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A Non-Linear Viscoelastic Viscoplastic Model for the Behaviour of Polyester Fibres

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Abstract: In this study the macroscopic analysis proposed by Schapery has been applied to model the non-linear viscoelastic and viscoplastic behaviour of polyester fibres. First, a special non-contact extensometry method has been developed in order to measure the fibre response without affecting the behaviour of these extremely anisotropic fibres. An accelerated identification method has then been developed. This enables a unique and independent set of parameters to be determined, which separate the reversible and non-reversible mechanisms responsible for measured strains. The model prediction for an arbitrary loading sequence corresponds closely to the measured strains. The model has been applied here at the yarn level but can equally well be used to predict rope behaviour, either by including the identified parameters in a rope structure analysis or by identifying parameters at the rope level.

Keywords: fibre, model, non-linear, polyester, viscoelastic, viscoplastic

1. Introduction

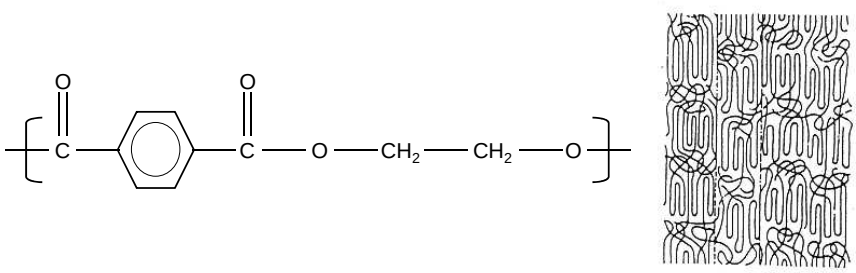
Cables made from synthetic fibres are being used increasingly to replace steel ropes. For deep sea applications such as offshore platform mooring lines, weight gain is the main attraction. However, the mechanical behaviour of synthetic cables differs from that of steel. Synthetic fibres show non-linear viscoelastic and viscoplastic behaviour. The elongation of these cables thus depends on the temperature, the loading time and the load level.
In the present study the behaviour of polyester fibres has been examined. These materials are finding many applications in the offshore industry on account of their low cost .
The geometry of a cable also contributes to its behaviour. Models which account for cable construction have been presented elsewhere [1] [2]. However, it is the polymeric nature of the fibres which dominates the overall mechanical response [3], particularly with respect to creep and plastic strains. It is therefore essential to develop a reliable model for the mechanical behaviour of polyester fibres if the strains in cables are to be predicted for varying load histories. Knowledge of these strains is essential for the long term safety of moored offshore platforms.
In a previous study the non-linear viscoelastic and viscoplastic behaviour of aramid fibres was successfully modelled [4]. The formulation presented then, based on the work of Schapery [5], is here modified to model a different fibre which shows much more pronounced non-linear behaviour.
A video extensometry technique will be presented which allows the mechanical behaviour of polyester fibres to be studied. Then the identification will be discussed, with particular emphasis on the need to obtain a unique set of parameters as quickly as possible. Finally the results from the model will be compared to experimental results.

2. Identification of the mechanical behaviour of polyester fibres

2.1. Material tested

The fibres studied here are Diolen 855TN polyester. Their linear weight was measured to be 225 tex, their diameter is approximately 30 μm. Tex is the generally accepted unit in the textile industry and corresponds to the weight in grams of one kilometre of fibre. Given the difficulty in measuring the diameter accurately the stresses are presented here in Newtons per tex (N/tex).
In contrast to the molecular arrangement of high performance fibres such as aramid, which have highly crystalline structures with an orientation distribution of crystallites concentrated along the fibre axis, the molecular organisation of polyester fibres is more disordered (see **Erreur ! Source du renvoi introuvable.**). As a result an amorphous phase is present (60-65 % [6]) with the crystalline phase. These fibres, manufactured by extrusion from the molten state, have relatively low modulus in the fibre direction of around 15 GPa and are more sensitive to temperature and rate effects than the aramid fibres. In order to simplify testing and to improve repeatability the fibres were tested in the form of yarns consisting of 192 fibres.

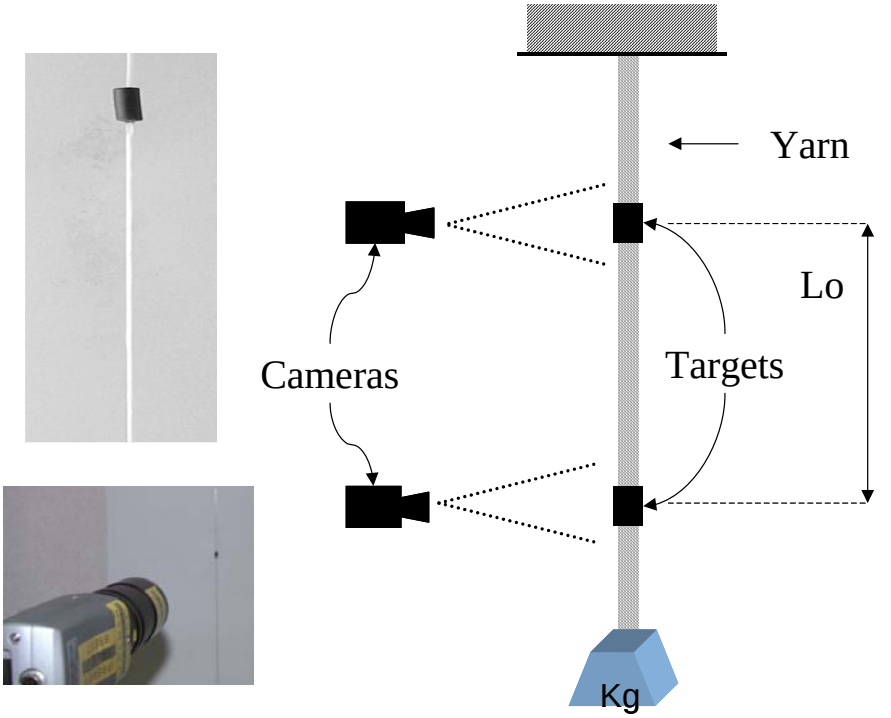
Figure 1. Chemical formula and representation of the molecular structure of polyester fibres



Exprimental methods

The fibres were clamped in special grips and loaded on a standard Instron test machine to obtain load-elongation plots. To perform creep and recovery tests a dedicated test frame was designed, (**Erreur ! Source du renvoi introuvable.**), located in a temperature (20°C) and humidity (50%) controlled laboratory. This involves dead weight loading of a 2500 mm long yarn, which enables the load to be applied very quickly for creep tests and removed rapidly to study recovery.

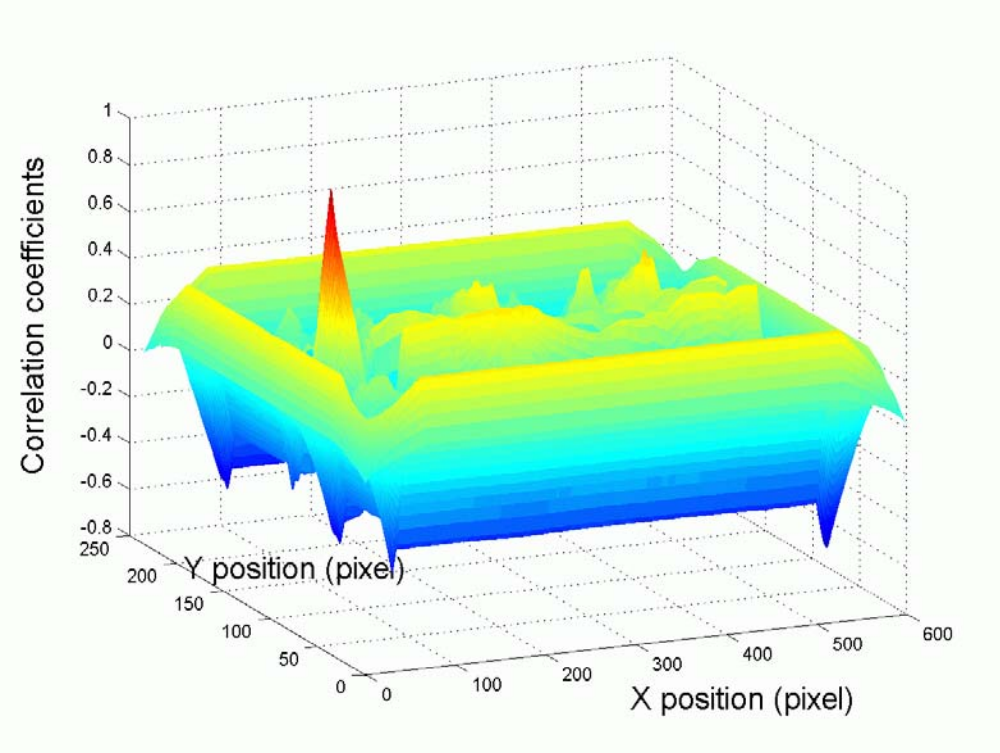
Figure 2. Diagram of experimental set-up.



In order to measure elongations accurately without disturbing the yarn a video extensometer was developed based on two CCD cameras. This allows photos to be taken at regular intervals of two cylindrical markers fixed to the yarn (**Erreur ! Source du renvoi introuvable.**). Image analysis by cross correlation enables the position of the markers to be determined to the nearest pixel (**Erreur ! Source du renvoi introuvable.**). After a calibration procedure the relative displacement of the two markers can thus be obtained and the strain is simply

calculated by dividing by the initial distance between the markers. The resolution depends on the magnification of the two cameras, which can be adjusted to the displacement of each marker. It is thus possible to achieve a resolution of 20 μm. The length of the yarns is selected to obtain displacements compatible with the camera resolution.

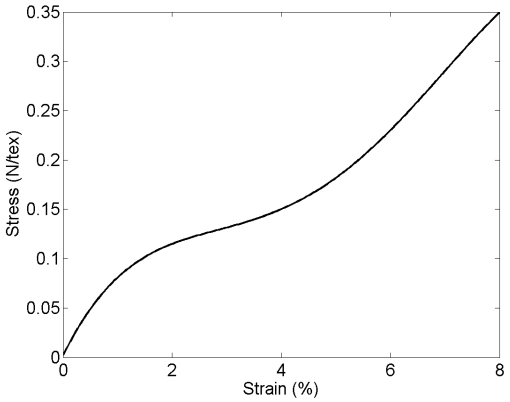
Figure 3. Image analysis by cross correlation to identify the position of the marker on the yarn



2.2. Mechanical behaviour

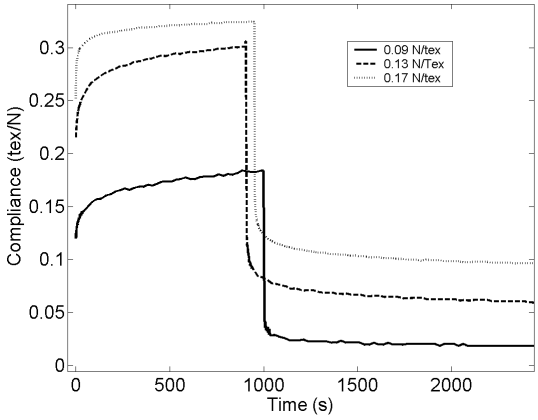
The tensile load-strain plot shows that the response of the new polyester yarn is highly non-linear (**Erreur ! Source du renvoi introuvable.**). Heuvel [6] proposed a macromolecular interpretation. The two-phase structure, amorphous and crystalline, is the source of these non-linearities. In zone 1, shown on **Erreur ! Source du renvoi introuvable.**, the material flow resulting in a loss of stiffness is due to a reorganisation of the entanglements et the unfurling of molecules (transition from gauche to trans conformation). Then the molecular chains which are aligned are loaded in turn (zone 2 in **Erreur ! Source du renvoi introuvable.**). The sequential arrangement of crystallites in the tensile direction – along the fibre axis – leads to a stiffening of the fibrilar structure. A third zone, not shown on **Erreur ! Source du renvoi introuvable.**, shows additional flow up to fibre rupture. This final step is attributed to the failure of aligned chains which are no longer able to align along the fibre axis.

Figure 4. Stress as a function of strain for a polyester yarn



Creep and recovery compliance curves for three load levels are shown in **Erreur ! Source du renvoi introuvable.**. It may be noted, as the tensile plot indicated, that the compliance depends on the load level. It may also be noted that the creep rate is non-linear with respect to the applied stress. In addition, a permanent plastic strain is observed even at low load. This plastic strain does not appear to result from chain breakage as shown by the increase in stiffness during the phase 2 (zone 2 in **Erreur ! Source du renvoi introuvable.**). According to Baltussen and Northolt [7], the irreversible strain results from the same mechanisms as the reversible process, namely the orientation of crystallites and rearrangement of entanglements.

Figure 5. Creep and recovery compliance for three applied stress levels



The behaviour of polyester fibres is therefore non-linear viscoelastic. The residual strain measured also appears to be non-linear. It remains to be determined whether it is time dependent. If so, the global behaviour will be non-linear viscoelastic and viscoplastic.

3. Model

In this section a model will be proposed which takes account of the non-linear viscoelastic and viscoplastic behaviour of polyester fibres. The range of loads considered extends up to 0.32 N/tex which is 50% of the failure stress. In order to separate the influence of time and load level the model is based on isothermal conditions. The test temperature for the identification and verification is therefore fixed at 20°C.

t	time
σ	stress
ε	strain
D_0	instantaneous compliance
$\Delta D = f(D_1, t)$	transient compliance
D_1	creep rate
g_0, g_1, g_2	non-linear factors
a_σ	acceleration factor
p	instantaneous plasticity factor
D_p	plasticity rate
$A_0 = g_0 \cdot D_0$	non-linear instantaneous compliance
$A_1 = g_2 \cdot D_1$	non-linear creep rate

3.1. Schapery's model

In a previous study it was shown that the macroscopic formulation of Schapery (see Equation 1) could be adapted to predict the mechanical behaviour of aramid fibres subjected to complex loading cycles [4]. The parameters of the model are determined from creep/recovery curves measured at different stress levels. The identification time for each stress level is about 90 minutes. The identification procedure selected separated the irreversible and reversible effects so that the model can take account of the mechanical history of the material. Reference [4] provides more details of the model and the procedure for identification of the model parameters.

$$\varepsilon = g_0(\sigma) \cdot D_0 \cdot \sigma + g_1(\sigma) \cdot \int_0^t \Delta D\big(\Phi(t) - \Phi(\tau)\big) \cdot \frac{d(g_2(\sigma) \cdot \sigma)}{d\tau} d\tau + F(p, D_p)$$

$$\text{avec } \Phi(t) = \int_0^t \frac{dt'}{a_\sigma} \text{ et } \Phi(\tau) = \int_0^\tau \frac{dt'}{a_\sigma}$$

Equation 1. Non-linear viscoelastic and viscoelastic model of Schapery

3.2. Identification procedure

For the aramid fibres considered previously [4], the identification of the non-linear viscoelastic and viscoplastic parameters was relatively straightforward, as two of these could be assumed to be constant throughout the stress range considered: $a_\sigma(\sigma) = g_1(\sigma) = 1$. This assumption, based on the regular macromolecular structure and highly crystalline nature of aramid fibres, is not justified for polyester fibres whose molecular structure is less ordered. As a result of this more complex molecular structure it is necessary to identify not four but 6 parameters. Indeed, for this material it is essential to consider the time-stress dependency (in a similar way to time-temperature relationships). In order to adjust the experimental curves to the equations from the model (Equation 2 and Equation 3) an optimisation with respect to these parameters is required for the polyester fibres. The need to take these parameters into account has been noted previously for other polymers such as polyethylene [8] .

The identification of the parameters from the creep and recovery curves is therefore less trivial for polyester fibres. Analysis of the creep and recovery equations indicates that certain parameters are not independent. thus, A_0 and p can not be determined independently of one another from the adjustment of Equation 2 to the experimental creep curve. In a similar way the adjustment of Equation 3 to the recovery curve does not enable A_0 to be obtained uniquely, as g_1 and a_σ are also functions of an entity independent of time (this operation was possible when g_1 and a_σ were constant and equal to 1).

$$\varepsilon_c(t) = A_0 \cdot \sigma_c + g_1 \cdot A_1 \cdot \sigma_c \cdot \log_{10}\left(\frac{t}{a_\sigma} + 1\right) + \sigma_c \cdot \left[p + D_p \cdot \log_{10}(t+1)\right]$$

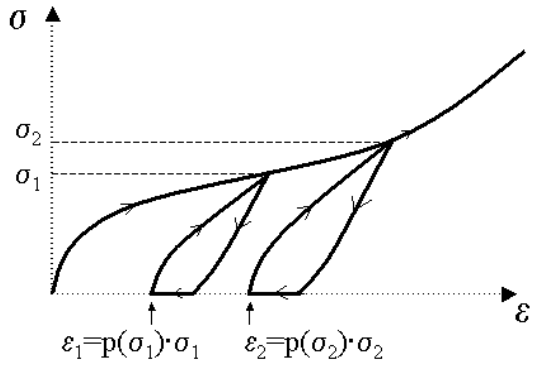
Equation 2. Schapery model for a creep test under applied stress σ_c .

$$\varepsilon_{cr}(t) = A_0 \cdot \sigma_c + A_1 \cdot \sigma_c \cdot \left[g_1 \cdot \log_{10}\left(\frac{t_1}{a_\sigma} + 1\right) + \log_{10}(t - t_1 + 1) - \log_{10}\left(\frac{t_1}{a_\sigma} + t - t_1 + 1\right)\right]$$

Equation 3. Schapery model for a recovery test after a period of creep t_1 under stress σ_c .

In order to identify a unique set of parameters with physical significance it was therefore decided to measure p by a simple test consisting of load-unload cycles at increasing stress on the same yarn sample (**Erreur ! Source du renvoi introuvable.**). Assuming that the viscoplastic phenomenon which is time dependent does not have time to be activated during a short period at the stress level σ_1, the residual strain measured after recovery corresponds to the instantaneous part of the plastic strain ($p(\sigma_1) \cdot \sigma_1$). The parameter p can then be deduced from the formula $p(\sigma_1) = \sigma_1 / \varepsilon_1$. The parameter p , determined for several stress levels, is shown on **Erreur ! Source du renvoi introuvable.**. The evolution of p shows an increase in irreversible strain up to 0,27 N/tex. Above this stress the residual strain remains constant. This "plastic plateau" was observed in a previous study of polyester mono-filaments tested in creep/recovery at several stress levels [3]. The value of p for zero stress was fixed arbitrarily.

Figure 6. Schematic diagram illustrating method to determine the parameter p for stress level



The identification of parameters begins, for each stress level, with the determination of the coefficient A_0 by adjusting the curves of Equation 2 to the experimental creep data. The parameter p, is assumed known and is not considered as a variable in this multi-variable optimisation. A preliminary non-unique set of parameters g_1, A_1, a_σ and D_p is obtained, as these values depend on the logarithm of time. Then, A_1, a_σ and D_p are uniquely determined by simultaneously adjusting the curves calculated from Equation 2 and Equation 3 to the creep and recovery data. It may be noted that these latter parameters are either only present in one of the equations or are differentiated with respect to their influence with respect to time in Equations 2 and 3.

3.3. Determination of parameters

3.3.1. Single specimen and multi-specimen methods

In the previously reported work [4] a new specimen was tested for each stress level (multi-specimen method). Here, in order to develop a simpler and more rapid identification procedure, an accelerated characterisation sequence was also tested. The parameters were determined using successive creep/recovery cycles on the same yarn at increasing stress levels. For this method the mechanical history of the fibres is considered to be removed when the sample is loaded to a higher stress level than that encountered in previous cycles. To validate this approach the identification procedure has been performed for both single specimen and multi-specimen. **Erreur ! Source du renvoi introuvable.** to **Erreur ! Source du renvoi introuvable.** show the results. Overall the two methods provide very similar values. Only a_σ appears to vary according to the type of identification employed. This suggests that the stress/time dependency is influenced by the mechanical loading history of the sample. It may also be noted that D_p becomes negative for stresses above 0.2 N/tex with the multi-specimen method. This evolution, physically impossible, results from the method used to determine p, because p was obtained by cycling a single specimen and increasing the stress level. This result, a consequence of including single specimen results in the multi-

specimen optimisation, shows the importance of the mechanical loading history on the viscoplastic behaviour of these fibres.

3.3.2. Analysis of the stress dependence of parameters

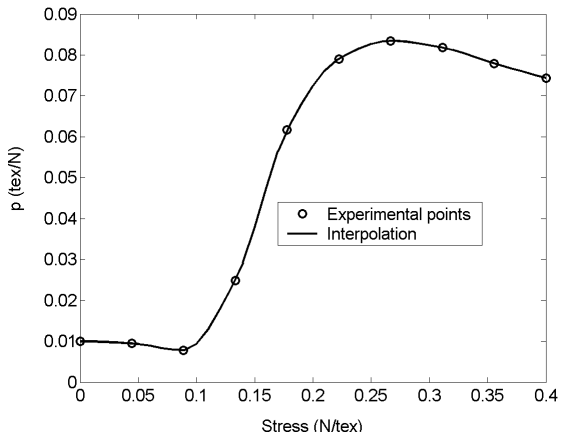
It should first be noted that all the parameters vary with applied stress. All mechanisms involved in the strain, both reversible and irreversible time dependent and independent, are thus affected by non-linearities. It is interesting to note that each parameter goes through a maximum. For A_0, A_1 and D_p, the transition extends from 0.1 N/tex to 0.2 N/tex. This transition is related to flow mechanisms followed by the increase in stiffness observed on **Erreur ! Source du renvoi introuvable.**.

The instantaneous creep compliance is simply the non-linear elastic behaviour shown in **Erreur ! Source du renvoi introuvable.**. Initially the compliance increases, corresponding to a drop in stiffness. Then, after going through a maximum value (between 0.15 et 0.2 N/tex) A_0 decreases while the fibre stiffens.

There are also correlations between the parameters. For example, A_1 and D_p both show a maximum between 0.1 and 0.15 N/tex, while g_1 and a_σ, have a maximum between 0.05 and 0.1 N/tex.

The delayed strain mechanism, both reversible (A_1) and irreversible (D_p) seems to be associated with the same molecular movement sequences. This agrees with the phenomenological model proposed by Northolt [7]. The strain rates A_1 et D_p both increase as the loading causes a reorganisation of entanglements and unfurling of molecules. At 0.12 N/tex the direction of the variation of these two strain rates change suddenly. The physical cause of this transition is not fully understood. The subsequent decrease is greater for the plastic strain rate which reaches zero at 0.3N/tex. As noted previously in **Erreur ! Source du renvoi introuvable.**, it appears that the capacity of the macromolecular network to deform irreversibly is exhausted above a certain stress level. In this same region ($\sigma > 0.12$ N/tex) the reversible strain rate decreases as the orientation of macromolecules tends towards the fibre axis. It may also be noted that the influence of D_p on residual strain is not negligible compared to the instantaneous plastic component. The behaviour of these fibres is thus truly viscoplastic.

Figure 7. Parameter p measured values (mean from 6 tests at each level) and interpolated curve.



3.4.Validation of the model

In order to validate the model proposed above a sequence of loading to different levels was applied to a polyester yarn. This sequence was also introduced into Equation 1. The identified parameters were incorporated into the model in their interpolated form (the continuous lines in **Erreur ! Source du renvoi introuvable.** to **Erreur ! Source du renvoi introuvable.**). As it is not possible to measure them at zero stress the values of the parameters p, A_0, A_1 and D_p were chosen arbitrarily with respect to the closest measured values (at 0.05 N/tex). The values of $g_1(0)$ et $a_\sigma(0)$ are fixed equal to 1, according to the assumptions of the model.

Figure 8. Parameter as a function of applied stress, and interpolation plot

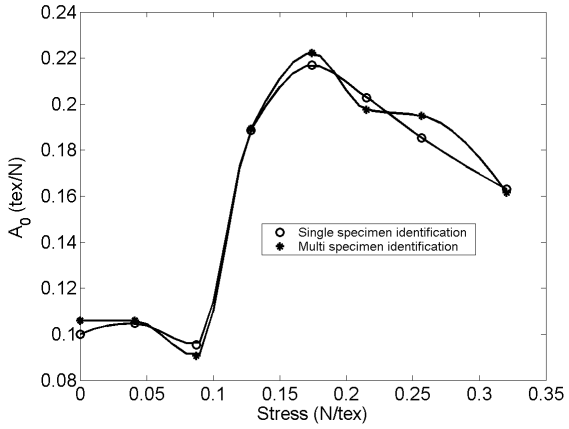


Figure 9. Parameter as a function of applied stress, and interpolation plot

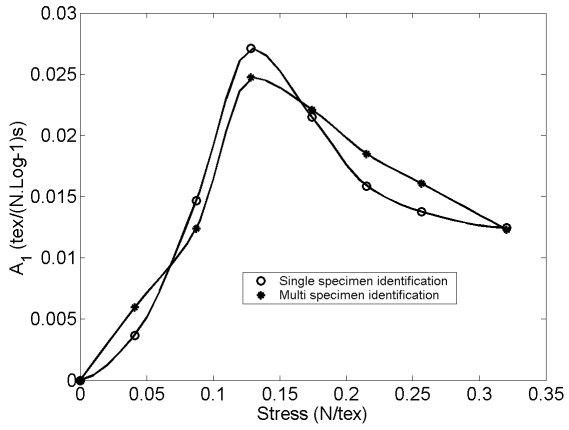


Figure 10. Parameter as a function of applied stress, and interpolation plot

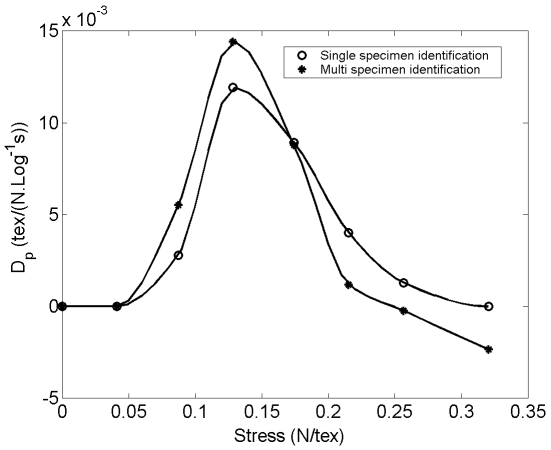


Figure 11. Parameter as a function of applied stress, and interpolation plot

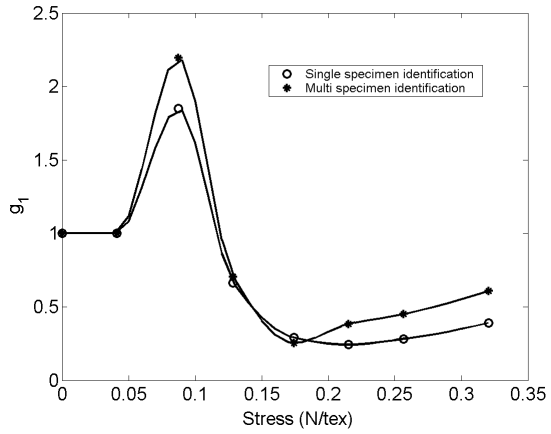


Figure 12. Parameter as a function of applied stress, and interpolation plot

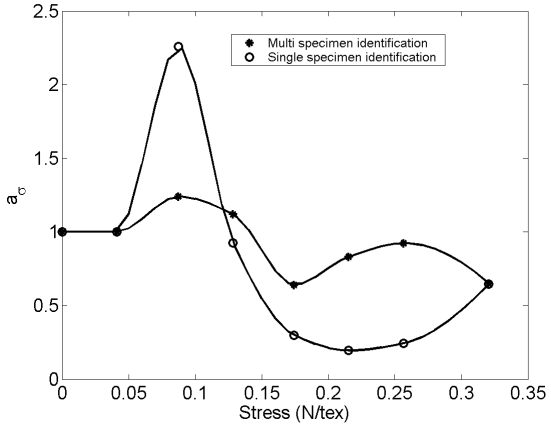
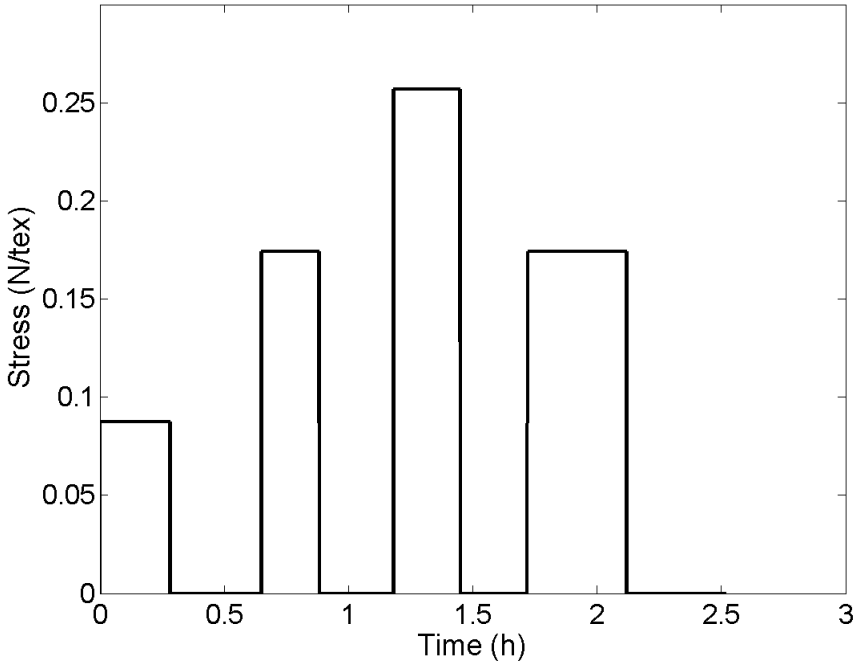
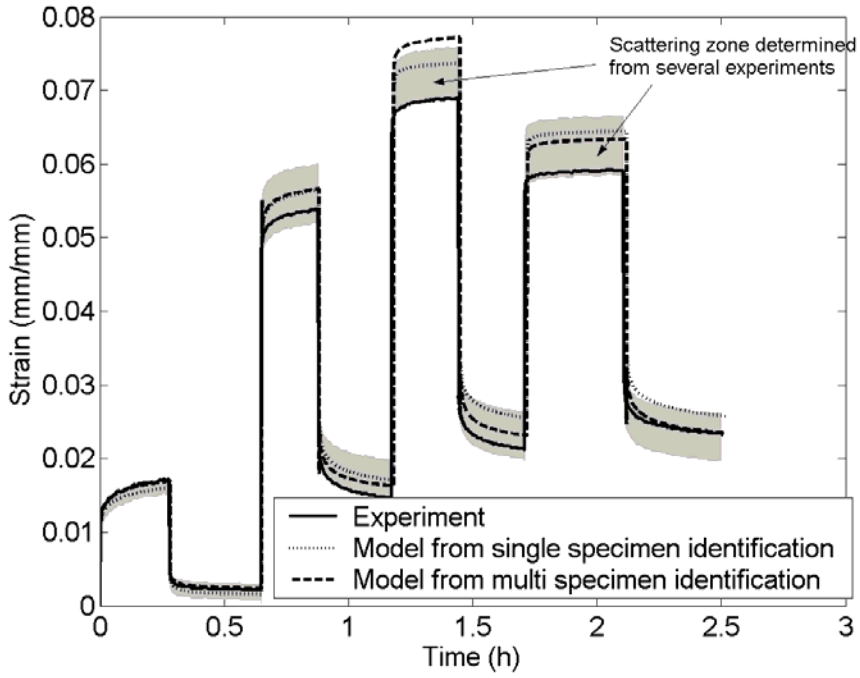


Figure 13. Sequence of loading for model validation



The measured strains and the model prediction are shown in **Erreur ! Source du renvoi introuvable.**. The predictions are given for two sets of parameters, one from the multi-specimen and one from the single specimen identification. The two predicted results are quite similar. However, the predicted residual strains at the end of the validation test are different. This is due to the fact that the parameter D_p was found to be negative above 0.2 N/tex in the multi-specimen identification which results in an under-estimation of the plastic strain.

Figure 14. Validation test for model. Comparison between measured and calculated strains



Overall, the model gives a good prediction of the strain in polyester fibres. It correctly predicts the non-linear behaviour and the accumulation of plastic strain. The strain is overestimated after a certain period, but the difference is within experimental scatter. Even though the viscoplastic phenomena and the acceleration factor are not characterised in a completely rigorous manner it appears that the accelerated identification may suffice to provide a good estimate of the mechanical behaviour of polyester fibres.

4. Conclusion

Schapery's macroscopic model, adapted to the non-linear viscoelastic viscoplastic behaviour of synthetic fibres, can be applied to the prediction of the mechanical behaviour of polyester fibres under a random loading sequence. The creep and recovery rates are modelled correctly and the predicted values of plastic strain correspond to measured values.
In order to determine the model parameters an experimental procedure has been developed.: This involves the use of video extensometry to allow strain measurements to be obtained without contact. This enables recovery to be followed without disturbance. The parameters are calculated by optimisation, adjusting the theoretical curves to the experimental points following a procedure which leads to a unique set of parameters.
The analysis of the parameters obtained reveals that for the polyester fibre studied here the non-linear mechanical behaviour is both viscoplastic and viscoelastic. The irreversible strain is a function of time so the behaviour is viscoplastic. However, the plastic strain no longer increases above a certain load level, indicating a limit to the irreversible molecular reorganisation. Also the analysis of the evolution of the reversible and irreversible strain rates shows that they are related to the same macromolecular phenomena. A fast identification procedure, using a single specimen has also been evaluated, and results have been validated using the model.
This model appears well suited to describe the behaviour of polyester fibres. It could thus be used as input to a model taking account of the construction of ropes and cables. Alternatively the identification procedure can be directly applied at the rope level to provide the basis for the prediction of the long term behaviour of large cables. This has been done and will be reported subsequently.

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