MODELLING MUD TRANSPORT
IN A MACROTIDAL ESTUARY

P. LE HIR - Ph. BASSOULLET - J. L'YAVANC
Afin de déterminer le devenir des particules fines et des contaminants adsorbés dans un petit estuaire à grand marage (estuaire de Morlaix), un modèle mathématique de transport de sédiment cohésif et d'évolution du fond, incluant un nouvel algorithme de tassement, a été développé. Après une analyse des processus d'érosion/transport/dépôt induits par la marée, une comparaison avec des mesures en nature est donnée. —

In order to determine the fate of the fine particles and adsorbed contaminants in a short macrotidal estuary, a mathematical modelling system for cohesive suspended matter transport and bottom evolution, including a new consolidation algorithm, is developed. An analysis of successive erosion, transport and deposition processes due to tidal forcing is given and comparisons with field data are provided. A description of mean-term bottom variations has been attempted and is partially validated by differential soundings. —

Mots-clés : Dynamique des sédiments cohésifs, estuaire Morlaix, tassement des vases, modèle mathématique, érosion, dépôt.

Key words : Cohesive sediment dynamics, estuary, erosion, deposition, soil consolidation, mathematical modelling.
MODELLING MUD TRANSPORT IN A MACROTIDAL ESTUARY

P. LE HIR, P. BASSOULLET, J. L'YAVANC
Institut Français pour l'Exploitation de la Mer (IFREMER), B.P. 70 29263 PLOUZANE, FRANCE.

Summary

In order to determine the fate of fine particles and adsorbed contaminants in a short macrotidal estuary, a mathematical modelling system for cohesive suspended matter transport and bottom evolution, including a new consolidation algorithm, is developed. An analysis of successive erosion, transport and deposition processes due to tidal forcing is given and comparisons with field data are provided. A description of mean-term bottom variations has been attempted and is partially validated by differential soundings.

1. Introduction

The urban sewage of Morlaix, a 30 000 inhabitants town on the northern coast of Brittany (France) is discharged in the upper part of an estuary. In order to know the fate of the effluent and its impact downstream, especially over shellfishing areas, a multidisciplinary environmental study has been carried out. As many pollutants may be adsorbed on the particles, turbidity variations and mud transport have to be considered.

For a better understanding of different integrated phenomena and to enable further calculations of concentrations for several adsorbed and/or dissolved constituents interacting with each other, mathematical modelling seems the best way to study the sedimentary processes. However, to face the lack of basic knowledge on the phenomena involved in erosion and sedimentation, many field data have to be collected and taken into account, either to calibrate the model, or to validate it.

2. The study area

Figure 1: Location of the Estuary of Morlaix.
2.1 Geographical presentation

The estuary of Morlaix stricto sensu is very short (about 6 km long) with a mouth situated in a bay opened to the English Channel (Figure 1). Upstream a weir closes a dock with a little harbour. An affluent so called Dourduff river joins the Morlaix river just before the bay. Figure 2 shows off typical cross-sections with extensive intertidal mudflats. Actually, at low tide, channel widths vary from 10 to 100 m in the two rivers, whereas at high tide the estuary is 50 m wide upstream and 1 000 m at the beginning of the bay. More often than not, cross-sections present a series of flat mud benches seperated with steep slopes (until 20 %).

The two rivers (Morlaix and Dourduff) have a small drainage basin (270 km²) so that their mean discharges are very low, respectively 3 m³.s⁻¹ and 0,8 m³.s⁻¹.

Figure 2 : Typical cross-sections of the Estuary of Morlaix. The intertidal area is represented by 5 bottom level references that are spread over the tidal variation.

2.2 Hydrodynamics

The main hydrodynamic forcing in the area is the tide : the level variation is 7,6 m for a mean spring tide and as the estuary is very shallow, which leads to empty and fill it up at each tide. Flood and ebb currents reach 1,2 m.s⁻¹ and 0,8 m.s⁻¹ respectively, with a classical but strong asymmetry induced by the tide propagation over depth varying area (see figure 3).

Water mass trajectories have been computed by the means of a mathematical model (Salomon and Breton, 6) : on spring tide, water leaving the weir upstream at high tide can reach the bay at low tide and come back within 500 m from the starting point during the next flood, depending on the river discharge.

Except on neap tide and for high freshwater input, the estuary is nearly well mixed and density currents are weak. However, current speed induced by the river flow at low water is never negligible, as the channel is very narrow.

Generally speaking, wind induced currents and short waves effects are weak because the bay is protected by a prominent peninsula and a series of little islands off the outlet. But some exceptional events may contribute to a redistribution of particles inside the bay.
2.3 Sedimentology

From the weir to the confluence with the Dourduff river, the channel mainly consists of pebbles. On both sides mudbanks are essentially constituted of particles less than 30 μm. However, near the outlet of the Dourduff river, the sediment becomes more silty. Everything goes on as though inputs from Morlaix river were "selected" by the dock located upstream the weir, most coarse particles settling in that dock (mean input to the estuary: 12 g.m⁻³) [Bassoulet et al., 1]. Concentration ranges of that surficial sediments expressed in dry weight reach 400 to 650 kg.m⁻³ at flood period and 300 to 480 kg.m⁻³ at ebb period according to the position in the transect.

Concerning the microgranulometry of suspended particulate matters, along the 6 first kilometres, particles are less than 10 μm for 90 % except at low water slack of spring tide situation where particles greater than 10 μm reached 70 to 80 per cent. Maxima contents of these suspended particulate matters are on average in the region of 40 g.m⁻³ (dry weight) but for low water slack on spring tide situation they can reach some grams per litre within the turbidity maximum (figure 3).

Figure 3: Estuary of Morlaix. In situ measurements at fixed stations. Spring tide - low river discharge (1 NTU ≈ 3 g.m⁻³). [See Fig. 1 for location].
3. Models description

3.1 Hydrodynamics

Considering the tidal forcing as the main factor of the estuarine circulation, vertically averaged models have been run in the area: a two-dimensional one in the bay and a one-dimensional model to represent the two estuaries and their connection, with realistic variable cross-sections computed from a refined soundings field work (Salomon and Breton, 6). With a mesh size of 250 m, the one-dimensional grid is 41 meshes long.

The downstream boundary condition is the tide level deduced from a previous running of the two coupled models (bay + estuary). The model is calibrated with a constant friction factor in order to simulate good flow velocities at station A and B (see figures 3 and 4).

3.2 Sedimentary processes

The principle of mud transport models is mass conservation of sediment. Two compartments can be distinguished, according to the environment of the particles:

3.2.1 Processes in the water column

Suspended matter can be calculated by solving an advection/diffusion equation that can be written in a one-dimensional form:

\[
\frac{\partial C}{\partial t} + \frac{\partial U C}{\partial x} = \frac{\partial}{\partial x} \left( K \frac{\partial C}{\partial x} \right) + (E + D) \ell
\]  

(1)

where:
- \( C \) = concentration of suspended matter
- \( \sigma \) = cross section area
- \( \ell \) = bed width
- \( U \) = mean current velocity
- \( K \) = diffusion coefficient
- \( E \) and \( D \) = erosion and deposition rate

Solving the equation (1) can fit the computation of \( U(x,t) \) and \( \sigma(x,t) \) with the one-dimensional hydrodynamic model.

The lateral averaging of the equation is justified by the weakness of lateral gradients of concentration that have been measured (see figure 3).

As for the vertical profile, figure 3 shows off high gradients, especially at mid-tide. To take them into account, a vertical distribution of suspended matters has been introduced to fit the deposition term \( D \) to the bottom concentration. This distribution results from the balance between the vertical exchanges due to the eddy diffusion and the settling velocity \( W_s \). If a logarithmic velocity profile and a same diffusivity for mass transfert and momentum are assumed, the concentration profile can be written:

\[
C(z) = C_{\text{bottom}} . e^{-\frac{W_s z}{K u^*}}
\]  

(2)
Boundary conditions

Upstream, as for any other outfall along the estuary, the concentration of suspended sediment has to be given, as well as the water discharge. Downstream, we only need a flood condition for \( C \). As the model results are sensitive to this condition (see 5 Discussion), the following one has been selected: at the beginning of the flood period, the boundary concentration is the same as at the end of the ebb, and then it decreases as the local cross-section increases, until the influx of suspended matter balances the previous outflux. By this way, the estuary system cannot be fed downstream, which is better for understanding the processes inside.

Deposition law

Most of authors use the Krone formula:

\[
\frac{\partial C}{\partial t} = W_s \cdot C \left(1 - \frac{\tau}{\tau_d}\right)
\]

where \( \tau = \rho u^* \) is the bottom shear stress \( (u^* : \text{friction velocity}) \) and \( \tau_d \) is the critical stress of deposition, which is often a constant.

The bottom stress can be deduced from the computed vertically averaged velocity \( U \) and a hypothetic rugosity length according to a logarithmic profile.

The settling velocity formulation has to take into account the flocculation phenomena. When the concentration is higher than 0.3 kg.m\(^{-3}\), this settling velocity is found to increase simultaneously until the concentration value of 3.5 kg.m\(^{-3}\), then it is hindered when aggregates form a continuous network (Krone, Thorn in Mehta (4); Migniot (5)). Besides, flocculation occurs in salted waters when salinity \( S \) exceeds 1 to 8 p.p.m. (Migniot, 5). Thus a general formula has been computed, with the following form:

\[
\begin{align*}
W_s &= W_0 \cdot f(S) \cdot C^{n_1} \quad \text{for} \ C < 3.5 \ \text{kg.m}^{-1} \\
W_s &= W_0 \cdot f(S) \cdot (1 - mC)^{n_2} \quad \text{for} \ C > 3.5 \ \text{kg.m}^{-3} \\
\end{align*}
\]

with \( f(S) = S/(S + \alpha_1 + \alpha_2 S^{-\alpha_3}) \)

\( \alpha_1, \alpha_2, \alpha_3, n_1, \) and \( n_2 \) are constant to be calibrated.

However the lack of experimental data on in situ settling velocities and the will to simplify the interpretation of the model results lead to first considering a constant settling velocity.

Erosion law

Following many authors as Owen, Onishi, Ariathurai and Krone, Baeyens et al (see De Nadaillac, 2) we use the classical formula:

\[
E = \frac{\partial C}{\partial t} = k_1 (\tau/\tau_e - 1)
\]

where \( k_1 \) is a constant.

The critical bottom shear stress of erosion, \( \tau_e \), is bound to the concentration of surficial sediment \( C_s \) according to the formula: \( \tau_e = k \cdot C_s^\alpha \) (6) which is used by several authors (Owen, Thorn and Parsons, Hayter in Mehta, 4; Migniot, 5). The knowledge of \( C_s \) needs modelling the soil behaviour and its consolidation.

Note that when \( \tau_d > \tau_e \ [= f(C_s)] \), erosion and deposition can occur at the same time: this point will not be discussed in this paper.
3.2.2. Processes in surficial sediment

Consolidation algorithm

From the modelling point of view, consolidation means time variation of the sediment concentration depending on the effective stress (weight of overlying sediment) and on the permeability which rates the expulsion of pore water.

Instead of considering typical sediment concentration profiles in several layers as many authors do, De Nadailliac (personal communication) suggested to manage a stock of elementary quantities of sediment ("quanta") which number varies with successive erosion and deposition events. Every quantum is identified by its concentration and may be followed along the time. Thus consolidation may be represented by a differential equation relative to the concentration, which is closer to the processes than a given profile of soil concentrations.

Up to now, the knowledge of consolidation processes does not fit a derivative form because of experimental strains and may be of the previous consolidation algorithm form. After experimental data from Migniot (5) who experienced consolidation in test-tubes, the following semi empirical law can be suggested:

\[- \frac{\partial C}{\partial t} = a_1 + b_1 \sigma' \quad \text{if } C \leq C_1, \text{ flooded bottom (slow consolidation)} \]
\[- \frac{\partial C}{\partial t} = a_2 + b_2 \sigma' \quad \text{uncovered bottom (fast consolidation)} \]
\[- \frac{\partial C}{\partial t} = \frac{A}{t} (a_3 + b_3 \sigma) \quad \text{if } C > C_2 \]
\[- \frac{\partial C}{\partial t} = 0 \quad \text{if } C > C_2 \text{ or } t > t_2 \]

where \( \sigma' \) is the effective stress (i.e.: N.Q where N and Q are the number and the dry weight of overlying quanta).

The first stage is related to the primary consolidation which consists of the expulsion of pore water, more or less easily according to the presence of water over the surficial sediment. The second stage (secondary consolidation) squares with a rearranging of the skeleton and induces a logarithmic increase of the concentration.

The \( a_i \) and \( b_i \) terms have been adjusted in order to restore the results of Migniot experiments:
\[ a_1 = 0.002 \; ; \; b_1 = 10^{-4} \; ; \; a_2 = 0.02 \; ; \; b_2 = 7 \times 10^{-4} \; ; \; a_3 = 50 \; ; \; b_3 = 2.5 \; (u. \; S.I.) \]
\[ C_1 = 230 \; kg.m^{-3} \; ; \; t_2 = 20 \; hours \]

The disadvantage of this new soil algorithm is the computer cost when a refined vertical description is advisable.
Spatial resolution

The exchanges between water column and sediment are drastically dependant on local water height and can considerably vary in a same section so that it becomes necessary to discretize the bottom according to the water depth. Actually, for each cross-section of the estuary, five reference levels have been considered, evenly distributed between the channel level and the bank level at high spring tide (see for example the bottom "cutting" near Locquenolé on figure 2). By this way a local water height and then a bottom shear stress, a consolidation rate and a mass transfer between sediment and suspended matter can be computed at any time. Note that any transversal change in channels or mud flats cannot be simulated : such an attempt would not be consistent with the one dimensional modelling of flow.

The three dimensional discretization of the soil is consistent with any development in considering vertical exchanges between quanta inside the sediment (for instance contaminant diffusion ...) or even horizontal ones (for instance surficial mass transfer in order to model mud collapses when the bank slope is steep).

4. Results

4.1 Reference simulations and parameters

The models have been calibrated with a few parameters (friction factor for hydrodynamics, rugosity length for the bottom stress, erosion rate). This calibration has been achieved with a periodic spring tide of constant amplitude. Other parameters have been selected within common limits suggested by various authors. Thus our "reference parameters" are the following :

- Friction factor (Strickler form) : 33 ; rugosity length $2.10^{-3}$ m
- Diffusion coefficient : $K = 10 + U.R_h \ (m^2.s^{-1})$, $R_h$ is the hydraulic radius.
- Quantum : 0,3 kg ; initial stock : 50 quanta.m$^{-2}$ ;
- Maximum computed stock : 150 quanta.m$^{-2}$
- Settling velocity : $10^{-3}$ m.s$^{-1}$ ; $u_d = (\tau_d/\zeta)^{0.5}$ : 0,01 m.s$^{-1}$
- Erosion rate : $k_e = 4.10^{-5} \ kg.m^2.s^{-1}$ ; $u_e = (\tau_e/\zeta)^{0.5}$ : 0,26.10$^{-4}$ (Cs)

In situ measurements have shown large variations of turbidity according to a realistic behaviour of the sediment. Thus a new simulation has been attempted : beginning on spring tide, the model is run during a whole spring/neap tidal cycle, until the following spring tide which related results will be discussed further.

The initial conditions are the same as for the previous simulation : nearly consolidated sediment profile with dry density varying from 380 kg.m$^{-3}$ near the surface to 500 kg.m$^{-3}$ 0,03 m below, everywhere in the estuary, even in the channel. The run begins at high tide, when suspended matter can be neglected.

4.2 Simulation analysis

The figure 4 shows the simultaneous variations of water level, depth averaged current, friction velocity, erosion and deposition rates, suspended matter and surficial sediment concentrations, and sediment stock during 25 hours, after 14 days simulation at station A.

The first result is a quasi periodicity : both figured tides are very similar and can be considered as a representative spring tide.
Figure 4: Estuary of Morlaix. Computed hydrodynamic and sedimentary parameters at station A (see Fig. 1 for location). Spring tide - low river discharge.
From the hydrodynamic point of view, the asymmetry between flood and ebb is very strong: actually, station A is in the upper part of the estuary. Secondarily we can observe a quasi-steady river flow at the end of the ebb: the fresh water discharge is only 1 m$^3$.s$^{-1}$ but the cross-section is so tiny at this time that velocities reach 0.5 m.s$^{-1}$. Consequently, friction velocities are relatively large because of the small depth and any fresh deposited sediment cannot stay during the ebb.

The tidal height diagram shows off the periods of uncovering benches according to their bottom level reference. Consequences on friction velocities are considerable: for instance over a mud flat which is flooded at mid-tide (level reference: 3), this friction velocity is at least twice less than in the channel.

Such differences between banks and channel can be seen on the erosion/deposition diagram. But they also depend on the sediment stock: for instance, in the channel no quantum is available during the flood period and no erosion occurs. But at the end of the flood, deposition is possible with suspended matter coming from lateral erosion. This deposition stops near high tide where no suspended matter is available. Then an erosion begins with the ebb but stops suddenly (at 14 h 30 on the figure) when the stock is over. On both sides of the estuary, processes are very different: deposition occurs at the end of the flood and erosion occurs when a strong velocity and a sufficient water height are concordant, that is within a short time during the flood or at the beginning of the ebb.

As for the sediment stock, its behaviour seems nearly stationary and far from the initial condition which consists of a uniform stock of 15 kg.m$^{-2}$. Thus the channel has been cleaned out, except during a brief moment at high tide. Just above the channel, the first banks are slowly eroded whereas deposition is maximum at the bottom level reference 3. Lastly, the upper banks are balanced because of the lack of suspended matter when they are flooded.

Many features can be seen on the suspended matter diagram: first the local effect of resuspension during the flood and the ebb, with relative maxima of concentration; secondly a concentration falling down at high tide. But the overall maximum occurs between low water and the flood turn: cross-sections are small and suspended matter comes from upstream erosion, according to the available sediment stock.

Lastly the vertical concentration gradient is important during the flood, just after the peak velocity. This gradient is computed for the channel water column and the surface concentration is taken into account for the lateral deposition: by this way the feeding of the upper banks is less important than in a fully vertically integrated system.

The evolution of the surficial sediment is more complicated. First the channel sediment is never consolidated, with always a small concentration. On the other hand, the first benches (bottom level reference 2) show large concentration variations.

Let us describe the processes related to these benches since the deposition time following the first flood. From 10 to 14 hours, the surficial concentration increases slowly, first because of a slow consolidation (flooded mudflat) and then because of an erosion of this fresh deposited matter. From 14 h to 14 h 30, deposition occurs when velocity decreases, just before the uncovering time: then fast consolidation sets up, until the concentration reaches 230 kg.m$^{-3}$ (at 16 h). Then a logarithmic secondary consolidation follows until the flood turn (at 19 h). After a short and small deposition that cannot be seen on the deposition diagram, the flood velocity induces erosion which reaches previous deposited layers (at 20 - 21 h). The tidal residual effect is this small excess erosion.
Analogous interpretations could be given for the other bottom level references.

Longitudinal variations of sedimentary parameters are not presented in this paper: processes are the same along the estuary, the only change being the relative weight of local processes and transport effects, especially for the suspended matter concentration, inducing time lags on maxima values.

Model validation

The computed suspended matter concentration on figure 4 can be compared to the data of station A plotted on figure 3. The same evolution is observed, with concentration relative maxima related to peak velocities, and an absolute maximum during the end of the ebb. This surprising feature, induced by upstream erosion, shows the importance of transport effects before local ones, as explained by the model.

4.3. Bottom evolution

The computed sediment stock is presented on figure 5, after a whole spring/neap tidal cycle simulation (14 days). The upstream shoaling is obvious, especially on the sides along 2 km. Downstream, these side benches seem stable (the initial stock was 15 kg.m⁻²). On the other hand the channel and the first lower benches are largely eroded, especially in the middle area of the estuary. The reason is less a longitudinal variation of current velocities than a simple exchange of sediment between the upper and the middle estuary. Actually, sediment is mainly eroded during the flood period, and then carried upstream, where it is deposited at high tide, everywhere along a cross-section. During the ebb, waters flow out of the lateral benches before the peak ebb current and erosion is very weak, except in the channel or just above. Then either these ebb eroded sediments are deposited farther in the channel and thus will be eroded during the next flood period, or they are ejected in the bay where they will eventually be deposited.

Figure 5: Estuary of Morlaix. Computed and observed bottom evolution.
So in the Morlaix estuary, the main sedimentary processes do not result from the asymmetry between ebb and flood, but rather from the morphological change of the estuary between high tide and low tide: schematically, erosion occurs at mid-tide with ebb/flood current and deposition occurs at high tide, preferentially upstream and over the lateral benches, whereas during low water, cross-sections are so small that ebb velocities remain high.

A comparison with observed bottom variations between 1929 and 1985 shows a qualitative agreement between measurements and computation (figure 5 and L'Yavanc, 3). However the real estuary seems in a steady state with a slight deposition rate upstream: this could be due to a non-modelized process that consist of steep slope collapses from lateral benches towards the channel. Besides, the simulated tidal cycle cannot be considered as a completely realistic events series: in particular a peak discharge from the river can increase the ebb erosion upstream and reduce the shoaling.

5. Discussion

Some points are worthwhile being developed from a general point of view.

5.1 Bottom friction evaluation

For smooth bottoms, a classic implicit logarithmic velocity profile has been fitted (for instance Migniot, 5): \( u = u^* \left( 2.5 \log \left( \frac{zu^*}{\nu} \right) + 5.5 \right) \) where \( \nu \) is the water viscosity. When applying this formula, we got a very steep velocity profile and consequently a small friction velocity, not in agreement with measured velocity gradients. Actually a rugosity length of \( 2.10^{-3} \) m (instead of \( 2.10^{-4} \) m, a common value for a mud bottom) is necessary to get realistic velocity gradients. This is likely due to large splits in the surficial sediments looking like bedforms for sandy bottoms. A special investigation should be carried out on these mud bedforms.

5.2 Spatial discretization validation

This modelling was achieved with a special care for simplifying as much as possible: thus a cross-section averaged model was chosen. However, the first tests have shown the necessity to distinguish several bottom levels, as processes are very dependent on the local water height and the uncovering time. The resulting mechanism of channel erosion and side deposition in the upper estuary warrants this lateral discretization.

Moreover, to prevent an excess deposition on the upper benches, a vertical gradient for suspended matter concentration had to be introduced. But the presently used formula (2) is fitted to a steady flow and does not take account accelerations and decelerations, or even the near bottom location of the exchange with sediment layers. For this reason as well as for simulating a stratified flow during high fresh water discharge and neap tide, a two dimensional model with vertical calculation for current and concentration could lead to a perceptible progress.
5.3 Model sensitiveness to boundary and initial conditions

The model results depend on the downstream concentration input during the flood. In the present model the condition has a conservative form, that is the input has not to exceed the output. Naturally, a better way would consist of modelling the transport and even simulating the sedimentary processes in the bay downstream.

With regard to initial conditions, the problem is more difficult. Of course during the first simulated tides, the model will be sensitive to the initial sediment concentration profile, but after several weeks, would the results become similar? Achieving tests on various initial conditions could give an answer and may be lead to a sedimentology time scale related to the study area.

6. Conclusions

The main objectives of this study were first to develop software for computing mud transports in coastal areas and then to improve the knowledge of sediment processes in macrotidal estuaries such as those opened to the English Channel.

Mud transport model
The model is operational, with an original algorithm for the sediment consolidation. It is suited to one dimensional flows but a lateral discretization of the bottom sediment allows to take into account the lateral benches uncovering at low tide. This software could easily be improved by running with two or three dimensional flows.

Sediment processes in the Estuary of Morlaix
The model is the best tool to show off the respective weights of transport phenomena and local erosion or deposition processes: thus it has been possible to explain the maximum turbidity at low water instead of during the maximum current, and to simulate the sediment exchange between the middle channel and the banks of the upper estuary.

But many other studies could be set by means of the model: for instance the fate of the upstream input of suspended matters, the trajectories of sediment samples during one tidal period, the impact of a high river discharge, the pore water output when the sediment consolidates (with environmental consequences), the effect of the highest spring tide and generally speaking the sensitiveness of the model to parameters and initial conditions.

7. Acknowledgements

The authors wish to thank G. De Nadaillac (Orga-Conseil, Lyon) for his work on elaborating the mud transport model. Thanks are also due to M. Breton (IFREMER) for her helping in running the model, J.P. Annézo and D. Guillerm (IFREMER) for drawing the figures, typing the manuscript and improving the general presentation.
8. References


