily derived from the computation of the level crossing rate $\mu(\beta)$.

Four main methods are considered to estimate the level crossing rate:

- the Rice formula for the level crossing rate of a random process under the Gaussian assumption (Rice, 1945) or a projection method which uses an approximation of the joint distribution of the response and its first derivative (Monbet et al., 1996);
- the Breitung's method (Breitung, 1988);
- Hagberd's method (Hagberg, 2004);
- and the time-domain simulation, which is accurate, provided the number of simulation is large enough, and is considered as the reference solution.

Let us mention also the existence of the saddle point method to compute the extreme statistics. This methods was already well investigated by Naess et al. (2006) and by Machado (2002), therefore it is no more discussed in the present study.

The aim of the present study is to investigate the methods under consideration through a comparison of their accuracy on an application example. First of all, the statistical description of the response process is given. Then, the methods for crossing rate approximation are successively presented. Finally, those methods are applied on a case study for comparison.

STATISTICAL ANALYSIS OF THE RESPONSE PROCESS

Although the extreme values of the response process are of more interest in this study, this values can only be predicted if a more complete statistical description of the response is available. The critical response of concern is the large amplitudes low frequency motions. It is known that these motions are the resonant response to low frequency second-order wave forces. As it is shown in the sequel, these responses can be computed using frequency domain analysis technics if the structural dynamic behaviour is assumed linear.

Second-order wave forces model

We consider a Gaussian sea-state, which is modelized in the first order by a sum of elementary waves $w_n(t,x) = exp\left[i(\omega_n t - \chi_n x)\right]$, where ω_n and χ_n satisfy the dispersion relation: $\omega_n^2 = g\chi_n \tanh(d\chi_n)$ (d is the water depth and g is the gravity acceleration).

$$\eta(t,x) = \sum_{n=-N}^{n=N} \frac{A_n}{2} w_n(t,x) \tag{2}$$

The amplitudes A_n are independent and identically distributed complex random variables. Their distributions are gaussian with zero mean value and their respective variances depend on the power spectral density of $\eta(t,x)$. In order to make $\eta(t,x)$ real, it is assumed that $A_{-n} = A_n^*$, $\omega_{-n} = \omega_n$ and $\chi_{-n} = \chi_n$, where $*$ denote the complex conjugate. Note that we can assume $x = 0$ without loss of generality. In the frequency domain, the power spectral density of $\eta(t)$ is $S_\eta(\omega) = |H_\eta(\omega)|^2$, where $H_\eta(\omega)$ is a linear transfer function. This way, the Gaussian sea considered appears as a linear transform of a Gaussian white noise $b(\omega)$.

The second order approximation of wave-induced loading can be expressed as $F(t) = F^{(1)}(t) + F^{(2)}(t)$, where the first and the second order terms reads:

$$\begin{aligned} F^{(1)}(t) &= \sum_{n=-N}^{n=N} \frac{A_n}{2} H_n(\omega_n) e^{i\omega_n t} \\ F^{(2)}(t) &= \sum_{m=-N}^{m=N} \sum_{n=-N}^{n=N} \frac{A_m}{2} \frac{A_n}{2} Q_{m,n} e^{i\omega_m t} e^{i\omega_n t} \end{aligned} \tag{3}$$

$H_F = \{H_n\}_{n\in\{-N,\ldots,N\}}$ and $Q = \{Q_{m,n}\}_{(m,n)\in\{-N,\ldots,N\}}$ are respectively a linear and a quadratic transform, which components depend on the structural properties. In the second order term, the elementary waves interaction produces waves which frequencies are $2\omega_m$, $2\omega_n$, $\omega_m+\omega_n$ or $\omega_n-\omega_m$. Thus, it is of practical interest to decompose the second order loading into a sum mode $F^{(+)}(t)$ and a difference mode $F^{(-)}(t)$:

$$\begin{aligned} F^{(-)}(t) &= 2\sum_{m=1}^{m=N} \sum_{n=1}^{n=N} \frac{A_m^*}{2} \frac{A_n}{2} Q_{m,n} e^{-i\omega_m t} e^{i\omega_n t} \\ F^{(+)}(t) &= 2\mathrm{Re}\left[\sum_{m=1}^{m=N} \sum_{n=1}^{n=N} \frac{A_m}{2} \frac{A_n}{2} Q_{m,n} e^{i\omega_m t} e^{i\omega_n t}\right] \end{aligned} \tag{4}$$

One can see that the low frequency motions, that produce the resonant behaviour of the structure, come from the difference mode of the quadratic approximation of the wave loading. The quadratic transfer function associated to that mode can be written $Q^{(-)}(\omega_n, -\omega_m)$.

Structural motion

Assuming a linear dynamic behaviour, the structural motion is governed by a linear mechanical system:

$$(M + M_a)\ddot{x}(t) + \alpha\dot{x}(t) + Kx(t) = F(t) \tag{5}$$

where M is the mass of the structure, M_a the added mass at zero pulsation, α is the damping coefficient and K is the stiffness of the mooring line. Let denote $H_x(\omega)$ the linear transfer function associated to this dynamical system:

$$\begin{aligned} S_X(\omega) &= |H_x(\omega)|^2 S_F(\omega) \\ H_x(\omega) &= \left[-(M+M_a)\omega^2 + K + j\alpha\omega\right]^{-1} \end{aligned} \tag{6}$$

Note that the response spectral density is all the higher as the frequency is closed to the structural resonant frequency ω_0.

$$\omega_0^2 = \frac{K}{M + M_a} \tag{7}$$

In summary, the structural response can be derived from a Gaussian white noise using a filter system which is the association of the linear filter to get the free sea elevation from a white noise, the quadratic filter to obtain the second order wave forces from the sea elevation and the linear filter associated to the dynamic behaviour of the structure to derive the structural motion. An overview of the filter system to get the structural motion from a Gaussian white noise is depicted in Figure 1. In this study, we assume that the low frequency motion is produced by the difference mode of the load quadratic transfer function $Q^{(-)}$ applied to the high frequency wave. We do not take into account the linear response of the low frequency waves. Thus, the total quadratic transfer function $Q_T(\omega_1, \omega_2)$ of the system of interest reads:

$$\begin{aligned} Q_T(\omega_1, \omega_2) =& H_x^{-1}(\omega_1 - \omega_2) Q^{(-)}(\omega_1, -\omega_2) \times \\ & H_\eta^{hf}(\omega_1) H_\eta^{hf}(-\omega_2) \end{aligned} \tag{8}$$

Using this total quadratic transfer function, we can now assess the statistical properties of the response process. Note that, in the sequel, we use the abreviation QTF to denote the total quadratic transfer function.

Statistical properties

Let us express the response process in a form suitable for statistical analysis. Thanks to its Hermitian property, the eigen decomposition of Q_T is:

$$Q_T(\omega_1, -\omega_2) = \sum_{j=1}^{N} \lambda_i \phi_i(\omega_1) \phi_i^*(\omega_2) \tag{9}$$

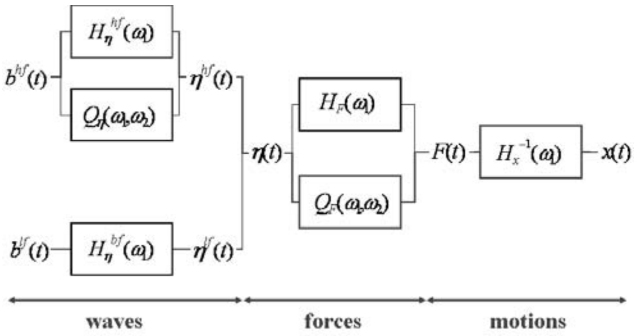

Fig. 1 Filter system for low frequency motions.

Denoting $B(\omega)$ the frequency representation of a Gaussian white noise, the process $x(t)$ can be written:

$$x(t) = 2\int_0^\infty \int_0^\infty \sum_{j=1}^{N} \lambda_i \phi_i(\omega_1)\phi_i^*(\omega_2) \times B(\omega_1)e^{\omega_1 t}B^*(\omega_2)e^{-\omega_2 t}d\omega_1 d\omega_2 \tag{10}$$

Eq.(10) turns out to be:

$$x(t) = \frac{1}{2}\sum_{j=1}^{N} \lambda_i \left(z_i(t)^2 + \tilde{z}_i(t)^2\right) \tag{11}$$

where $\tilde{z_i}(t)$ is the Hilbert transform of $z_i(t)$ and:

$$z_i(t) = \int_{-\infty}^{\infty} \phi_i(\omega_1)B(\omega_1)e^{\omega_1 t}d\omega_1 \tag{12}$$

Another expression of the process $x(t)$ is useful for the computation of its statistical properties. Let us denote $\Delta = diag(\lambda_1, ..., \lambda_N)$ and:

$$Z(t) = \begin{bmatrix} z(t) \\ \tilde{z}(t) \end{bmatrix} \text{ and } D = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta \end{bmatrix} \tag{13}$$

Then, our process can be written:

$$x(t) = \frac{1}{2}Z(t)^T DZ(t) \tag{14}$$

From this expression it is easy to get the mean and the variance of the process:

$$\mathrm{E}[x(t)] = \frac{1}{2}\mathrm{E}\left[Z(t)^T DZ(t)\right] = \sum_{n=1}^{n=N} \lambda_n \tag{15}$$
$$\mathrm{V}[x(t)] = \frac{1}{4}\mathrm{V}\left[Z(t)^T DZ(t)\right] = \sum_{n=1}^{n=N} \lambda_n^2$$

For the prediction of the extreme value statistics, it is useful to compute the joint characteristic function of the process and its first derivative. It is shown (Naess, 2001) that for the second order approximation of the response process that joint characteristic function can be given in closed form. To achieve this, consider the vector $\left(Z(t)^T\ \dot{Z}(t)^T\right)^T$, where $\dot{Z}(t)$ is the first derivative of $Z(t)$. One can obtain the covariance matrix of $\left(Z(t)^T\ \dot{Z}(t)^T\right)^T$ by the following expression:

$$\Sigma = \begin{bmatrix} I & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \tag{16}$$

where I is the $2N \times 2N$ identity matrix, $\Sigma_{12} = \mathrm{E}\left[Z\dot{Z}^T\right]$, $\Sigma_{21} = \mathrm{E}\left[\dot{Z}Z^T\right]$ and $\Sigma_{22} = \mathrm{E}\left[\dot{Z}\dot{Z}^T\right]$. Now let us compute the moment generating function of the process $(x(t), \dot{x}(t))$ given by:

$$M(u,v) = \mathrm{E}\left[e^{ux+v\dot{x}}\right] = \int_{R^2} f_{X\dot{X}}(x_1,x_2)e^{ux_1+vx_2}dx_1dx_2 \tag{17}$$

When we report in Eq.(17) the expression of $x(t)$ in Eq.(14) and we use the fact that $f_{\dot{Z}Z}(\dot{z},z) = f_Z(z)f_{\dot{Z}|Z}(\dot{z}|z)$, the moment generating function reads:

$$M(u,v) = \int_{R^{4N}} f_Z(z)f_{\dot{Z}|Z}(\dot{z}|z)e^{\frac{1}{2}uz^T Dz + v\dot{z}^T Dz}d\dot{z}dz \tag{18}$$

It has been shown that the random variable $\dot{Z}|Z = z$ is normally distributed with a mean value $\Sigma_{12}z$ and a covariance matrix $V = \Sigma_{22} - \Sigma_{21}\Sigma_{12}$ (Anderson, 1958). One deduces that the variable $Z_w = \dot{Z}^T DZ|Z = z$ is also normally distributed with a mean value $\bar{Z}_w$ and a covariance matrix V_w given by:

$$\bar{Z}_w = z^T\Sigma_{12}Dz \tag{19}$$
$$V_w = z^T DVDz$$

Invoking the expression of the characteristic function of a Gaussian variable: $\mathrm{E}\left[e^{vZ_w}\right] = e^{\bar{Z}_w v + \frac{1}{2}V_w v^2}$, Eq.(18) reads:

$$M(u,v) = \int_{\Re^{2N}} e^{\bar{Z}_w v + \frac{1}{2}V_w v^2} e^{\frac{1}{2}uz^T Dz} f_Z(z)dz \tag{20}$$
$$M(u,v) = \frac{1}{(2\pi)^N}\int_{\Re^{2N}} e^{z^T\left[\Sigma_{12}Dv + \frac{1}{2}DVDv^2 + Du - I\right]z}dz$$

Finally, the derivation of the formula for the characteristic function is based on an integral equality cited by (Cramer, 1946), which states that:

$$\int_{R^n} e^{-\frac{1}{2}z^T\Gamma z}dz = \frac{(2\pi)^{n/2}}{\sqrt{\det(\Gamma)}} \tag{21}$$

Denoting $\Gamma = I - 2\Sigma_{12}Dv - DVDv^2 - Du$, the characteristic function of the process $X(t)$ given by:

$$M(u,v) = \frac{1}{\sqrt{\det(\Gamma)}} \tag{22}$$

From the expression of the moment generating function given in Eq.(22) one can derive the cross moment of $(x, \dot{x})$:

$$\mathrm{E}[x^i\dot{x}^j] = \frac{\partial^{i+j}M(u,v)}{\partial u^i \partial v^j} \tag{23}$$

STATISTICAL ANALYSIS OF EXTREME VALUES

The statistics of the extreme values of a random process are in general derived from the crossings of the process thanks to the relation given in Eq.(1) for stationary random process. Moreover, instead of the up-crossing it is convenient to compute the crossing rate, which makes no difference since by stationariry the first is half the second.

Time-domain simulation

From the QTF, it is possible to generate time series of that process with a proper simulation methods. The general procedure for the simulation of $x(t)$ is based on the expression of $x(t)$ given in Eq.(11). An overview of this procedure is shown on Figure 2. Note that for cost reduction issues one have to reduce the number of eigen values to the sufficently larger ones.

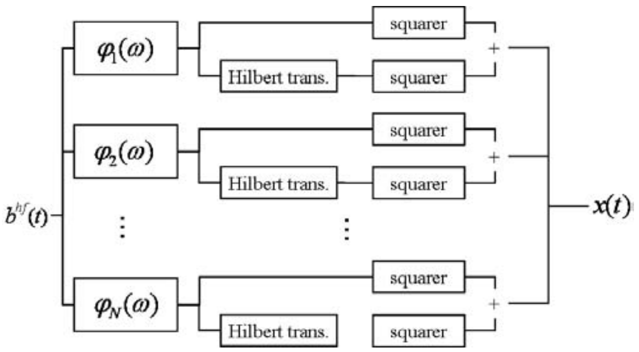

Fig. 2 Simulation procedure of process $x(t)$.

The empirical estimation of the up-crossing rate of the level β consits in counting how many times each sample of $x(t)$ cross the level β and divide that number with the time duration of the sample. Denoting δt the sample path, the empirical estimation of the up-crossing rate of the level β reads:

$$\mu^+(\beta) = \frac{1}{T}\sum_{j=1}^{n} \mathbf{1}_{\{x((j-1)\delta t)\leq\beta\}\cap\{x(j\delta t)>\beta\}} \tag{24}$$

where $\mathbf{1}$ is the indicator function. The time-domain simulation, which is accurate provided the number of simulation is large enough, is considered as the reference solution.

Rice formula and Projection method

The general formula of Rice for any level up-crossing rate is expressed as follows:

$$\mu^+(\beta) = \int_0^\infty \dot{z} f_{Z\dot{Z}}(\beta,\dot{z})d\dot{z} \tag{25}$$

If $x(t)$ is supposed to be a Gaussian process, denoting m_2 its second order spectral moment, the up-crossing rate of any level β is given by:

$$\mu^+(\beta) = \frac{\sqrt{m_2}}{2\pi}e^{-\frac{\beta^2}{2}} \tag{26}$$

Some other methods use an approximation of the joint distribution of the response and its first derivative. Among them is the projection method (Monbet et al., 1996). Let us assume that the couple $\left(X(t),\dot{X}(t)\right)$ has the same joint distribution for any t and that distribution can be written as follows:

$$f_{X,\dot{X}}(x,\dot{x}) = s(x,\dot{x})p_1(x)p_2(\dot{x}) \tag{27}$$

where p_1 and p_2 are parametric density functions that have to fit the marginal distributions of x and $\dot{x}$. Some rules for a good choice of p_1 and p_2 will be explained in the sequel. The main idea behind the projection method is to approximate the function $s(x,\dot{x})$ by a polynomial function $\tilde{s}(x,\dot{x})$ which preserve the value of the cross-moments $m_{ij} = \mathrm{E}\left[X^i\dot{X}^j\right]$ for any $0 \leq i+j \leq L$ for a given power degree L. To achieve this, consider P_L the space of two dimensional polynomial functions which degrees are less or equal to L. Consider also $\{\varphi_{i,j}(x,\dot{x})\}_{0\leq i+j\leq L}$ a given base of polynomial on this space. The approximation function $\tilde{s}(x,\dot{x})$ can be written in the form:

$$\tilde{s}(x,\dot{x}) = \sum_{k=0}^{L}\sum_{j=0}^{k} d_{j,k-j}\varphi_{j,k-j}(x,\dot{x}) \tag{28}$$

where $d_{j,k-j}$ are unknown coefficients to be determined. Recalling that this approximation must preserve the value of the cross-moments, the coefficients $d_{i,j}$ for any $0 \leq i+j \leq L$ are obtained solving the system of equations:

$$\int \varphi_{j,k-j}(x,\dot{x})\tilde{s}(x,\dot{x})p_1(x)p_2(\dot{x})dxd\dot{x} = \mathrm{E}\left[\varphi_{j,k-j}(x,\dot{x})\right] \tag{29}$$

It appears from the Eq.(29) that $\tilde{s}(x,\dot{x})$ is the projection of $s(x,\dot{x})$ on the space P_L for a scalar product defined by:

$$< a,b >= \int a(x,\dot{x})b(x,\dot{x})p_1(x)p_2(\dot{x})dxd\dot{x} \tag{30}$$

In practice, we have some observed time series of $X(t)$. We can also have some observed time series of $\dot{X}(t)$, if not they can be computed by finite difference with the time series of $X(t)$. Using those observations it is possible to get empirical estimations of $\mathrm{E}\left[\varphi_{j,k-j}(x,\dot{x})\right]$ for any $0 \leq i+j \leq L$. One can therefore solve the system of Eqs.(29) to obtain the coefficients $d_{i,j}$. At this stage, it is important to notice that the density functions p_1 and p_2 should be chosen as parametric distributions that fit the best the observations of $X(t)$ and $\dot{X}(t)$. Moreover, the form of those parametric distributions should be chosen such that the moments of order less or equal to L exist. Once an approximation of the joint density of $X,\dot{X}$ is computed, the Rice formula in Eq.(25) is applied to get any level up-crossing rate.

Breitung's method

The underlying results of Breitung's method (Breitung, 1988) is an asymptotic approximation of the expected number of crossings of a vector process through a hypersurface, when that vector process is assumed Gaussian, stationary and differentiable. Consider $X(t) = (X_1(t),...,X_n(t))$ a vector process. Let $S \in R^n$ be a region defined by the function $g(x)$ such that $S = \{x \in R^n; g(x) > 0\}$. When associated to a physical system, the function $g(x)$ describes whether the system fails or not with the following conventions:

$$\begin{aligned} S &= \{x \in R^n; g(x) > 0\} \equiv \text{safe region} \\ F &= \{x \in R^n; g(x) < 0\} \equiv \text{failure region} \\ G &= \{x \in R^n; g(x) = 0\} \equiv \text{limite-state surface} \end{aligned} \tag{31}$$

Then, the probability that the system does not fail during the time $[0,T]$ is given by $P_S = \Pr\{g(X(t)) > 0 \text{ for all } t \in [0,T]\}$. The basic idea under the approximation of P_S is to study the process not during the whole time duration $[0,T]$, but only at the points where the process has an outcrossing. The mean number of those points (i.e. points of out crossing) for a differentiable stationary Gaussian process $x(t)$ is under some regularity conditions given by a surface integral over the surface G (Lindgren, 1980).

$$\mu(\beta) = \frac{\beta^{n-1}}{(2\pi)^{n/2}} \times \qquad (32)$$

$$\int_G \mathrm{E}\left[\left|n(x)^T \dot{y}(t)\right|; y(t) = \beta x\right] e^{\left(-\frac{\beta^2}{2} x^T x\right)} ds(x)$$

where β is the distance of G from the origin, $n(x)$ is the unitary normal vector on G and $s(x)$ is the surface area mesure on G.

Now, we suppose that there are only m points $y_1, ..., y_m$ on G with $|y_i| = \beta = \min_{y\in G}|y|$. Breitung's derived from the expression in Eq.(32) an asymptotic formula of the mean number of outcrossings for large values of β in the form:

$$\mu(\beta) = \Phi(\beta) \sum_{i=1}^{m} d_i^{-1/2} \qquad (33)$$

$$d_i = \prod_{j=1}^{n-1} (1 - \kappa_{i,j})$$

where $\kappa_{i,j}$ are the main curvatures of G at y_i. This solution is also suitable for the evaluation of the level crossing rate of a random process with the required properties. To this end, one have to define the function g as follows:

$$g(Z) = \frac{1}{2} Z(t)^T D Z(t) - \beta \qquad (34)$$

for the vector process $Z(t)$ and any level β.

Asymptotic expansion of crossing rate and Monte-Carlo integration

The method presented here stem from the PhD report of Oskar Hagberg (2004). In his research work, Hagberg derived a formula for the level crossing rate and proposed two different approaches to approximate that formula. The first one is an asymptotic expansion of the given formula of the level crossing rate. The coefficients of that expansion depend on the joint correlation structure of the process and its derivative. The second one is a Monte-Carlo integration scheme to compute the crossing rate by the given formula.

Let us consider a random process $x(t)$ which is express as a quadratic form of an n-dimensional stationary Gaussian vector process $z(t)$. We assume that the first derivative $\dot{z}(t)$ exists. As it was stated above, the joint distribution of the couple $(z(t), \dot{z}(t))$ reads:

$$\begin{bmatrix} z(0) \\ \dot{z}(0) \end{bmatrix} \in \aleph_{2n} \left(0, \begin{bmatrix} I & -\Sigma_{21} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}\right) \qquad (35)$$

Let us denote $z_j(t)$ the components of $z(t)$. The process $x(t)$ can be written:

$$x(t) = \sum_{j=1}^{n} \lambda_j z_j^2(t) \qquad (36)$$

where

$$\lambda_j \neq 0 \text{ for all } j \text{ and } \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n = 1 \qquad (37)$$

We define the number k as the largest integer such that $\lambda_j < 1$ when $j \leq k$, and the matrix $\Gamma_k = \mathrm{diag}(\lambda_1, \ldots, \lambda_k)$. Using the Lindgren's formula Eq.(32) (1980) and the fact that the variable $\dot{y}(t)|y(t) = x$ is normally distributed with mean value $\Sigma_{21}x$ and correlation matrix V, Hagberg derive the following formula for the crossing rate of the level β^2:

$$\mu(\beta^2) = A(\beta) \int_{t\in S_{n-k-1}} I(t,\beta) ds_u(t) \qquad (38)$$

where S_{n-k-1} is the unit sphere in R^{n-k} and $s_u(t)$ is the surface area mesure on S_{n-k-1}. $I(t,\beta)$ is expressed as follows:

$$I(t,\beta) = \int_{s^T\Gamma_k s<\beta^2} Q(s,t,\beta) \exp\left[\frac{1}{2} s^T (I - \Gamma_k) s\right] ds \qquad (39)$$

The expression of $A(\beta)$ and $Q(s,t,\beta)$, which are complex are not reported in this paper, but they can be found in Hagberg's PhD report.

In the first approach to approximate the crossing rate, Hagberg use the theory of asymptotic expansions to show that the integral $I(t,\beta)$ can be expanded in powers of $1/\beta^2$ for larger values of β. Then, using this results, he states the existence of an asymptotic expansion of the crossing rate $\mu(\beta^2)$ in the form:

$$\mu(\beta^2) \to A(\beta) \sum_{j=0}^{\infty} c_j \frac{1}{\beta^{2j}} \text{ when } \beta \text{ tends to } \infty \qquad (40)$$

In particular, he gives the complete expression of the first coefficient c_0 and the second one c_1. It doesn't worth reporting these expression in the present paper. The reader could find them in Hagberg's PhD report. However, it is important to mention that the first order expansion coincides with Breitung's formula.

In the second approach, one can see that the inner integral $I(t,\beta)$ can be considered as an expectation of a particular function $\tilde{Q}(s,t,\beta)$ for a normally distributed centered random vector s with a covariance matrix $(I - \Gamma_k)^{-1}$. Likewise, the outer integral in Eq.(38) can be regarded as the expectation of a particular function for t, a uniformly distributed random vector on S_{n-k-1}. Note that s and t are independent. It is therefore possible to compute the crossing rate $\mu(\beta^2)$ using a Monte-Carlo integration procedure with samples of variables s and t. Hagberg's scheme for Monte-Carlo integration of Eq.(38) is explained in his PhD report.

APPLICATION

Now, let us consider a LNG carrier in shallow water with ten mooring lines as shown on Figure 3. The QTF of the surge motion is used to compute the low frequency structural response. We suppose that the structure is subjected to unidirectional random waves modelized as a Jonswap which parameters are: $H_S = 5$m, $T_P = 12$s and $\gamma = 10$. The resonant period of the moored system is 125s.

In order to perform the extreme value analysis, except the direct application of the Rice formula, the other methods requires a diagonalization of the QTF. For CPU reason, one have to reduce the number of eigenvalues to the sufficiently largest ones. As a preliminary analysis, one checked the convergence of the empirical moments of the response process in terms of the number of largest eigenvalues retained. This study revealed that from ten retained largest eigenvalues the

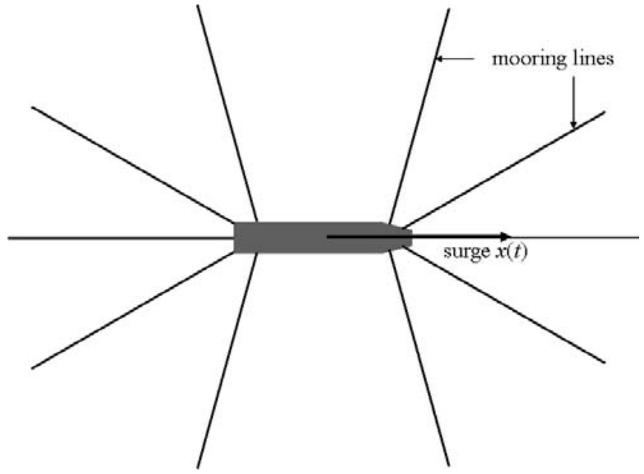

Fig. 3 Surge motion of the LNG carrier.

convergence is achieved. In this study, we reduce the number of eigenvalues to the 20 largest ones.

The given methods are then applied to compute the crossing rate in terms of the motion levels. For the time-domain method, we use 40000 simulations of a Gaussian sea state of 1.5 hours. Concerning the Projection method, the marginal parametric density p_1 and p_2 are estimated in order to preserve the statistical moments of x and $\dot{x}$ using the maximum entropy method (Guiasu & Shenitzer 1946). Note that those moments are computed with their expression given in Eq.(23). The degree of the polynomial function $\tilde{s}(x, \dot{x})$ is $L = 3$. As stated above, the Breitung's result coincides with the first order term of Hagberg's asymptotic expansion. Finally, note that Hagberg's Monte-Carlo integration is performed with 5000 samples of random vector $(s^T t^T)^T$.

The results are plotted in Figure 4. They show a large divergence of the Rice formula under Gaussian assumption for the larger motion level, while the other methods are closed to the time-domain simulation. The Figure 5 is a zoom of the previous for larger motion levels and the Figure 6 shows the relative error of the methods with respect to the time-domain simulation.

One can see that the Breitung's method, the second order asymptotic expansion and Hagberg's Monte-Carlo integration remain closed to the time-domain simulation results for larger motion levels. The Projection method corrects the Rice formula with the Gaussian hypothesis but still deviates from the time-domain simulation for high levels (Fig.6). One can see in Figure 6 that the second order asymptotic expansion improves the first order (i.e. Breitung's method) for higher levels. But, Hagberg's Monte-Carlo integration appears as the best approximation.

CONCLUSIONS

In this study, we have discuss various alternative methods for the statistical analysis of extreme motion of moored floating bodies. The methods which were discuss are the Rice formula with either a Gaussian assumption or an approximation of the joint distribution of the response and its first derivative by a projection method; the Breitung's formula; the Hagberg's approaches using either an asymptotic expansion or a Monte-Carlo integration and finally the time domain simulation which serve as the reference method. Those methods were applied to an industrial application given by the Bureau Veritas and which consist in the estimation of the distibution of extreme low frequency surge motion of a moored floating body in shallow water. The results reveals a large deviation from the reference solution of the Rice formula with a Gaussian assumption. The other methods appears closed to the reference solution, but the Hagberg's Monte-Carlo integration gives the best approximation. Finally, note that these methods can serve as interesting alternatives to the saddle point method.

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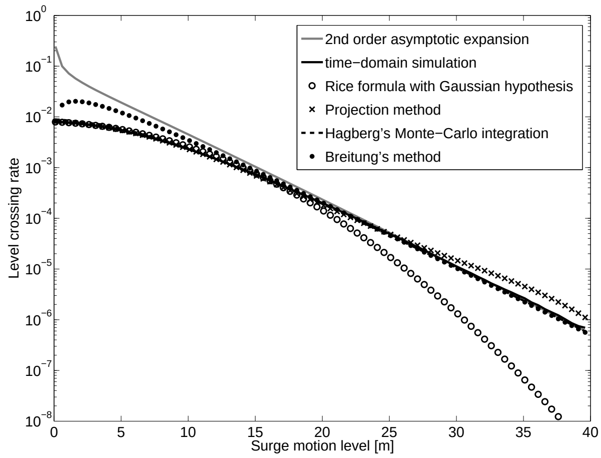

Fig. 4 Level crossing rate with respect to surge motion level by the different methods.

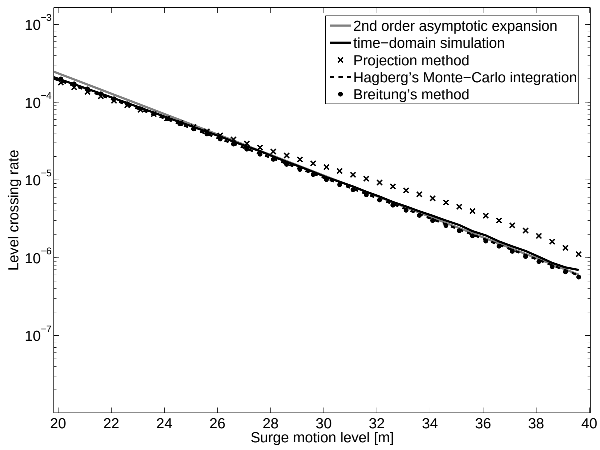

Fig. 5 Level crossing rate with respect to surge motion level for larger level.

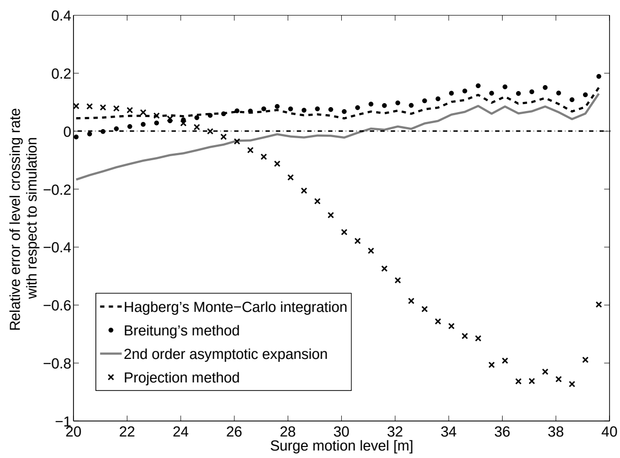

Fig. 6 Relative error of the crossing rate with respect to the time-domain simulation