# Estimating the North Atlantic mean surface topography by inversion of hydrographic and lagrangian data

Topography
Mean
North Atlantic
Atimetry
Inverse

Topographie
Moyenne
Atlantique Nord
Atimétrie
Inverse

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ABSTRACT

The non-linear inverse model described in Mercier *et al.* (1992) is used to estimate the mean surface topography of the North Atlantic and its formal error. The surface is based on a large hydrographic and Lagrangian data set and has spatial resolution of 2° in latitude by 2.5° in longitude. The formal error is between 4 and 8 cm. Contrary to previous estimations, this takes into account the errors with regard both to the density field and to the estimation of the velocity field at a reference level.

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Estimation de la topographie moyenne de surface de l'Atlantique Nord par inversion de données hydrographiques et de flotteurs lagrangiens

Le modèle inverse non-linéaire décrit dans Mercier *et al.* (1992) est utilisé pour estimer la topographie moyenne de surface de l'Atlantique nord et son erreur formelle. Cette surface est estimée à partir d'un nombre important de données hydrographiques et de données de flotteurs Lagrangiens et a une résolution spatiale de 2° en latitude par 2.5° en longitude. L'erreur formelle est comprise entre 4 et 8 cm. Contrairement aux estimations précédentes, cette erreur prend en compte l'erreur sur le champ de densité et l'erreur sur l'estimation du champ de vitesse à un niveau de référence.

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## INTRODUCTION

In a recent paper, Mercier *et al.* (1992) produced a realistic description of the North Atlantic general circulation between 20°N and 50°N, using a non-linear inverse model. Employing the same inverse formalism, it is a fairly straightforward matter to estimate the corresponding mean surface (dynamic) topography and its error. Such estimations are directly relevant to the use of satellite altimetry in oceanography and geodesy. Up to now, due to the lack of a precise geoid at smaller scales, estimations of the general (mean) circulation by altimetry have been limited to the largest wavelengths of the oceanic circulation (> 4 000 km) (*e. g.* Tai and Wunsch, 1984; Nerem *et al.*, 1990). This is why it has not been possible to extract a sufficiently accurate mean oceanic signal from altimetry, although the variable signal is fairly easy to extract. A mean surface topography derived from *in situ* data is thus required to map the absolute signal that is generally necessary for assimilating altimeter data in oceanic models [*e. g.* Holland *et al.*, 1991; Verron, 1992 (note, however, that the mean oceanic altimetric signal does not generally correspond to the *in* *situ* mean surface topography)]. The additional error induced should be taken into account in the assimilation. It is also useful in geodesy for an accurate estimation of the geoid in combination with an altimeter mean surface. Altimeter mean surface accuracies (over a few years) are typically 10 to 20 cm using data generated by past altimeter missions, and will probably reach a few centimetres using ERS-1 and Topex/Poseidon data (*e. g.* Mazzega and Houry, 1989; Blanc and Le Traon, 1992). As far as geoid estimation is concerned, the mean surface topography (which typically varies within a range of one metre) is thus a very significant correction.

One of the main advantages of the Mercier *et al.* (1992) model is that it explicitly accounts for errors in the data, especially in the density field, as well as errors in the dynamics and the *a priori* statements about the circulation. Below, we show that this leads to a larger estimation of the error than in, for instance, the linear model by Martel and Wunsch (1992 *a* and *b*), which only accounts for errors in the reference level velocities. A realistic estimation of the error is necessary, especially when the surface topography is to be combined with other data (*e. g.* altimeter mean surface for geoid estimation). The error also determines the minimum accuracy required in the combined altimetry/geoid system to extract useful information on the general circulation (*e. g.* Wunsch and Gaposhkin, 1980; Roemmich and Wunsch, 1982; Martel and Wunsch, 1992 *b*).

## ESTIMATING MEAN SURFACE TOPOGRAPHY

The inversion of the North Atlantic Ocean between 20°N and 50°N discussed in Mercier *et al.* (1992) used a database of 1872 high quality hydrographic stations (Fig. 1), surface and deep Lagrangian float trajectories amounting to approximately 45 000 and 35 000 float days respectively. The inverse model is a non-linear finite difference model with a resolution of 2° in latitude by 2.5° in longitude. Hydrographic and Lagrangian data were combined to describe the three-dimensional circulation assuming geostrophy, Ekman pumping, transport constraints and conservation of planetary vorticity, mass, heat and salt. Basically, the model seeks an optimal estimation of the two components of the velocity field at a reference level (uo,vo) and of ten Empirical Orthogonal Functions (EOFs) coefficients of the density field ($a_i$) for each grid point given dynamical and observational constraints. Errors on these constraints are explicitly taken into account, which makes the model non-linear. The inverse formalism is as proposed by Tarantola and Valette (1982).

The model was modified so that the surface topography $\zeta$ and its error could be estimated. Technically, this was done by considering $\zeta$ as an additional parameter (like uo, vo and $a_i$) and by applying the following (linear) constraints at each velocity grid point:

$$- g\, \partial\zeta/\partial y = f\, uo + f\, \Sigma_{i=1,10}\, \partial a_i/\partial y \int_{z_0}^{z_s} \phi_i(z)\, dz \qquad (1)$$

$$g\, \partial\zeta/\partial x = f\, vo - f\, \Sigma_{i=1,10}\, \partial a_i/\partial x \int_{z_0}^{z_s} \phi_i(z)\, dz \qquad (2)$$

where g is gravity and f is the Coriolis parameter (f varies with latitude). $a_i$ is the coefficient of EOF number i $\phi_i$ (z). (1) and (2) define $\zeta$ to within a constant bias. This indetermination was solved by constraining the mean (over all grid points j) of $\zeta$ to be exactly equal to zero :

$$\Sigma_j\, \zeta = 0 \qquad (3)$$

An *a priori* value of zero and an associated *a priori* standard deviation of one metre were chosen for $\zeta$. Constraints (1), (2) and (3) are applied exactly. The choice of an *a priori* value for $\zeta$ permits the definition of a unique surface using only the constraints (1) and (2). However, without the constraint (3), the estimation error on the mean of $\zeta$, which depends only on the *a priori* variance of $\zeta$, contributes to the *a posteriori* error on $\zeta$. With the constraint (3) added, the estimation of $\zeta$ and of the *a posteriori* error do not depend significantly on the *a priori* choices. This was verified by performing two additional inversions: the first with an *a priori* error on $\zeta$ of 0.5 m; the second with an *a priori* error on $\zeta$ of 5 m. The surfaces obtained from the three inversions differ on the average by less than 0.5 cm and the *a posteriori* errors by less than 0.1 cm.

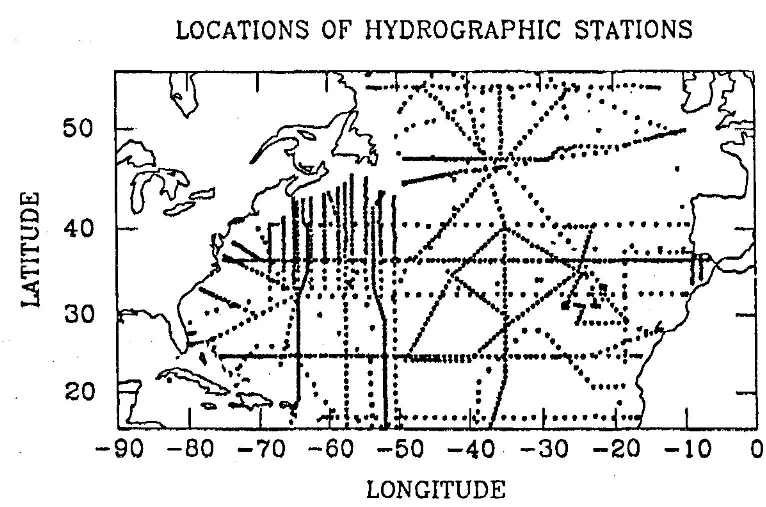


Figure 1

*Hydrographic stations used in the Mercier* et al. *(1992) inversion.*

Stations hydrographiques utilisées dans l'inversion de Mercier *et al.* (1992).

Finally, it should be remembered that our estimation of the surface topography and of its error has the resolution of the model (2° in latitude by 2.5° in longitude). The estimation of the *a posteriori* error accounts for smaller scale processes only through the imbalances permitted in the large-scale dynamical constraints.

## RESULTS AND DISCUSSION

The mean surface topography and the corresponding error are shown in Figures 2 and 3 respectively. The surface topography has an rms of 40 cm and varies between - 90 and 90 cm. Major features of the general circulation (the Gulf Stream and its recirculations, the North Atlantic drift, the subtropical gyre ...) are well represented and the smaller spatial scales of the general circulation are better resolved than in previous estimations (*e. g.* Levitus, 1982; Martel and Wunsch, 1992 *b*). For a detailed description of the surface geostrophic circulation, the reader is referred to Mercier *et al.* (1992).

The error estimates merit additional discussion. Formal error on $\zeta$ is typically 5 to 6 cm rms with a maximum of 8 cm rms in the Northeast Atlantic where there are few hydrographic data. *A priori* errors for the reference velocity and density field lead to an *a priori* mean error of about 15 cm for $\zeta$. This shows the additional contribution of Lagrangian floats and dynamical constraints. The structure of the off-diagonal part of the $\zeta$ covariance error matrix also reflects the importance of the dynamic and hydrographic and float data constraints. The correlation between adjacent grid points is thus generally below 0.2 apart from the edges. In the Gulf Stream area, where there is a high data density, it is almost zero, while it is about 0.2 in the Northeast Atlantic where hydrographic data are few. This shows that there are sufficient data and dynamic constraints to resolve $\zeta$ at the 2° in latitude by 2.5° in longitude grid. It also suggests that $\zeta$ is observable at a higher resolution near the Gulf Stream.

Most of the *a posteriori* error reflects uncertainties in the *a posteriori* estimation of the density field rather than errors on the estimation of the reference velocity field. This explains why our error estimations are larger than those obtained by Martel and Wunsch (1992 *a* and *b*), who performed a similar calculation in the North Atlantic but with a linear inverse model (a linear inverse model does not expli-

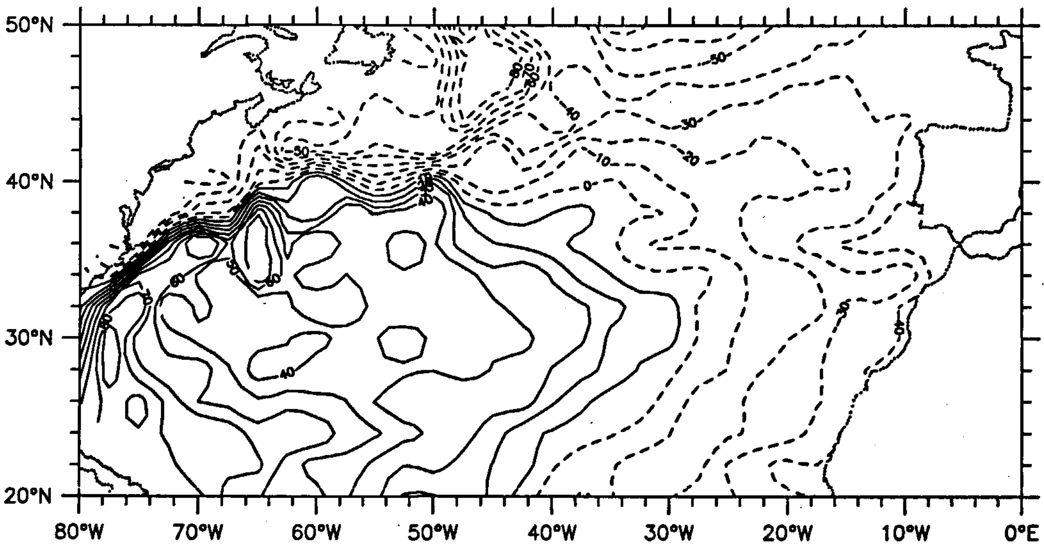


Figure 2

*Mean sea surface topography in the North Atlantic obtained after inversion. Contour interval is 10 cm.*

Topographie moyenne de la surface de la mer dans l'Atlantique Nord obtenue à partir du modèle inverse. L'intervalle de contour est de 10 cm.

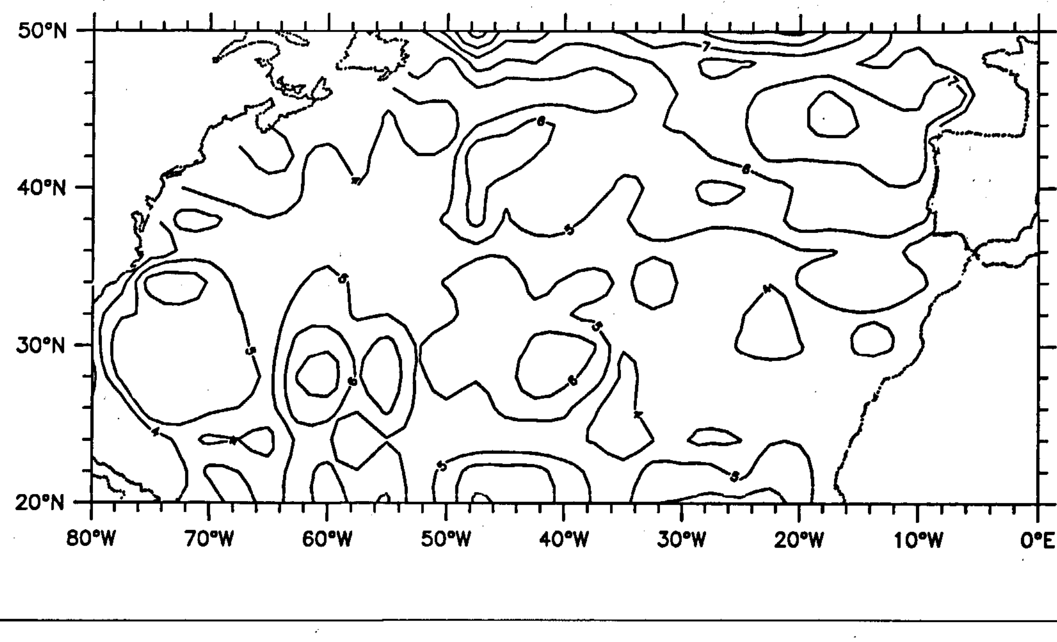


Figure 3

*Corresponding rms error. Contour interval is 1 cm.*

Écart-type de l'erreur correspondante. L'intervalle de contour est de 1 cm.

citly account for errors in the density field which consequently is not adjusted by inversion). Their error estimate yields an accuracy of 2 to 3 cm rms which reflects only the error on the velocity field at the reference level. With a linear inversion, we find an even smaller error of less than .5 cm rms, because our estimation is constrained with a larger data set. An independent confirmation of the large contribution of the density field error to the total error budget can be obtained as follows. Variance of surface topography $\langle h'^2 \rangle$ (the noise for our estimation) as given by Geosat satellite altimetry ranges from 5 cm to more than 30 cm rms in the North Atlantic (*e. g.* Le Traon *et al.*, 1990). Assuming that all the hydrographic data are independent (an optimistic view) and that reference velocities are perfectly known, a rough estimation of the rms accuracy of a mean surface topography $e_i$ with 2° by 2.5° resolution is simply equal to $\sqrt{\langle h_i'^2 \rangle / N_i}$, where $\langle h_i'^2 \rangle$ is the variance of surface topography (as given by Geosat) and $N_i$ the number of data points in 2° by 2.5° box number i. This leads to nonnegligible errors comprised between 3 and 10 cm.

## CONCLUSION

The next step to enhance this estimation is to feed altimeter data directly in the inverse model, or more exactly to add the difference between an altimeter mean surface and an independent geoid and the error covariance of that difference. As shown by Martel and Wunsch (1992 *b*), geoids are not yet accurate enough - and altimeter mean surfaces may not be, either - to enhance this estimation particularly in the North Atlantic which is the ocean which has been studied the most. It is hoped, however, that with the advent of Topex/Poseidon accurate mean surfaces will be obtained in combination with other altimeter data (Topex/Poseidon data will provide strong constraints at large wavelengths) and that geoids will progress similarly. However, a dedicated gravity mission such as Aristoteles would be most welcome. In any case, the error estimates must be accurate so that the inverse model and its dynamics can extract the relevant information from conventional, geoid and altimeter data.

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