

# Influence of polarization filtering on image registration precision in underwater conditions

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Underwater images often suffer from poor visibility due to photon scattering. However, in some cases, optical polarization filtering techniques can decrease the contribution of the scattered light and improve the visual image quality. In this Letter, the influence of these techniques for underwater image registration is analyzed, particularly when backscattered light is the main perturbation induced by the submarine environment. This analysis is performed using the Cramer–Rao bound and relies on a standard image formation model, taking into account various kinds of noises. © 2012 Optical Society of America

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Object location and image registration are two key tasks in optical underwater imaging, especially for applications such as mosaicing or autonomous underwater vehicle positioning [1–3]. These tasks are in general challenging since underwater imaging suffers from poor image visual quality due to various sources of noise and to light scattering in sea water [4–6]. Polarization filtering techniques have previously been used to improve underwater visibility by reducing the contribution of backscattered light [7–10] (i.e., light that has been scattered by the medium directly to the camera; see Fig. 1). In that case, it is generally necessary to illuminate the scene with polarized light and to analyze the different polarization components of the signal received by the camera. However the extent of polarization filtering impact on image registration is yet to be studied. This Letter addresses this issue in the context of underwater optical imaging under artificial illumination. For that purpose, a standard image formation model is considered that takes into account backscattered light fluctuations, photon noise on the image sensor, and electronic additive noise. The main parameters that influence the image registration precision are then discussed with the Cramer–Rao bound (CRB) [11–13], which allows one to get an analysis independently of any particular unbiased registration technique.

For the sake of clarity, the one-dimensional (1D) case is considered, and the image registration issue comes down to estimating a 1D translation parameter. The results of the analysis can, however, be generalized in two dimensions, but at the expense of a more tedious formalism not useful for the purpose of this Letter. Let  $i(x)$  be the measured image where  $x$  is the spatial coordinate. According to standard Jaffe–McGlamery model [14–16],  $i(x)$  can be seen as the sum of a contribution  $s(x)$ , resulting from the light that propagates to the camera after being reflected by the scene, and a backscattering contribution  $b(x)$  due to light directly scattered toward the sensor (Fig. 1). In standard intensity imaging, the mean value  $\langle i(x) \rangle$  of  $i(x)$  (i.e., in the absence of noise) can be written as

$$\langle i(x) \rangle = s(x - \eta) + b(x), \quad (1)$$

where  $\langle \rangle$  stands for the statistical (or ensemble) average and  $\eta$  is the unknown translation parameter to estimate. Since the backscattering only relies on the medium properties and on the detection-illumination configuration,  $b(x)$  has thus been assumed to be independent of the translation of the observed scene in Eq. (1).

If the source light is totally polarized and the image is captured through a polarization splitting system (PSS),  $s(x)$  can be written as the sum of the copolarized intensity  $s_{\parallel}(x)$  and the crossed-polarized intensity  $s_{\perp}(x)$ . Using such an imaging system has the interest of improving the image contrast either by reducing the veiling light due to backscattering or by revealing polarization contrasts intrinsic to the scene. However, the latter phenomenon is not specific to underwater imaging and does not allow one to separate the gain due to the scene properties and the gain specific to underwater imaging. In order to characterize only the effect of polarization filtering in the presence of backscattered light (independently of the scene properties), it will be assumed that the same reflectivity contrast is available with both standard and polarimetric techniques. In that case, the degree of polarization (DOP)  $P$  of the scene contribution  $s(x)$  is thus constant

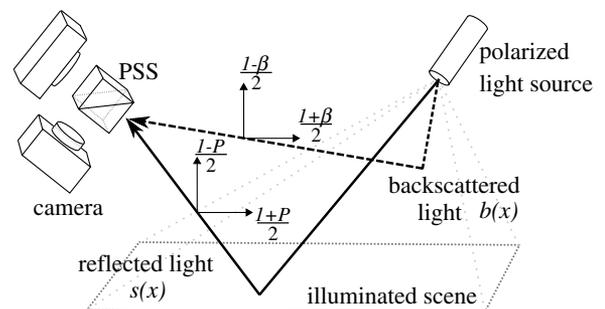


Fig. 1. Image formation model and setup. A polarization splitting system (PSS) allows one to select one or both polarization states. The “intensity imaging” configuration refers to the case where the PSS is not used.

over the image leading to  $s_{\parallel}(x) = \frac{1+P}{2}s(x)$  and  $s_{\perp}(x) = \frac{1-P}{2}s(x)$ , with  $P \in [0, 1]$ . Similarly,  $\beta$  is defined as the DOP of the backscattered light and is assumed to be constant over the image [17]. The mean value of the backscattering contribution polarized parallel (resp. perpendicular) to the incident light can then be written as  $\frac{1+\beta}{2}b(x)$  (resp.  $\frac{1-\beta}{2}b(x)$ ) with  $\beta \in [0, 1]$ . The mean value  $\langle i_{\parallel}(x) \rangle$  of the intensity  $i_{\parallel}(x)$  measured in the same polarization state as the source is thus

$$\langle i_{\parallel}(x) \rangle = \frac{1+P}{2}s(x-\eta) + \frac{1+\beta}{2}b(x). \quad (2)$$

The mean value  $\langle i_{\perp}(x) \rangle$  in the orthogonal state is then easily deduced from Eq. (2) with the substitutions  $\beta \rightarrow -\beta$  and  $P \rightarrow -P$ .

In the study, three types of noises are assumed to corrupt the measurements: white electronic noise, signal fluctuations of the backscattering contribution, and photon noise. Additive electronic noise can be modeled with a Gaussian probability density function (pdf) of variance  $\sigma_0^2$ . Moreover, in order to take into account backscattering signal fluctuations (due, for example, to turbulence [18] or to small variations of the scattering particle density), the backscattering signal  $b(x)$  is also assumed to be corrupted with an additive Gaussian noise with variance  $\sigma_b^2(x)$ . Finally, photon noise is generally described with a Poisson law, which can generally be approximated with a Gaussian pdf with a variance equal to its mean value, as most of the image acquisition conditions correspond to high intensity limit. At high photon level, the pdf of  $i(x)$  can thus be written

$$\Lambda[i(x)|\eta] = \frac{\exp\left(-\frac{[i(x)-\langle i(x) \rangle]^2}{2[\sigma_0^2 + \sigma_b^2(x) + \langle i(x) \rangle]}\right)}{\sqrt{2\pi[\sigma_0^2 + \sigma_b^2(x) + \langle i(x) \rangle]}}.$$

Given this model, the precision on the estimation of the unknown translation parameter  $\eta$  can now be discussed with its CRB [12]. Indeed the variance of any unbiased estimator cannot be smaller than the CRB which is equal to  $I_F^{-1}$  where the Fisher information  $I_F$  is defined by

$$I_F = -\langle \partial^2 \log \Lambda[\mathbf{i}|\eta] / \partial \eta^2 \rangle,$$

where  $\mathbf{i} = \{i(1), i(2), \dots, i(N)\}$  if the image has  $N$  pixels. The CRB, and thus the image registration precision, can therefore be discussed through the analysis of  $I_F$ .

Without PSS, since the noise is assumed to be spatially independent,  $\log \Lambda[\mathbf{i}|\eta] = \sum_{x=1}^N \log \Lambda[i(x)|\eta]$ , and the Fisher information is thus  $I_F \simeq \sum_x [1 + A][s'(x-\eta)]^2 / [\sigma_0^2 + \sigma_b^2(x) + \langle i(x-\eta) \rangle]$  with  $A^{-1} = 2[\sigma_0^2 + \sigma_b^2(x) + \langle i(x-\eta) \rangle]$  and  $u'(x) = \partial u(x) / \partial x$ . In the considered high intensity limit  $\sigma_0^2 + \sigma_b^2(x) + \langle i(x-\eta) \rangle \gg 1$ , and defining  $\xi(x) = \sigma_b^2(x) + b(x)$ , it leads to

$$I_F = \sum_x \frac{[s'(x-\eta)]^2}{\sigma_0^2 + \xi(x) + s(x-\eta)}. \quad (3)$$

Let now  $I_F^{\parallel}$  (resp.  $I_F^{\perp}$ ) be the Fisher information for the image acquired in a polarization state parallel

(resp. perpendicular) to the source one. It can be shown that

$$I_F^{\parallel} = \frac{(1+P)^2}{4} \sum_x \frac{[s'(x-\eta)]^2}{\sigma_0^2 + \frac{1+\beta}{2}\xi(x) + \frac{1+P}{2}s(x-\eta)}. \quad (4)$$

$I_F^{\perp}$  is also easily deduced from Eq. (4) with the substitutions  $\beta \rightarrow -\beta$  and  $P \rightarrow -P$ . If both components are measured, the noise on each component is independent and the Fisher information is equal to  $I_F^T = I_F^{\parallel} + I_F^{\perp}$ .

As mentioned above, the precision of an image registration estimator can be characterized by its variance. For unbiased and efficient estimators, the variance is equal to the CRB and thus to  $I_F^{-1}$ . Using polarization imaging may decrease this variance. In particular, when a two component polarimetric setup is used instead of a standard intensity imaging system, the variance of such estimators is decreased by a factor  $G^T = I_F^T / I_F$ . This factor  $G^T$  is denominated as the registration ‘‘precision gain’’ in the following. Therefore a registration precision gain equal to 5 corresponds to a decrease of the variance by a factor 5 for unbiased and efficient estimators.

Let  $m = \min(P, \beta)$ . In that case,  $(1+m)/2 \leq (1+\beta)/2 \leq 1$  and  $(1+m)/2 \leq (1+P)/2 \leq 1$ . Combining Eqs. (4) and (3), one obtains  $(1+P)^2 I_F / 4 \leq I_F^{\parallel} \leq (1+P)^2 I_F / (1+m)$ . A similar expression is obtained for  $I_F^{\perp}$  substituting  $(\beta, P, m)$  by  $(-\beta, -P, -M)$  with  $M = \max(P, \beta)$ . Therefore, using the definition of  $G^T$ ,

$$\frac{1+P^2}{2} \leq G^T \leq \frac{1}{2} \left[ \frac{(1+P)^2}{1+m} + \frac{(1-P)^2}{1-M} \right]. \quad (5)$$

These bounds, which only depend on the polarimetric properties of the measured signals, correspond to the less and most favorable situations that may be encountered using a polarimetric setup. This result is illustrated in Fig. 2(a) (dashed and plain lines for the lower and upper bounds, respectively). As can be seen in Eq. (5), the upper bound diverges if  $\beta$  is equal to 1. However, this situation corresponds to the case where the backscattering light  $b(x)$  is totally filtered by the PSS in one polarization state of the imaging system, which generally does not correspond to practical situations and will not be considered in the following.

When  $G^T \simeq (1+P)^2 / 2 \leq 1$  [dashed line in Fig. 2(a)], no gain can be expected. This situation occurs when the

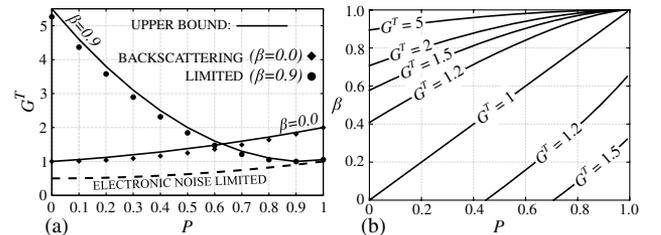


Fig. 2. (a) Variations as a function of  $P$ , of the lower (dashed line) and upper (plain line) bounds of the precision gain  $G^T$  given by Eq. (5) and of  $G^T$  given by Eq. (6) (black diamonds and black dots), for  $\beta = 0$  and  $\beta = 0.9$ . (b) Map of  $G^T$  for backscattering limited imaging [see Eq. (6)] as a function of  $P$  and  $\beta$ .

electronic noise is the main source of noise. Moreover if  $P = 0$ , the location precision described by the CRB decreases by a factor of 2. On the other hand, gain in location precision can be obtained when the main source of noise comes from fluctuations induced by the back-scattered light ( $\xi(x) \gg s(x)$ ,  $\xi(x) \gg \sigma_0^2$ ) as in the case of long range imaging or when the source is close to the camera. In that case

$$G^T \simeq 1 + (P - \beta)^2 / (1 - \beta^2), \quad (6)$$

and the gain is thus always greater than 1 [cf. Fig. 2(b) and black markers in Fig. 2(a)], which shows that using a full polarimetric setup is advantageous in that situation. In particular, the higher the difference between  $P$  and  $\beta$  is, the higher the gain can be, particularly when the back-scattering light is partially polarized. For example, if  $\beta = 0.9$  and  $P = 0$ , then  $G^T \simeq 5$  [see Fig. 2(b) and black dots in Fig. 2(a)]. No gain is, however, obtained if  $\beta = P$ , which is easily understandable since both backscattered and reflected lights have the same effect on polarization measurements in that case.

In the above analysis, image quantization effects have been assumed negligible. However, they can severely degrade the registration quality. At first approximation, quantization noise can be modeled [19] by adding a variance  $\sigma_Q^2 = (A/n)^2/12$  (where  $n$  is the number of quantization levels and  $A$  is proportional to the maximum value of the image) to  $\sigma_0^2$  in Eq. (3). In the backscattering limited case,  $A \propto \max \xi(x)$ , which changes into  $(1 + \beta)A/2$  (resp.  $(1 - \beta)A/2$ ) when acquiring the image in the parallel (resp. perpendicular) polarization state. Moreover if quantization effects are the main source of noise, Eqs. (3) and (4) now lead to  $G^T \approx (1 + P)^2 / (1 + \beta)^2 + (1 - P)^2 / (1 - \beta)^2$ , which can be shown to be always greater than in Eq. (6). Polarization filtering techniques are thus favorable to the backscattering limited case, and particularly when quantization noise becomes important.

Nevertheless, in all this study, the scene is considered to be illuminated with a polarized light. If thermal light is used, polarizing the light decreases the intensity by a factor of 2, which could be avoided when no PSS is required and which can degrade the precision (as with electronic and Poisson noise limited imaging) limiting the cases where  $G^T \geq 1$ : A careful study should then be done to determine whether polarization filtering might be beneficial. However, it has also been assumed that  $P$  is independent of  $x$ . Therefore if the scene demonstrates polarization contrasts (as when imaging man-made objects on a natural background), the gain on registration precision will necessarily increase, which can compensate for a precision loss due to the source intensity decrease mentioned above.

In conclusion, this Letter has presented a theoretical framework to quantify the gain that can be achieved on image registration precision in the context of underwater optical imaging. It has been shown that notable improvements can be expected if the images are limited by the perturbations induced by backscattered light, which is a canonical context for underwater polarization imaging. In that case, the amplitude of the precision increase is directly linked to the relative DOP of each contribution of the image.

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