On the use of time-frequency domains for the improvement of the Stochastic Matched Filter Pulse Compression scheme with a High Speed Computing Architecture

Philippe Courmontagne[1]
[1]ISEN Toulon - IM2NP
UMR CNRS 6242
Equipe Signaux et Systèmes
Maison des Technologies, Place George Pompidou
83000 TOULON
FRANCE
Email: <firstname.lastname>@isen.fr

Gregory Julien[1,2], Marie Edith Bouhier[2]
[2]IFREMER
Zone portuaire de Brégaillon
BP 330
83507 LA SEYNE SUR MER
FRANCE
Email: <firstname.lastname>@ifremer.fr

***Abstract*—In several domains of engineering technologies, such as telecommunication, sonar imaging, positioning systems, radar, medical imaging ..., the main problem is to identify a transmitted useful pulse in a noise-corrupted received signal. A solution to this problem consists in using a modulated pulse for emission and a matched filter for reception. Such a concept is known as pulse-compression. Taking into account the main assumptions of the matched filter theory, the use of the bandpass transmitted pulse as matched filter's impulse response is only available if the useful signal is well known and if the noise is white, which is not the case in practice. For this reason, it has been recently proposed an alternative to the classical pulse-compression scheme taking into account the random nature of the useful signal and the coloration of the noise. This new principle is known as the SMF-PC. Although, this method allows a great improvement in terms of signal to noise ratio compared to the classical pulse-compression technique, the SMF-PC appears under optimal due to stationary assumptions. In this context, the purpose of this paper is to propose an improvement of the SMF-PC by coupling this technique with a time-frequency method. Results obtained on synthetic and real data are proposed and discussed.**

***Index Terms*—Pulse-compression; matched filter; stochastic matched filter; Wigner-Ville transform; linear chirp**

I. INTRODUCTION

Pulse Compression technique is used in several domains of engineering such as telecommunication, sonar imaging [1], acoustic positioning systems, radar [2], [3], medical imaging [4]. In those applications, the main problem is to identify a pulse in a noisy environment. The well-known Pulse Compression technique consists in using a modulated pulse (a chirp for example) for emission and a matched filter for reception [5]. This technique enhances the range resolution and the signal to noise ratio (SNR). Nevertheless, to ensure the theoretical properties of the pulse compression, in terms of range resolution and SNR gain, the useful signal has to be well known and the noise must be a realization of a white noise. In [6], it has been shown that these two main assumptions are not available in an underwater acoustic propagation context and more especially because of the accuracy of the electronic components, the Doppler effect and the transmission channel itself, the useful signal must be considered as the realization of a stochastic process. For these reasons, the use of this Pulse Compression technique is more often under optimal. As an example, let us consider the experiment proposed in figure 1, where two signals are pulse-compressed using the same method.

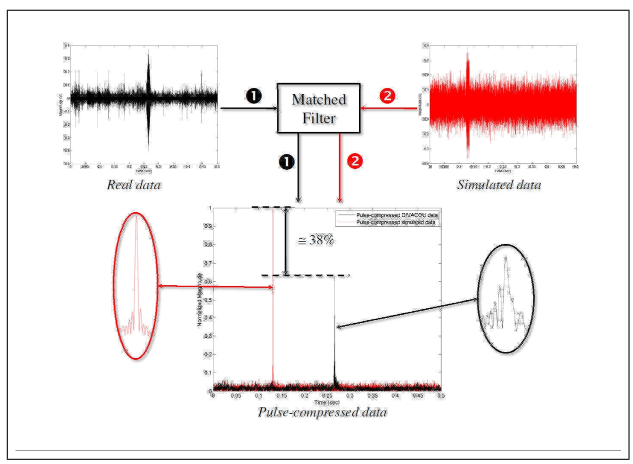

Fig. 1. *Main differences between a real pulse-compressed data and a simulated one using the classical matched filter approach*

The first signal (the black one) represents a real data recorded during an IFREMER campaign (the DIVACOU campaign). It corresponds to a received linear chirp (the emitted pulse has the following characteristics: $T = 1\,s$, $\Delta f = 1.5\,kHz$ and $f_0 = 1.25\,kHz$), disturbed by a colored noise. The second signal (the red one) represents a simulated data corresponding to the theoretical emitted linear chirp used during the DIVACOU campaign disturbed by a white Gaussian noise. For the two signals, the signal to noise ratio is the same. An analysis of the pulse-compressed data reveals on the one hand a significant loss in detection range (as the main lobe magnitude in the real data case is $38\,\%$ smaller than the theoretical one equal to $T\Delta f$) and, on the other hand, a main lobe deformation corresponding to a range resolution deterioration. Taking into account the real characteristics of the useful signal and the noise, a new technique called the Stochastic Matched Filter Pulse Compression (SMF-PC) has been proposed in [6]. Results obtained on real data coming from an IFREMER campaign have revealed the efficiency of such an approach, allowing a far reduction of the noise level in the pulse-compressed data. Nevertheless, two key points could be improved:

- The SNR gain: when the useful input signal's power is too low, the SMF-PC is ineffective because the useful information is destroyed;
- The range resolution: the main-lobe width of the compressed signal by the SMF-PC method is the same than using the classical pulse compression method and so presents a higher width than the optimal compressed one.

A strong assumption of the Stochastic Matched Filter theory is the stationarity of the signals (i.e. each sample of the studied signals carries the same frequency information). But a modulated pulse can not be considered as a realization of a stationary process. So, in some ways, the use of the SMF to process the pulse compression is under optimal and this explains the two previous referenced key points. In this context, the well known time-frequency methods are a good approach. For this reason, the purpose of this paper is to propose an improvement of the SMF-PC by coupling this technique with a time-frequency method. Indeed, as the SMF-PC is applied using a sliding sub-window processing and due to the SMF stationary assumption, each sample of the studied window is processed the same way; the use of a time-frequency approach allows to answer the non-stationary characteristic of the useful signal, each sample being processed taking into account the frequencies it carries.

After a brief recall of the SMF-PC principle and a discussion on some results, we present in a third section the new pulse-compression scheme based on the use of the Wigner-Ville transform coupled with the SMF-PC. Next, in a fourth section, we confront this new concept to simulated and to real data coming from an IFREMER campaign. Unfortunately, an important drawback of this method is the computing cost. For this reason, we propose in the last part of this article to implement this new pulse-compression scheme using a parallel architecture.

II. The Stochastic Matched Filter Pulse Compression scheme (SMF-PC)

The principle of the SMF-PC, described in [6], is to modify the classical pulse-compression scheme using a de-noising process, known as the stochastic matched filtering method [7].

A. The Stochastic Matched Filter (SMF)

The Stochastic Matched Filter, first proposed by Cavassilas in [7], consists in expanding a noise-corrupted signal into series of functions with uncorrelated random variables for decomposition coefficients. The basis functions are obtained by maximizing the signal to noise ratio, described as a generalized Rayleigh quotient, where the kernels are the signal and the noise autocorrelation functions. So that, each basis function contributes to an improvement of the signal to noise ratio after processing.

Let us consider a noise-corrupted signal $\boldsymbol{Z}$, made of M successive samples and corresponding to the superposition of a signal of interest $\boldsymbol{S}$ with a colored noise $\boldsymbol{N}$. If we consider the signal and noise variances, σ_S^2 and σ_N^2, we have:

$$\boldsymbol{Z} = \sigma_S \boldsymbol{S_0} + \sigma_N \boldsymbol{N_0}, \tag{1}$$

with $E\left\{\boldsymbol{S_0}^2\right\} = 1$ and $E\left\{\boldsymbol{N_0}^2\right\} = 1$. In the previous relation, reduced signals $\boldsymbol{S_0}$ and $\boldsymbol{N_0}$ are assumed to be independent, stationary and with zero mean.

It can be shown that an approximation $\widetilde{\boldsymbol{S}}_{\boldsymbol{Q}}$ of the signal of interest (the filtered noise-corrupted signal) can be obtained as follows:

$$\widetilde{\mathbf{S}}_{\mathbf{Q}} = \sum_{m=1}^{Q} z_m \boldsymbol{\Psi}_{\mathbf{m}}, \tag{2}$$

where basis vectors $\boldsymbol{\Psi}_{\mathbf{m}}$ are given by:

$$\boldsymbol{\Psi_m} = \boldsymbol{\Gamma_{N_0 N_0}} \widetilde{\boldsymbol{\Phi}}_{\boldsymbol{m}}, \tag{3}$$

with:

$$\widetilde{\boldsymbol{\Phi}}_{\boldsymbol{m}} = \boldsymbol{\Phi_m} / \sqrt{\boldsymbol{\Phi_m}^T \boldsymbol{\Gamma_{N_0 N_0}} \boldsymbol{\Phi_m}} \tag{4}$$

and where the basis $\{\boldsymbol{\Phi}_{\mathbf{m}}\}$ ensures a maximization of the signal to noise ratio and is made up with M-dimensional deterministic vectors solution of the following generalized eigenvalues problem:

$$\boldsymbol{\Gamma}_{\mathbf{S_0 S_0}} \boldsymbol{\Phi}_{\mathbf{m}} = \lambda_m \boldsymbol{\Gamma}_{\mathbf{N_0 N_0}} \boldsymbol{\Phi}_{\mathbf{m}}, \tag{5}$$

$\boldsymbol{\Gamma}_{\mathbf{S_0 S_0}}$ and $\boldsymbol{\Gamma}_{\mathbf{N_0 N_0}}$ corresponding to the signal and noise reduced covariances respectively.

The decomposition coefficients z_m in (2) are some random variables coming from the noisy data:

$$z_m = \boldsymbol{Z}^T \widetilde{\boldsymbol{\Phi}}_{\boldsymbol{m}}. \tag{6}$$

The decomposition order Q in (2) is chosen in order to ensure a mean square error minimization between the useful signal $\boldsymbol{S_0}$ and its approximation $\widetilde{\mathbf{S}}_{\mathbf{Q}}$. It has been shown in [8] that this mean square error is minimized when Q corresponds to the number of eigenvalues λ_m verifying:

$$\lambda_m \left.\frac{S}{N}\right|_{\boldsymbol{Z}} > 1, \tag{7}$$

$\left.\frac{S}{N}\right|_{\boldsymbol{Z}}$ denoting the signal to noise ratio before process.

Consequently, if the noisy data presents a high enough signal to noise ratio, many $\boldsymbol{\Psi_m}$ will be considered for the filtering (so that $\widetilde{\boldsymbol{S}}_{\boldsymbol{Q}}$ tends to be equal to $\boldsymbol{Z}$), and in the opposite case, only a few number will be chosen inducing a strong smoothing effect.

B. The SMF-PC principle

Let K be the number of samples of the useful signal used at emission and let consider the K-dimensional vector $\boldsymbol{Z_k}$ corresponding to the data extracted from a window centered on index k of the noisy data, i.e.:

$$\boldsymbol{Z_k}^T = \left\{ Z\left[k - \frac{K-1}{2}\right], \ldots, Z[k], \ldots, Z\left[k + \frac{K-1}{2}\right] \right\}.$$

As the SMF is applied using a sliding sub-window processing, only the sample located in the middle of the window is estimated, so that relation (2) becomes:

$$\widetilde{S}_{Q[k]}[k] = \sum_{m=1}^{Q[k]} z_{m,k} \Psi_m \left[\frac{K+1}{2}\right], \tag{8}$$

with:

$$z_{m,k} = \boldsymbol{Z_k}^T \widetilde{\boldsymbol{\Phi}}_{\boldsymbol{m}} \tag{9}$$

and where $Q[k]$ corresponds to the number of eigenvalues λ_m times the signal to noise ratio of window $\boldsymbol{Z_k}$ greater than one, i.e.:

$$\lambda_m \left. \frac{S}{N} \right|_{\boldsymbol{Z_k}} > 1. \tag{10}$$

To estimate the signal to noise ratio of window $\boldsymbol{Z_k}$, the signal power is directly computed from the window's data and the noise power is estimated on a part of the received data $\boldsymbol{Z}$, where no useful signal *a priori* occurs. This estimation is realized using the maximum likelihood principle.

Using relations (8) and (9), the estimation of the de-noised sample value is realized by a scalar product:

$$\widetilde{S}_{Q[k]}[k] = \boldsymbol{Z_k}^T \underbrace{\sum_{m=1}^{Q[k]} \Psi_m \left[\frac{K+1}{2}\right] \widetilde{\boldsymbol{\Phi}}_{\boldsymbol{m}}}_{\boldsymbol{h_{Q[k]}}}. \tag{11}$$

Using these values, a K-dimensional vector $\widetilde{\boldsymbol{S}}_{\boldsymbol{k}}$ made up with samples $\widetilde{S}_{Q[m]}[m]$, $\forall m \in \mathbb{N}$ such as $m \in \left[k - \frac{K-1}{2}; k + \frac{K-1}{2}\right]$ is constructed.

Let $\boldsymbol{C}$ be the useful signal used at emission. The pulse-compression principle consists in making the correlation between $\boldsymbol{C}$ and the received data $\boldsymbol{Z}$. As the use of the SMF modifies the nature of the data, it is necessary to modify the same way the useful signal $\boldsymbol{C}$.

Let $\widetilde{\boldsymbol{C}}_{\boldsymbol{q}}$ be the result obtained applying the SMF on $\boldsymbol{C}$, considering for the whole signal the same number q of eigenvalues, i.e.:

$$\widetilde{C}_q[k] = \boldsymbol{C_k^0}^T \boldsymbol{h_q} \quad \forall k = 1 \ldots K, \tag{12}$$

where:

$$\boldsymbol{C_k^0}^T = \left\{ C^0\left[k - \frac{K-1}{2}\right], \ldots, C^0[k], \ldots, C^0\left[k + \frac{K-1}{2}\right] \right\},$$

$\boldsymbol{C^0}$ being a $(2K-1)$-dimensional vector constructed from $\boldsymbol{C}$ completed with zero on its edges.

Next the k^{th} pulse-compressed sample $Z^{\text{SMF-PC}}[k]$ is obtained using the scalar product between vector $\widetilde{\boldsymbol{S}}_{\boldsymbol{k}}$ and $\widetilde{\boldsymbol{C}}_{\boldsymbol{q=Q[k]}}$, i.e.:

$$Z^{\text{SMF-PC}}[k] = \widetilde{\boldsymbol{S}}_{\boldsymbol{k}}^T \widetilde{\boldsymbol{C}}_{\boldsymbol{q=Q[k]}}. \tag{13}$$

Doing so, first, window $\boldsymbol{Z_k}$ is de-noised using, for each sample and depending on the signal to noise ratio, the most appropriate number of eigenvalues and, next, the data is compressed using the pulse approximation obtained using the same number of eigenvalues than for the middle sample of the studied window.

C. Experiments

To illustrate the SMF-PC, let us consider, as received pulse, a linear chirp (duration: $10.5\,ms$, central frequency: $26\,kHz$, bandwidth: $8.6\,kHz$). This one is proposed at the figure 2 and corresponds to a real data directly recorded at the output of an hydrophone. The simulated received data $\boldsymbol{Z}$ corresponds to an additive mix of the useful signal $\boldsymbol{S}$ (the pulse) and a colored noise $\boldsymbol{N}$ corresponding to some marine signatures coming from an IFREMER campaign. This disturbing signal is presented on the figure 3.

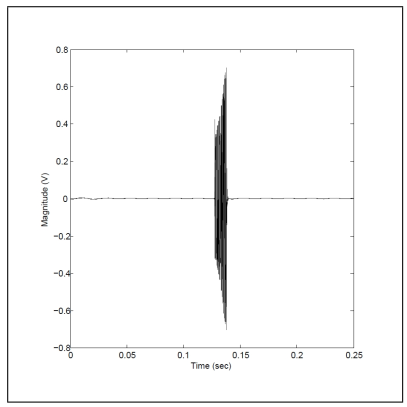

Fig. 2. *Useful signal $\boldsymbol{S}$ corresponding to a real data directly recorded at the output of an hydrophone*

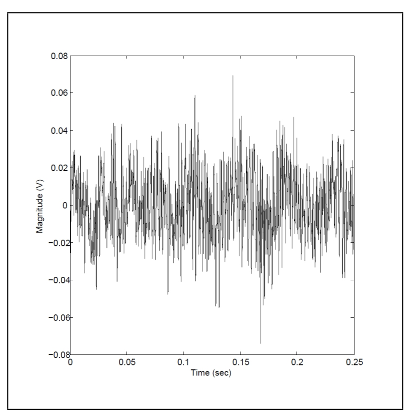

Fig. 3. *Colored noise $\boldsymbol{N}$ corresponding to some marine signatures*

In these conditions, the received pulse appears $0.127\,s$ after the beginning of the received data. In order to control the signal to noise ratio, we introduce a parameter g as follows:

$$\boldsymbol{Z} = \boldsymbol{S} + g\boldsymbol{N}.$$

The signal to noise ratio of the received data corresponds to the ratio between the signal and the noise powers in the same spectral bandwidth, i.e.:

$$\frac{\displaystyle\int_{\Delta\nu} \gamma_{SS}(\nu)d\nu}{\displaystyle\int_{\Delta\nu} g^2\gamma_{NN}(\nu)d\nu},$$

where $\Delta\nu$ corresponds to the intersection of the signal and noise spectral bandwidths and with $\gamma_{SS}(\nu)$ and $\gamma_{NN}(\nu)$ as signal and noise power spectral densities. While varying the control parameter g, several realizations of the observation were built.

The received signal is bandpass filtered in the useful signal bandwidth (i.e. for $\nu \in [21.7\,kHz; 30.3\,kHz]$) and is sampled using a $100\,kHz$ sampling frequency. Doing so the number of samples to describe the pulse is equal to 1001 and will correspond to the K value afterwards.

Let us consider first a $5\,dB$ SNR case. The bandpass filtered received signal is presented on figure 4.

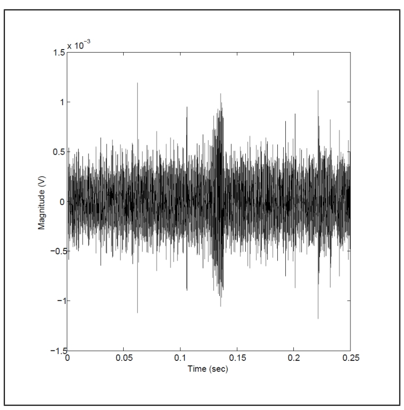

Fig. 4. *Bandpass filtered received signal in a $5\,dB$ SNR case*

Applying the SMF-PC on this signal allows to obtain the pulse-compressed data presented on figure 5 in linear magnitude and on figure 6 in dB-magnitude. In order to quantify the relevance of this process compared to the classical approach, the pulse-compressed data obtained by the use of the matched filter theory are presented on the same graphs in black lines. These results show, on the one hand, that the new process allows to locate the received pulse the same way than using the classical approach and, on the other hand, a far better noise rejection with an average noise level $12\,dB$ lower than using the classical pulse-compression.

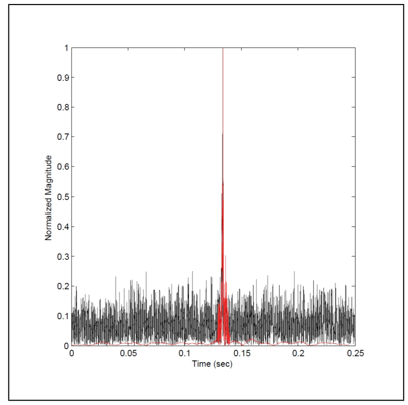

Fig. 5. *Normalized pulse-compressed data obtained applying the classical PC in black lines and using the SMF-PC in red lines; in both cases, a peak near the 0.127^{th} second reveals the useful signal presence, the main difference between the two methods being the noise level*

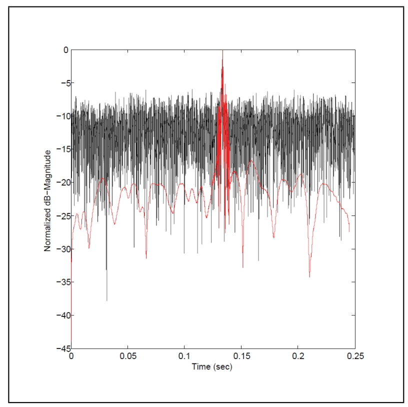

Fig. 6. *Normalized pulse-compressed data in dB magnitude; the SMF-PC method allows to obtain an average noise level $12\,dB$ lower than using the classical approach*

Nevertheless, when we consider a more unfavorable SNR case, such as $0\,dB$ for example (see figure 7 for the noisy received data), the SMF-PC reaches its limits (see figure 8 for the normalized pulse-compressed data in dB magnitude).

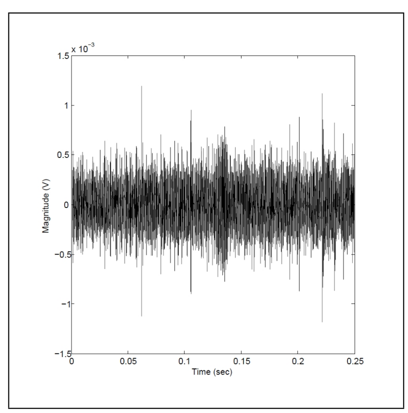

Fig. 7. *Bandpass filtered received signal in a $0\,dB$ SNR case*

Although a peak near the 0.127^{th} second reveals the useful signal position, but the width of this peak is very huge compared to the one obtained applying the classical approach and the noise rejection is not clearly evident; so that both the range resolution and the detection range are degraded.

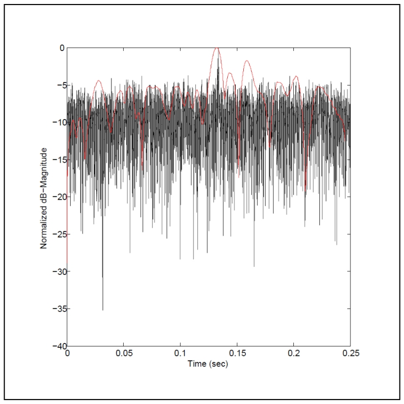

Fig. 8. *Normalized pulse-compressed data in dB-magnitude in red lines for the SMF-PC and black lines for the classical approach*

Several experiments for different received signal configurations have revealed that this is always the case when the SNR is lower or equal than $0\,dB$. Such a problem can be simply explain, on the one hand, if we consider the SMF assumptions and, on the other hand, if we take care of the SMF-PC key point. Indeed, for the first point, by assumptions the useful signal and the noise are considered as stationary signal, but a linear chirp can not be considered as a stationary signal (i.e. each sample of the signal carries the same frequencies). For the second point, the number of eigenvalues retained to process the studied window is conditioned by the sub-window signal to noise ratio; if this SNR is too low, only one eigenvector will be retained to process the data, which corresponds to a strong smoothing of the observation and so a cancellation of the received pulse. For the SNR lower than $0\,dB$ the received pulse power is not sufficiently significant to lead to a high number of eigenvalues.

To bypass this problem, it appears of great interest to interact the SMF-PC with a time-frequency approach. Indeed, it is well known that time-frequency approaches are well appropriate to study non-stationary signals. Furthermore, the use of the time-frequency plane associated to the received signal will allow to estimate the SNR using a strategy based on line integration. In the scientific literature, it exists several ways to compute a time-frequency plane, one of them, the Wigner-Ville transform [9] is perfectly localized when applied to a linear chirp [10]; for this reason, we will use this transform to enhance the SMF-PC performances.

III. The Wigner-Ville Stochastic Matched Filter Pulse-Compression scheme (WV-SMF-PC)

A. The Wigner-Ville and Pseudo-Wigner-Ville distributions

The Wigner-Ville distribution is defined by

$$W_s(t,\nu) = \int_{\mathcal{R}} s\left(t+\frac{\tau}{2}\right) s^*\left(t-\frac{\tau}{2}\right) \exp(-2i\pi\nu\tau)d\tau, \quad (14)$$

where $*$ designates the complex conjugate. This distribution presents many interesting and well-known properties [10], especially it presents a perfect localization for linear chirp signals. Indeed, if we consider a linear chirp signal having a bandwidth Δf, a central frequency ν_0 and a unitary length:

$$p(t) = \exp\left[2i\pi\left(\nu_0 t + \frac{\Delta f}{2}t^2\right)\right], \quad (15)$$

its Wigner-Ville distribution is given by:

$$W_p(t,\nu) = \delta(\nu - (\nu_0 + t\Delta f)). \quad (16)$$

Unfortunately, if we consider two signals $s_1(t)$ and $s_2(t)$, as the Wigner-Ville distribution is a bilinear function, the quadratic superposition principle gives:

$$W_{s_1+s_2}(t,\nu) = W_{s_1}(t,\nu) + W_{s_2}(t,\nu) + 2\Re\left\{W_{s_1,s_2}(t,\nu)\right\}, \quad (17)$$

where:

$$W_{s_1,s_2}(t,\nu) = \int_{\mathcal{R}} s_1(t+\frac{\tau}{2}) s_2^*(t-\frac{\tau}{2}) \exp(-2i\pi\nu\tau)d\tau. \quad (18)$$

So that, when the studied signal is made up with several signals of various kinds, some interference terms appear, which can be troublesome for the signal interpretation.

To limit the influence of these interference terms, one can use the Pseudo-Wigner-Ville distribution defined by the following relation:

$$PW_s(t,\nu) = \int_{\mathcal{R}} h(\tau) s\left(t+\frac{\tau}{2}\right) s^*\left(t-\frac{\tau}{2}\right) \exp(-2i\pi\nu\tau)d\tau, \quad (19)$$

where $h(t)$ is a regular window (an Hamming window, for example). Thus, because of their oscillating nature, the interferences will be attenuated in the pseudo-Wigner-Ville distribution compared to the Wigner-Ville distribution.

B. The WV-SMF-PC principle

The basic idea of the WV-SMF-PC is to apply the stochastic matched filter in the time-frequency plane obtained by the way of the pseudo-Wigner-Ville transform in order to take into account the non-stationarity of the useful signal. Furthermore, as the determination of the truncature order Q depends on the SNR of the studied window extracted from the noisy data, this SNR will be estimated using the time-frequency plane.

A well known property of the Wigner-Ville transform is the energy conservation: by integrating the Wigner-Ville plane of a signal $\boldsymbol{S}$ over the time-frequency plane, we obtain the energy E_S of $\boldsymbol{S}$, i.e.:

$$E_S = \int_{\mathbb{R}} \int_{\mathbb{R}} W_S(t,\nu)dtd\nu. \quad (20)$$

As the Wigner-Ville distribution presents a perfect localization for linear chirp signals, a way to estimate the useful signal power is to integrate the pseudo-Wigner-Ville plane of the noisy data over the time-frequency domain of the chirp and next to subtract from the obtained result the noise power estimated on the same domain, i.e.:

$$E\left\{\boldsymbol{S}^2\right\} = \int_{\mathbb{R}}\int_{\mathbb{R}} PW_Z(t,\nu)W_p(t,\nu)dtd\nu \dots$$
$$\dots - \int_{\mathbb{R}}\int_{\mathbb{R}} PW_N(t,\nu)W_p(t,\nu)dtd\nu, \qquad (21)$$

where $PW_Z(t,\nu)$ and $PW_N(t,\nu)$ designate the pseudo-Wigner-Ville plane of the noisy received data and the noise, respectively ($W_N(t,\nu)$ being previously estimated in an homogeneous area of the noisy data) and where $W_p(t,\nu)$ represents the Wigner-Ville plane of the transmitted pulse.
This way, in the case of a linear chirp with a duration T and considering (16), it comes:

$$E\left\{\boldsymbol{S}^2\right\} = \int_0^T PW_Z(t,\nu_0+t\Delta f)dt - \int_0^T PW_N(t,\nu_0+t\Delta f)dt. \qquad (22)$$

Let $E\left\{\boldsymbol{N}^2\right\}$ be the noise power estimated in an homogeneous area of the received data by the way of its pseudo-Wigner-Ville plane:

$$E\left\{\boldsymbol{N}^2\right\} = \int_0^T PW_N(t,\nu_0+t\Delta f)dt. \qquad (23)$$

In these conditions, the decomposition order $Q(t)$ retained to process the data received at an instant t will correspond to the number of eigenvalues λ_m verifying the following inequality:

$$\left(\frac{\int_t^{t+T} PW_Z(\tau,\nu_0+\tau\Delta f)d\tau}{E\left\{\boldsymbol{N}^2\right\}} - 1\right)\lambda_m > 1. \qquad (24)$$

One can show that the classical pulse-compression can be achieved by integrating the pseudo-Wigner-Ville plane of the received noisy data over the time-frequency domain of the transmitted pulse, i.e.:

$$Z^{PC}(t) = \int_t^{t+T} PW_Z(\tau,\nu_0+\tau\Delta f)d\tau. \qquad (25)$$

In the same way, the pulse-compression of the received data using the stochastic matched filter theory can be realized as follows in the case of a linear chirp signal:

$$Z^{\text{WV-SMF-PC}}(t) = \int_t^{t+T} H_{Q(t)}(\nu_0+\tau\Delta f)PW_Z(\tau,\nu_0+\tau\Delta f)d\tau, \qquad (26)$$

or more generally:

$$Z^{\text{WV-SMF-PC}}(t) = \int_t^{t+T} H_{Q(t)}(\nu)PW_Z(\tau,\nu)W_p(\tau,\nu)d\tau d\nu, \qquad (27)$$

where $H_{Q(t)}(\nu)$ corresponds to the frequency response associated to $h_{Q(t)}(t)$ constructed from the $\{\boldsymbol{\Phi_m}\}$ and $\{\boldsymbol{\Psi_m}\}$ basis coming from the SMF theory:

$$H_{Q(t)}(\nu) = \int_{\mathbb{R}} h_{Q(t)}(\tau)\exp(-2i\pi\nu\tau)d\tau, \qquad (28)$$

with $h_{Q(t)}(t)$ such as:

$$h_{Q(t)}(\tau_1-\tau_2) = \sum_{m=1}^{Q(t)} \Phi_m(\tau_1)\Psi_m(\tau_2). \qquad (29)$$

C. The WV-SMF-PC algorithm

The algorithm leading to the pulse-compression of the M-dimensional noisy data $\boldsymbol{Z}$, by the way of the WV-SMF-PC, is presented below.

1) Computation or estimation of reduced covariances $\boldsymbol{\Gamma_{S_0S_0}}$ and $\boldsymbol{\Gamma_{N_0N_0}}$ of signal of interest and noise respectively.
2) Determination of eigenvectors $\boldsymbol{\Phi_m}$ by solving the generalized eigenvalue problem described in (5).
3) Normalization of eigenvectors $\boldsymbol{\Phi_m}$ in respect of relation (4).
4) Computation of basis vectors $\boldsymbol{\Psi_m}$ according to (3).
5) Determination of impulse response $h_{Q(t)}(\tau)$, described in (29), for all $Q(t)$ values (raised by the number of eigenvalues λ_m).
6) Computation of the frequency response $H_{Q(t)}(\nu)$ (see relation (28)), for all $Q(t)$ values.
7) Estimation of the noise power in an homogeneous area of noisy data $\boldsymbol{Z}$ by the way of relation (23).
8) For $k = 1$ to M do:
   a) Sub-window $\boldsymbol{Z_k}$ data acquisition.
   b) Computation of the sub-window pseudo-Wigner-Ville plane.
   c) Determination of the number Q retained to process the sub-window (relation (24)).
   d) Determination of the pulse-compressed data $\boldsymbol{Z}^{\text{WV-SMF-PC}}$ according to (26).

D. Experimentations

As a first experiment, we have applied the WV-SMF-PC on the simulated received signal presented on figure 4. The result obtained is presented on figure 9 in normalized linear magnitude and on figure 10 in normalized dB-magnitude. An analysis of these results compared to those obtained applying the SMF-PC (see figures 5 and 6) reveals without ambiguity the contribution of the Wigner-Ville transform in terms of noise rejection. Indeed, the average noise level is $32\,dB$ lower than using the classical approach and so $20\,dB$ lower than using the SMF-PC. If we consider now the $0\,dB$ SNR case (see figure 7 for the simulated data), we can see that, contrary to the SMF-PC method, the WV-SMF-PC remains effective with a noise level near $30\,dB$ lower that using the classical approach (see figure 11 for the pulse-compressed data in dB-magnitude).

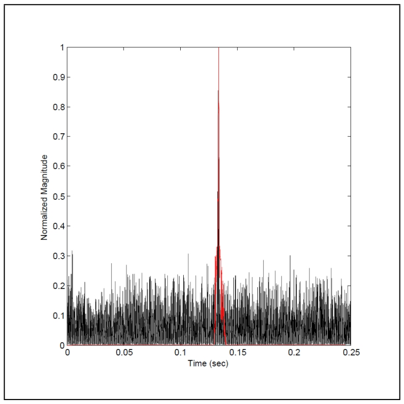

Fig. 9. *Normalized pulse-compressed data obtained applying the classical PC in black lines and using the WV-SMF-PC in red lines; the SNR of the simulated received data is equal to* $5\,dB$

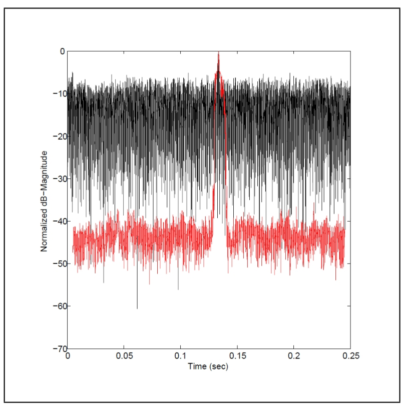

Fig. 10. *Normalized pulse-compressed data in* dB*-magnitude obtained applying the classical PC in black lines and the WV-SMF-PC in red lines*

In order to estimate the limit of this new process, it has been applied on a simulated received data with a SNR equal to $-5\,dB$. This signal is presented on figure 12. As shown on figures 13 in linear magnitude and 14 in dB-magnitude, for such a SNR and contrary to the WV-SMF-PC, it is impossible to identify the peak corresponding to the useful signal without false alarms when the pulse-compression is realized using the classical matched filter approach. Furthermore, the graph on figure 15 reveals that the width of the main lobe corresponding to the useful signal is really thin. Consequently, the use of the WV-SMF-PC for the pulse-compression allows to improve both the range detection and the range resolution. For SNR smaller than $-5\,dB$, several experimentations have revealed that the limits of the process are reached, the useful signal power being too small compared to the noise power. It may be possible to dismiss these limits by considering a reallocated spectrogram [11] in place of the pseudo-Wigner-Ville transform. Indeed, it is well known that the Wigner-Ville transform renders imperfect information about the energy distribution of the signal in the time-frequency plane (relation (16) is only a theoretical point of view) and a solution can consist on reconcentrate the signal's energy spread out by the transform. But such an operation could be really expensive in terms of computing time.

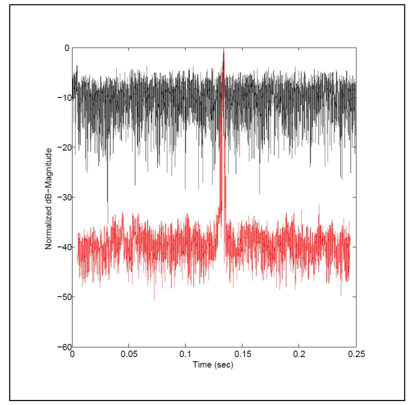

Fig. 11. *Normalized pulse-compressed data in* dB*-magnitude obtained applying the classical PC in black lines and using the WV-SMF-PC in red lines; the SNR of the simulated received data is equal to* $0\,dB$

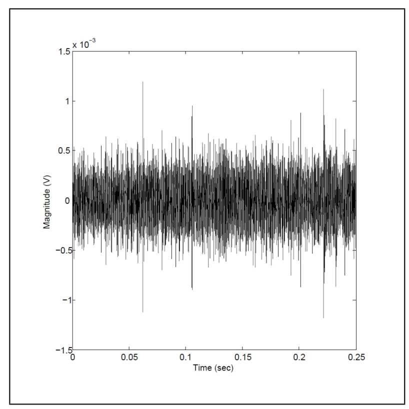

Fig. 12. *Simulated received data in a* $-5\,dB$ *SNR case*

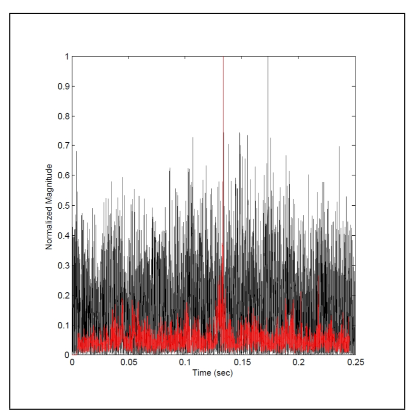

Fig. 13. *Normalized pulse-compressed data obtained applying the classical PC in black lines and using the WV-SMF-PC in red lines; the SNR of the simulated received data is equal to* $-5\,dB$

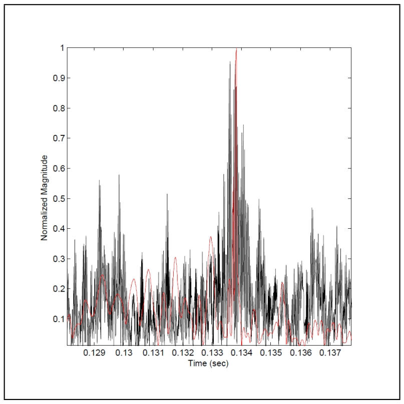

Fig. 14. *Normalized pulse-compressed data in dB-magnitude obtained applying the classical PC in black lines and using the WV-SMF-PC in red lines*

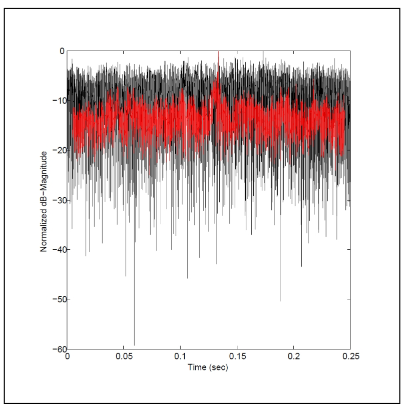

Fig. 15. *Detail extracted from the graph on figure 13*

Let consider now a real signal coming from an IFREMER (the French research institute for the sea exploitation) campaign: the DIVACOU campaign realized in September 2009. This signal corresponds to a measurement of a communication between an underwater beacon and a ship, in order to give to the ship the possibility to locate the beacon. The distance between the ship and the beacon in the horizontal plane was $300\,m$, the beacon immersion was $490\,m$. The beacon emits the linear chirp signal presented on figure 2 with a power of $190\,dB$. At reception, this pulse appears near the 0.25^{th} second with a SNR near $13\,dB$ (see figure 16). We present on the figures 17 and 18, the results obtained applying the classical pulse-compression scheme, the SMF-PC and the WV-SMF-PC in linear magnitude and dB-magnitude respectively. These results confirm the previous observations made on simulated signals, namely a strong improvement of the range detection.

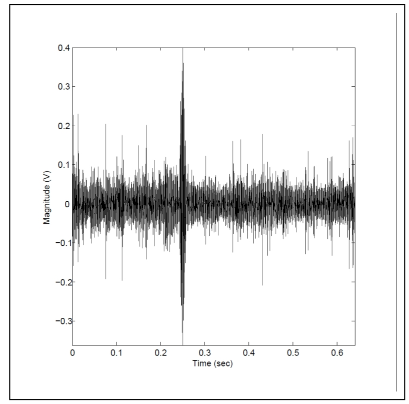

Fig. 16. *Received signal coming from the DIVACOU campaign; this signal has been sampled at $75\,kHz$, the pulse happens near the 0.25^{th} second; the SNR is approximatively equal to $13\,dB$.*

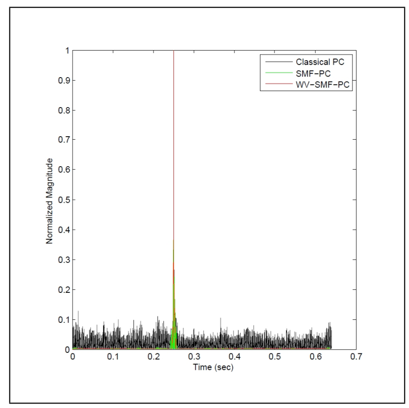

Fig. 17. *Normalized pulse-compressed data obtained applying the classical PC in black lines, the SMF-PC in green lines and the WV-SMF-PC in red lines; in all cases, a peak near the 0.25^{th} second reveals the useful signal presence, the main difference between the methods being the noise level.*

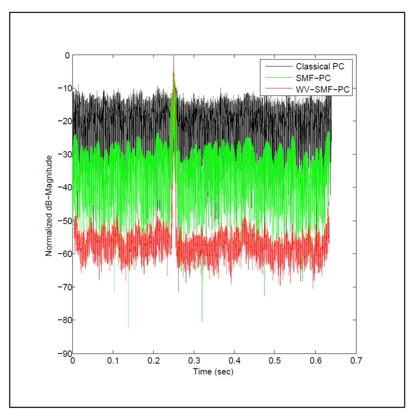

Fig. 18. *Normalized pulse-compressed data in dB magnitude; the WV-SMF-PC method allows to obtain an average noise level $40\,dB$ lower than using the classical approach and $20\,dB$ lower than using the SMF-PC.*

IV. High speed computing architecture

The WV-SMF-PC technique gives good results but its implementation is quite expensive in terms of computing time. For this reason, we propose in this section to implement it using a parallel architecture.

First of all, let's remark that only a small part of the time-frequency plane is required for the pulse-compression. Indeed, taking into account the characteristics of the transmitted pulse, only the frequencies surrounded by $\nu_{\min} = \nu_0 - \Delta f/2$ and $\nu_{\max} = \nu_0 + \Delta f/2$ have to be studied. Furthermore, as the duration of the pulse is equal to T and the sampling frequency equal to F_s, the pulse is described by K samples, with K equal to TF_s+1. It follows that K frequencies should be studied between $\nu_{\min}$ and $\nu_{\max}$, which corresponds to a frequency increment $\Delta\nu$ equal to $\Delta f/(K-1) = \Delta f/(TF_s)$. In this context, it appears not interesting to use a fast Fourier transform algorithm to compute the Wigner-Ville transform. Indeed, to obtain such a frequency increment using a FFT algorithm and taking into account that the sub-window size to compute the Wigner-Ville transform is equal to K, it will be necessary to pad with zeros the signal described by the sub-window, in order to obtain N_{FFT} samples, with N_{FFT} equal to $F_s^2 T/\Delta f$. For this reason, the computation of the pseudo-Wigner-Ville transform will be achieved directly applying the discrete form of the relation (19). Let $\boldsymbol{Z_k}$ be the following column vector:

$$\boldsymbol{Z_k} = \{Z[k],\, Z[k+1],\, \ldots,\, Z[k+K-1]\}^T\,, \tag{30}$$

$\boldsymbol{Z_k^{inv}}$ the following column vector:

$$\boldsymbol{Z_k^{inv}} = \{Z[k+K-1],\, Z[k+K-2],\, \ldots,\, Z[k]\}^T\,, \tag{31}$$

$\boldsymbol{H_m}$ a column vector made up with the coefficients of a Hamming window and $\boldsymbol{t}$ and $\boldsymbol{\nu}$ be two column vectors defined as follows:

$$\begin{cases} \boldsymbol{t} = \{0,\, 1/F_s,\, 2/F_s,\, \ldots,\, T\}^T \\ \boldsymbol{\nu} = \{\nu_{\min},\, \nu_{\min}+\Delta\nu,\, \ldots,\, \nu_{\max}\}^T \end{cases}. \tag{32}$$

All these vectors are K-dimensional.

In these conditions, the k^{th} row of the time-frequency plane described as a matrix (the row corresponding to the times and the column to the frequencies) is computed as follows:

$$PW_Z[k] = \left(\boldsymbol{H_m}.*\boldsymbol{Z_k}.*\boldsymbol{Z_k^{inv}}\right) \times \exp\left[-4i\pi(\boldsymbol{t}\otimes\boldsymbol{\nu}^T)\right], \tag{33}$$

where $.*$ denotes the term by term product and $\otimes$ the tensorial product.

Let us remark that the matrix described by the exponential could be previously computed and stored in a register.

Each computed row is stored in a $K \times K$ shift register. Next, a summation is realized on the time-frequency domain of the transmitted pulse (in the case of a linear chirp, the summation is realized on the diagonal of the $K \times K$ matrix corresponding to the pseudo-Wigner-Ville transform of a T-duration part of the noisy data $\boldsymbol{Z}$) to access to the signal power or to obtain the pulse-compressed k^{th} sample when the time-frequency plane has been previously multiplied by the frequency response $H_{Q[k]}(\nu)$ defined by (28).

In order to reduce the computing time, this can be done using several FPGA and shift registers. Let us suppose that the number of registers (or FPGA) is L; in this condition the dimension of the shift registers is $K \times (K/L)$. Each FPGA computes a part of the WV-SMF-PC, the vector $\boldsymbol{\nu}$ being divided into L blocks of dimension K/L, where the first block $\boldsymbol{\nu_1}$ corresponds to:

$$\boldsymbol{\nu_1} = \{\nu_{\min},\, \nu_{\min}+\Delta\nu,\, \ldots,\, \nu_{\min}+(K-1)\Delta\nu/L\}^T\,, \tag{34}$$

the second one to:

$$\boldsymbol{\nu_2} = \{\nu_{\min}+K\Delta\nu/L,\, \ldots,\, \nu_{\min}+(2K-1)\Delta\nu/L\}^T\,, \tag{35}$$

and so on. This way, we obtain L pulse-compressed data (one for each block), the summation of these results giving the final pulse-compressed data. Such an approach divides by L the computing time of this new pulse-compression scheme.

V. Conclusion

In this article, we have proposed a new pulse-compression principle taking into account the random nature of the transmitted pulse and the coloration of the disturbing signal. This new concept corresponds to an improvement of the SMF-PC method presented in [6] and is based on the use of the pseudo-Wigner-Ville transform (WV) coupled to the stochastic matched filter (SMF), which corresponds to the noisy data projection onto a basis ensuring a maximization of the signal to noise ratio. The use of the Wigner-Ville transform allows, on the one hand, to bypass the stationary assumptions required by the stochastic matched filter theory and, on the other hand, to better estimate the decomposition order of the noisy received signal expansion introduced by the stochastic matched filter. This way and at the same moment, the received data is de-noised and compressed taking into account the non-stationarity of the useful signal. Results obtained on real and synthetic data reveal the effectiveness of such a process. Indeed, on the one hand, the signal to noise ratio of the WV-SMF-PC pulse-compressed data appears far lower than the one obtained using either the classical method based on the matched filter notion or the SMF-PC technique used alone and, on the other hand, the range resolution remains the same or is slightly improved. Future works concerning this approach is the development of an electronic card dedicated to pulse-compression and based on the WV-SMF-PC theory using the parallel architecture in order to be compatible with real time constraints.

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