Estimating the North Atlantic mean surface topography by inversion of hydrographic and lagrangian data

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ABSTRACT

The non-linear inverse model described in Mercier et al. (1992) is used to estimate the mean surface topography of the North Atlantic and its formal error. The surface is based on a large hydrographic and Lagrangian data set and has spatial resolution of 2° in latitude by 2.5° in longitude. The formal error is between 4 and 8 cm. Contrary to previous estimations, this takes into account the errors with regard both to the density field and to the estimation of the velocity field at a reference level.


Estimation de la topographie moyenne de surface de l'Atlantique Nord par inversion de données hydrographiques et de flotteurs lagrangiens

Le modèle inverse non-linéaire décrit dans Mercier et al. (1992) est utilisé pour estimer la topographie moyenne de surface de l'Atlantique nord et son erreur formelle. Cette surface est estimée à partir d'un nombre important de données hydrographiques et de données de flotteurs Lagrangiens et a une résolution spatiale de 2° en latitude par 2.5° en longitude. L'erreur formelle est comprise entre 4 et 8 cm. Contrairement aux estimations précédentes, cette erreur prend en compte l'erreur sur le champ de densité et l'erreur sur l'estimation du champ de vitesse à un niveau de référence.


INTRODUCTION

In a recent paper, Mercier et al. (1992) produced a realistic description of the North Atlantic general circulation between 20°N and 50°N, using a non-linear inverse model. Employing the same inverse formalism, it is a fairly straightforward matter to estimate the corresponding mean surface (dynamic) topography and its error. Such estimations are directly relevant to the use of satellite altimetry in oceanography and geodesy. Up to now, due to the lack of a precise geoid at smaller scales, estimations of the general (mean) circulation by altimetry have been limited to the largest wavelengths of the oceanic circulation (> 4 000 km) (e. g. Tai and Wunsch, 1984; Nerem et al., 1990). This is why it has not been possible to extract a sufficiently accurate mean oceanic signal from altimetry, although the variable signal is fairly easy to extract. A mean surface topography derived from in situ data is thus required to map the absolute signal that is generally necessary for assimilating altimeter data in oceanic models [e. g. Holland et al., 1991; Verron, 1992 (note, however, that the mean oceanic altimetric signal does not generally correspond to the in
Altimeter mean surface accuracies (over a few years) are typically 10 cm. Information on the general circulation (e.g., Mazzega and Houry, 1989; Blanc and Le Traon, 1992). As far as geoid estimation is concerned, the mean surface topography (which typically varies within a range of one metre) is thus a very significant correction.

One of the main advantages of the Mercier et al. (1992) model is that it explicitly accounts for errors in the data, especially in the density field, as well as errors in the dynamics and the a priori statements about the circulation. Below, we show that this leads to a larger estimation of the error than in, for instance, the linear model by Martel and Wunsch (1992a and b), which only accounts for errors in the reference level velocities. A realistic estimation of the error is necessary, especially when the surface topography is to be combined with other data (e.g., altimeter mean surface for geoid estimation). The error also determines the minimum accuracy required in the altimeter mean surface for geoid estimation). The error on the mean of the density field (ai) for each grid point given dynamical and observational constraints. Errors on these constraints are explicitly taken into account, which makes the model non-linear. The inverse formalism is as proposed by Tarantola and Valette (1982).

The model was modified so that the surface topography ζ and its error could be estimated. Technically, this was done by considering ζ as an additional parameter (like uo, vo and ai) and by applying the following (linear) constraints at each velocity grid point:

\[ -g \frac{\partial \zeta}{\partial y} = f uo + f \sum_{i=1,10} a_{ij} \frac{\partial}{\partial y} \int_{z_0}^{z_e} \phi_i(z) \, dz \]  
\[ -g \frac{\partial \zeta}{\partial x} = f vo - f \sum_{i=1,10} a_{ij} \frac{\partial}{\partial x} \int_{z_0}^{z_e} \phi_i(z) \, dz \]

where g is gravity and f is the Coriolis parameter (f varies with latitude), a_{ij} is the coefficient of EOF (z). (1) and (2) define ζ to within a constant bias. This indetermination was solved by constraining the mean (over all grid points j) of ζ to be exactly equal to zero:

\[ \sum_j \zeta = 0 \]

An a priori value of zero and an associated a priori standard deviation of one metre were chosen for ζ. Constraints (1), (2) and (3) are applied exactly. The choice of an a priori value for ζ permits the definition of a unique surface using only the constraints (1) and (2). However, without the constraint (3), the estimation error on the mean of ζ, which depends only on the a priori variance of ζ, contributes to the a posteriori error on ζ. With the constraint (3) added, the estimation of ζ and of the a posteriori error do not depend significantly on the a priori choices. This was verified by performing two additional inversions: the first with an a priori error on ζ of 0.5 m; the second with an a priori error on ζ of 5 m. The surfaces obtained from the three inversions differ on the average by less than 0.5 cm and the a posteriori errors by less than 0.1 cm.

The model seeks an optimal estimation of the two components of the velocity field at a reference level (uo, vo) and of ten Empirical Orthogonal Functions (EOFs) coefficients of the density field (ai) for each grid point given dynamical and observational constraints. Errors on these constraints are explicitly taken into account, which makes the model non-linear. The inverse formalism is as proposed by Tarantola and Valette (1982).

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ESTIMATING MEAN SURFACE TOPOGRAPHY

The inversion of the North Atlantic Ocean between 20°N and 50°N discussed in Mercier et al. (1992) used a database of 1872 high quality hydrographic stations (Fig. 1), surface and deep Lagrangian float trajectories amounting to approximately 45 000 and 35 000 float days respectively. The inverse model is a non-linear finite difference model with a resolution of 2° in latitude by 2.5° in longitude. Hydrographic and Lagrangian data were combined to describe the three-dimensional circulation assuming geostrophy, Ekman pumping, transport constraints and conservation of planetary vorticity, mass, heat and salt. Basically, the model seeks an optimal estimation of the two components of the velocity field at a reference level (uo, vo) and of ten Empirical Orthogonal Functions (EOFs) coefficients of the density field (ai) for each grid point given dynamical and observational constraints. Errors on these constraints are explicitly taken into account, which makes the model non-linear. The inverse formalism is as proposed by Tarantola and Valette (1982).

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Figure 1

**Hydrographic stations used in the Mercier et al. (1992) inversion.**

Stations hydrographiques utilisées dans l'inversion de Mercier et al. (1992).
Finally, it should be remembered that our estimation of the surface topography and of its error has the resolution of the model (2° in latitude by 2.5° in longitude). The estimation of the \textit{a posteriori} error accounts for smaller scale processes only through the imbalances permitted in the large-scale dynamical constraints.

**RESULTS AND DISCUSSION**

The mean surface topography and the corresponding error are shown in Figures 2 and 3 respectively. The surface topography has an rms of 40 cm and varies between -90 and 90 cm. Major features of the general circulation (the Gulf Stream and its recirculations, the North Atlantic drift, the subtropical gyre ...) are well represented and the smaller spatial scales of the general circulation are better resolved than in previous estimations (e.g. Levitus, 1982; Martel and Wunsch, 1992 b). For a detailed description of the surface geostrophic circulation, the reader is referred to Mercier \textit{et al.} (1992).

The error estimates merit additional discussion. Formal error on \( \zeta \) is typically 5 to 6 cm rms with a maximum of 8 cm rms in the Northeast Atlantic where there are few hydrographic data. \textit{A priori} errors for the reference velocity and density field lead to an \textit{a priori} mean error of about 15 cm for \( \zeta \). This shows the additional contribution of Lagrangian floats and dynamical constraints. The structure of the off-diagonal part of the \( \zeta \) covariance error matrix also reflects the importance of the dynamic and hydrographic and float data constraints. The correlation between adjacent grid points is thus generally below 0.2 apart from the edges. In the Gulf Stream area, where there is a high data density, it is almost zero, while it is about 0.2 in the Northeast Atlantic where hydrographic data are few. This shows that there are sufficient data and dynamic constraints to resolve \( \zeta \) at the 2° in latitude by 2.5° in longitude grid. It also suggests that \( \zeta \) is observable at a higher resolution near the Gulf Stream.

Most of the \textit{a posteriori} error reflects uncertainties in the \textit{a posteriori} estimation of the density field rather than errors on the estimation of the reference velocity field. This explains why our error estimations are larger than those obtained by Martel and Wunsch (1992 \textit{a} and \textit{b}), who performed a similar calculation in the North Atlantic but with a linear inverse model (a linear inverse model does not expli-
Conclusively account for errors in the density field which consequently is not adjusted by inversion. Their error estimate yields an accuracy of 2 to 3 cm rms which reflects only the error on the velocity field at the reference level. With a linear inversion, we find an even smaller error of less than .5 cm rms, because our estimation is constrained with a larger data set. An independent confirmation of the large contribution of the density field error to the total error budget may not be obtained as follows. Variance of surface topography \( \langle h'^2 \rangle \) (the noise for our estimation) as given by Geosat satellite altimetry ranges from 5 cm to more than 30 cm rms in the North Atlantic (e. g. Le Traon et al., 1990). Assuming that all the hydrographic data are independent (an optimistic view) and that reference velocities are perfectly known, a rough estimation of the rms accuracy of a mean surface topography \( \langle h'^2 \rangle \) is simply equal to \( \sqrt{ \langle h'^2 \rangle / N_i } \), where \( \langle h'^2 \rangle \) is the variance of surface topography (as given by Geosat) and \( N_i \) the number of data points in a 2° by 2.5° box number \( i \). This leads to non-negligible errors comprised between 3 and 10 cm.

**REFERENCES**


