Numerical simulation
of the turbidity maximum transport
in the Gironde estuary (France)

Zhan Hua LI(1), Kim Dan NGUYEN(2), Jean-Claude BRUN-COTTAN(3) and
Jean-Marie MARTIN
Institut de Biogéochimie Marine, URA C.N.R.S 386, URM Ifremer 6,
École Normale Supérieure, 1, rue Maurice Arnoux, 92120 Montrouge, France
(1) New address : Institute of Mechanics, Chinese Academy of Sciences,
15, Zhong Guan Cun Road, Beijing 100080, China.
(2) Author for correspondence.
(3) New address : Centre de Géomorphologie et Transports de Surface
C.N.R.S., 24, rue des Tilleuls, 14000 Caen, France.
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ABSTRACT
A coupled two-dimensional vertical width-integrated hydrodynamic and sediment
transport numerical model is presented to simulate the "turbidity maximum"
formation and dynamics in the Gironde estuary. A turbulence closure sub-model is
used to parametrize the turbulent mixing process. The numerical results of a
five-day simulation are compared with field measurements carried out at several
locations. The formation and the displacement of the turbidity maximum during
one tidal cycle are correctly simulated. The intensity of the "turbidity maximum" is
strongly related to the turbulent energy field. The influence of the tidal amplitude
on the suspended matter transport is also studied during the simulation.

RÉSUMÉ
Simulation numérique du bouchon vaseux dans l’estuaire de la Gironde
Un modèle numérique 2D vertical, intégré sur la largeur, couplant l’hydrodynamique et le transport sédimentaire, est présenté pour simuler le "bouchon vaseux" dans l’estuaire de la Gironde. Un sous-modèle de fermeture de la turbulence est utilisé pour paramétriser les échanges turbulents. Les résultats numériques obtenus pour une simulation de cinq jours, sont comparés à ceux obtenus par des mesures directes en plusieurs sites. La formation et le déplacement du "bouchon vaseux" au cours d’un cycle de marée est bien décrit. Sa concentration est fortement reliée au champ de l’énergie turbulente. L’influence du coefficient de marée sur le transport sédimentaire a aussi été étudiée.

INTRODUCTION
The transport of fine-grained sediment is very important to
the prediction of the flux of pollutants in estuaries and the
design of dredging strategies for navigation channels. The
main feature of fine-grained sediment transport in macrotidal
estuaries is the "turbidity maximum", a phenomenon
that has long been observed in various macrotidal estuaries
(Baumgartnen (1836-1844) ; Glangeaud, 1939; Inglis and
Allen, 1957). This turbidity maximum, characterized by a
high suspended matter concentration zone in the estuary, moves with the tide and its location depends on the river flow. The residence time of the particles that constitute the maximum is at least several months, and probably of the order of years for the Gironde estuary (Martin et al.,1986).
Turbidity maximum transport in estuaries, a complex
dynamic process which involves various time scales
(Dyer, 1988), depends on tidal dynamics, river discharge
and the erosion and settling of sediment, as well as the ero-
dibility of the bottom sediment. The time scales consist of
half-day (short lunar tidal period), two-week (long lunar tidal period) and seasonal scales (influence of river discharge). The Gironde estuary (France) is a partially mixed and macrotidal estuary (Allen, 1973), with well-developed turbidity maximum extending for 20-60 km of its length as a permanent feature. Total Suspended Matter (TSM) concentration can exceed 50 g l⁻¹ near the bottom (Allen, 1973). Some hypotheses have been advanced in the past to explain the formation and the displacement of the turbidity maximum. They mainly involved the role of the salt water motion at the bottom of the estuary and the tide-averaged residual circulation (introduction of a so-called "null point"). The role of the flocculation processes at the fresh and salt water interface, and more recently, the effects of the global dynamic interactions have also been invoked. Mathematical models of the TSM and mud transport in estuaries have been developed since the early 1970s: for instance, Odd and Owen (1972) for the Thames estuary; Du Penhoat and Salomon (1979) for the Gironde estuary, Rodger and Odd (1985) for the Brisbane estuary and Le Hir and Karlikow (1991) for the Loire estuary. In the most basic and commonly used models, the movement of mud in an estuary is considered to be governed by a cycle of erosion, transport in suspension with vertical mixing, settling and deposition. Most of the advances in recent studies have involved refining the physical processes: the representation of the processes of erosion and deposition of sediment (Falconer and Owens, 1990); and the simulation of the effect of sediment-induced stratification on the bottom boundary layer dynamic and on the erosion processes (Sheng and Villaret, 1989).

The aim of this paper is to describe a two-dimensional vertical model that simulates turbidity maximum behaviour in the Gironde estuary, to show the effect of some important hydrodynamic and sedimentological processes and to improve the simulation of turbulent vertical mixing by introducing a turbulence closure sub-model.

MAIN FEATURES OF THE GIRONDE ESTUARY

The Gironde estuary, located in southwestern France, is a product of the confluence of the Garonne and Dordogne rivers (Fig. 1). Its geometry from Bordeaux to Pauillac, except for islands such as the Ile Verte, is considered to be regular. The water-surface width is about 2-3 km and the depth 6-8 m in the navigation channel. However, from Pauillac to the mouth, the geometry becomes more com-

Figure 1

Chart of the Gironde estuary showing the sampling sections (PK: Kilometres downstream from Bordeaux).
plex due to an increase of width and the presence of the Saintonge channel on the right bank. The width ranges from 4 km at Pauillac to 14 km at Richard. The depth is 10 m in the navigation channel in this region. From Richard to the mouth of the estuary, the bottom depth increases from 10 m to 30 m over a distance of 10 km.

The total drainage area of the estuary is about $8 \times 10^4$ km$^2$, and the annually-averaged river discharge (1961-1970) amounts to 760 m$^3$.s$^{-1}$ (Jouanneau and Latouche, 1981). Seasonal variations in river flow occur with maximum values, usually in January (1,500 m$^3$.s$^{-1}$ monthly mean), and minimum values in August (200 m$^3$.s$^{-1}$) (Jouanneau and Latouche, 1981). The estuary is partially mixed and macrotidal, with a 12 hour and 25 minutes tidal lunar period and a tidal amplitude of 1.5 to 5 m at the mouth. Tide effects are felt up to 150 km upstream from the mouth during periods of low river discharge, whilst the upper limit of the salinity intrusion is about 75 km upstream for low river discharge, and 40 km for high river discharge (Allen, 1973). The annually-averaged solid discharge to the estuary (1959-1965) is about 2.17 10$^9$ tons, two-thirds of which are contributed by the Garonne river (Migniot, 1971).

The sedimentological structure of the bottom varies from the mouth to the upstream boundaries of the tidal intrusion in the Garonne and Dordogne rivers.

- At the mouth of the estuary, the bottom is covered by sand.
- From Richard (25 km upstream from the mouth) to Bordeaux, a "fluid mud" layer, constituted by silt and mostly by clay minerals, can cover the bottom in patches, mainly in the navigation channel. These patches are mostly 10 to 20 km long, and up to 2-3 metres thick. Close to Bordeaux when the river flow is low, they drift seaward to Richard when the river flow is high.
- From Bordeaux to the upstream boundary (70 km upstream from Bordeaux) and for the part of the Dordogne river taken into account in the computed domain, the river bed constituted by non-consolidated soft mud with patches of sand.

River solid discharge combined with macro-tidal conditions causes the formation of a well-developed "turbidity maximum". This turbidity maximum and the fluid mud are located in the same area of the estuary, and can be considered as a coupled system, the maximum total mass of which is of the order of 2-5 10$^8$ tons (close to the annual solid river discharge). When the turbidity maximum is fully developed, during the high tidal amplitude period, the total mass of the fluid mud in the estuary decreases or disappears. The reverse phenomenon occurs during the small tidal amplitude period. Within the turbidity maximum, the TSM concentration increases from the top to the bottom of the water column. When this concentration exceeds 150 g.l$^{-1}$, the top of the fluid mud layer is reached, with the concentration corresponding to a change of the mechanical properties of the fluid from Newtonian to non-Newtonian. The concentration of the fluid mud layer varies from 150 g.l$^{-1}$ at the top to 400 g.l$^{-1}$ at the bottom. With a greater concentration, the consolidated bottom is considered as reached. Some experiments with radio-tracers (Jouanneau and Latouche, 1981) show that the fluid mud has no horizontal velocity, its displacement, when the river flow varies, is due to the displacement of the turbidity maximum; iii material resuspended from the fluid mud then settles at a new location. The top of the fluid mud layer is thus considered as the mathematical bottom boundary. Only the resuspended sediment transport is described in this paper. However, when determining the bottom conditions of the erosion, some mechanical properties of the fluid mud are taken into account (see section: sedimentological parameters).

MATHEMATICAL AND NUMERICAL BACKGROUND

The basis of the model described here is a two-dimensional width-integrated model first proposed by Nguyen (1987) to simulate estuarine water circulation. The sigma-coordinate system transformation on the z axis is thus used in order to take into account significant variations of the bottom topography and the tidal water surface when setting the computing point-grid (the number of grid points on the vertical remains constant). The computational domain includes the Garonne and Dordogne rivers. A numerical technique to couple the confluence of the two rivers with the Gironde estuary is then used. A turbulence closure sub-model is introduced to calculate the turbulent viscosity coefficients.

**Governing equations**

The governing equations are the Navier-Stokes ones, integrated over the whole width, with the following hypothesis: (i) Hydrostatical approximation for the vertical distribution of pressure; (ii) Boussinesq's approximation for the density distribution so that a constant reference density $\rho_0$ may, except for the buoyancy term, be substituted for the water density $\rho$ without any serious errors for all terms of the governing equation; (iii) Lateral variation of the mass and momentum distributions are negligible.

Let the coordinate origin be at the mean water level. The $x$ axis is horizontal pointing landward and the $z$ coordinate is taken positive upward (Fig. 2). $b$ is the width of the

![Figure 2](image-url)  
**Figure 2**  
Coordinate system.
The friction term \( F_s \) in equation (2), along the two lateral sides of a river cross-section is defined as:

\[
F_s = \frac{1}{\gamma} \rho \frac{\partial \tau_{xy}}{\partial y} dy = \frac{g}{C_h} u_0 \left[ \left| \frac{\partial y}{\partial x} \right| + \left| \frac{\partial y}{\partial z} \right| \right]
\]

where \( \tau_{xy} \) is the side-wall stress; \( \frac{\partial K}{\partial z} \) the lateral-side slope in the z direction of the cross-section (Fig. 3); \( C_h \) is the Chezy coefficient.

Estuarine circulation is not completely driven by tidal forcing. Some non-tidal processes have an important effect and must be taken into account, the principal ones among them being: river discharge; density gradients; turbulent energy exchanges in fluid flows; wind stress on the water surface; and bottom and lateral-wall friction. The complexity of the topography and the morphology precludes the simple use of a rectangular geometry for each cross-section in the estuary to calculate the bottom and side-wall friction. In the proposed model, the width of every cross-section thus depends on the \( x \) and \( z \) coordinates. The maximum water depth \( H_{max}(x) \) and the relative width \( b_{max}(x,z) \) are given. The instantaneous width \( b(x,z) \) is calculated by a linear interpolation corresponding to the real water depth (Fig. 4).

### Boundary Conditions

#### At the water surface

For the momentum equation, the shear stress \( \tau_w \) at the water surface, due to the wind effect, implies:

\[
\rho K_s \left( \frac{\partial u}{\partial z} \right)_{wn} = \tau_w
\]

For the salt transport equation,

\[
K_s \left( \frac{\partial S}{\partial z} \right)_{wn} = 0
\]

For the TSM transport equation,

\[
\left( K_s \frac{\partial C}{\partial z} + w_s C \right)_{wn} = 0
\]

#### At the bottom

The bottom shear stress can be expressed as follows:

\[
\tau_s = \rho K_s \left( \frac{\partial u}{\partial z} \right)_{wn}
\]

Near the bottom, because of the boundary layer effects, a direct simulation of the current velocity and the bed shear stress would imply an excessive local refinement of the computing grid. Moreover, the velocity profile in the tur-
TURBIDITY MAXIMUM TRANSPORT IN AN ESTUARY

Figure 4
Schematic cross-section of the estuary width as a function of x and z. (For each section of the estuary, the width at the surface (B_i0) corresponds to the maximum water depth (Hmax). Hmax is divided in 10 layers of the same thickness. The real width at each point of the mesh (b_k = 1, n_z where n_z is the point grid number over the vertical), related to the real water depth H, is interpolated based on the corresponding layer.)

Buoyant boundary layer is known as a logarithmic law (Dyer, 1986). The current velocity \( u_a \) at point a very near the bottom is thus expressed as follow (Bradshaw, 1978):

\[
\frac{u_a}{u_*} = \frac{1}{K} \ln \left( \frac{z_a}{z_0} \right) + C_i
\]

(10)

where \( z_a \) is the distance from the point a to the bottom, \( K \) is the von Karman constant (= 0.4), \( u_* \) the friction velocity, \( z_0 \) the bottom roughness and \( C_i \) an integration constant (= 5). \( u_* \) is unknown, but from the derivative of \( u \) with respect to \( z \), one obtains:

\[
\frac{\partial u}{\partial z}_{z=-h} = \frac{u}{Kz_0 C_z}
\]

with \( C_z = \frac{1}{K} \ln \left( \frac{7.7 z_a}{z_0} \right) \)

(11)

This expression is applied as the boundary condition for the momentum equation. \( z_0 \) is calculated with the formula given by Ramette (1981): \( z_0 = H(26.4/Ch)^8 \). This method was used by Sauvaget (1987), to deal with the boundary condition in an estuarine flow modelling.

For the salt transport equation:

\[
K_w \left( \frac{\partial S}{\partial z} \right)_{z=-h} = 0
\]

(12)

For the TSM transport equation:

\[
\left( -K_w \frac{\partial C}{\partial z} - w_z C \right)_{z=-h} = F_e - F_d
\]

(13)

\( F_e \) and \( F_d \) are the erosion and deposition flux functions, integrated on the width \( b \).

At the open boundaries

At the estuary mouth, the water elevation is permanently given by the recorded values of the tidal elevation at the locality known as "Pointe de Grave". At the two upstream ends, the river discharges of the Garonne and the Dordogne, i.e. at la Réole and Pessac respectively, are provided by the recorded data (IGBA, 1975).

Regarding the salinity at the mouth, values based on measured data and revised, according to tidal amplitudes, are used. At the upstream ends, the salt concentration value is assumed to be 0.1 g l\(^{-1}\), which corresponds to the freshwater content in salts; the sea-water salinity effect is null.

Based on the annually-averaged river solid discharges of 2.2 \( 10^6 \) tons and the river discharges of 30 Gm\(^3\), the turbidity at the upstream ends is assumed: 0.03 g l\(^{-1}\) for the Garonne river and 0.02 g l\(^{-1}\) for the Dordogne river. At the mouth, the turbidity of the seawater is set to 10 mg l\(^{-1}\) during the flood, while the turbidity is calculated by the model during the ebb.

Sedimentological parameters

In the Gironde estuary, the fine-grained sediments are for the most part cohesive. The main parameters governing the erosion flux \( F_e \) and the deposition flux \( F_d \) of TSM on the bottom are the critical erosion shear stress \( \tau_{ce} \), the erosion flux constant \( M \) and the critical bottom deposition shear stress \( \tau_{cd} \). The vertical flux \( F_e \) of the eroded sediment is calculated by using the formula of Partheniades (1962):

\[
F_e = M \left( \frac{\tau_{ce}}{\tau_{cd}} - 1 \right)
\]

(14)
The deposition flux $F_d$ of the TSM is calculated using the formula of Einstein-Krone (1962):

$$F_d = w_s C \left(1 - \frac{\tau_b}{\tau_{ed}}\right)$$  

(15)

When $\tau_b \geq \tau_{ed}$, sediment erosion occurs. The eroded sediment flux is proportional to the constant $M$. When $\tau_b \leq \tau_{ed}$, TSM deposition occurs.

Numerous results of laboratory measurements, reviewed by Metha (1988), showed that, under steady and turbulent flows, the rate of deposition depends mainly on the flow conditions, the TSM concentration and the nature of the sediment surface, while the rate of erosion is determined by the bed properties: the bed shear strength and the state of the sediment consolidation. However, in tidal estuaries, where the tidal currents are usually unsteady and where the bottom is not so flat as in the laboratory, the results coming from laboratory experiments must be used with care.

Measurements in the Gironde have shown that particle size ranges from 2 to 300 $\mu$m at the salinity front and that a large fraction (>70%) of the TSM could have a particle size over 100 $\mu$m (flocs) (Eisma et al., 1991). It is not reasonable therefore, to calculate the settling velocity based on a unique mean particle diameter. Moreover, in the Gironde estuary, where the concentration near the bottom is very high, particle trajectory interactions occur. The hindered settling phenomenon is obvious. The mean TSM settling velocity $w_s$ is thus not calculated with reference to particle size distribution, but is estimated as a function of the TSM concentration. Following the results given by Thorn (1981), in Metha (1986), $w_s$ used in the calculation is:

$$w_s = \begin{cases} K_1 C^n & (C < 3 \text{ g.}^{-1}) \\ w_{so} (1 - K_2 C)^\beta & (C > 3 \text{ g.}^{-1}) \end{cases}$$

(16)

where $w_{so} = 2.6 \times 10^{-3} \text{ m.s}^{-1}$, $K_1 = 0.513$, $n = 1.3$, $K_2 = 0.008$, $\beta = 4.65$. According to this formula, $w_s = 2 \text{ mm.s}^{-1}$ when $C = 3 \text{ g.}^{-1}$.

**Linking condition at the confluence**

An equation is used for the conservation of the salt mass at the confluence of the Dordogne and Garonne rivers:

$$\frac{\partial (\Omega N S_N)}{\partial t} + \frac{\partial}{\partial x} \left( w_N S_N - K_z \frac{\partial S_N}{\partial z} \right) dz |_{\Omega N} = \sum_{i=1}^{N} \int u_i S_i + K_n \frac{\partial S_i}{\partial X} b_i dz$$

(17)

with $\Omega N$ the horizontal water surface area of the confluence zone, $w_N$ the vertical current velocity, $S_N$ the salinity in the confluence area (independent of $x$), $u_i$ the longitudinal current velocity and $S_i$ the salinity in each branch $i$, just outside the confluence zone; the $N$ denotes the confluence variables (Fig. 5). A similar expression is assumed for the turbidity, which means that the erosion and deposition fluxes are neglected at the confluence of the rivers.

**Turbulence-closure model**

The equation system (1)-(5) is not yet closed because the values of the turbulent viscosities and diffusion coefficients are unknown. A turbulence-closure model which permits the determination of these values, must therefore be introduced.

Some turbulence models applied in the oceanic domain have been proposed, such as empirical formulas of turbulent viscosities (Hamilton, 1975), mixing-length models (Boerick and Hogan, 1977) and more frequently one- or two-equation models (Mellor and Yamada, 1982; Blumberg and Mellor, 1987; Nihoul et al., 1989; Galperin and Mellor, 1990) which are mainly based on the transport equations of the turbulent state parameters. In the mixing-length model, the turbulent viscosity $K_z$ is assumed to be proportional to the mean-velocity gradient and a "mixing-length" scale parameter. In the shear-layer flows as boundary layer flows and jet flows, this model gives very good results. But in some complex situations, e.g. recirculating flows or unsteady flows, this model is not very suitable, for two reasons:

- the close link with the velocity gradient in the mixing-length model implies a local equilibrium of the turbulent energy. That means that the turbulent energy is locally dissipated at the same rate as it is produced. The turbulent transport and history effects are thus neglected (ASCE, 1988).
it is quite difficult to specify the mixing-length distribution. A trial-and-error process is therefore generally required to adjust the mixing-length values, in order to obtain a good agreement between the computed results and the observed ones. There is a lack of universality for these models. In order to overcome the limitation of the mixing-length models, equation models are developed, which consider transport and history effects of the turbulent quantities.

The essence of the one- and two-equation models is: (i) the local state of the turbulence is characterized by several parameters: the kinetic energy $k$, the energy dissipation rate $\varepsilon$ or the length-scale of the turbulence $l$, the dissipation rate $\varepsilon$ can be expressed: $\varepsilon \sim k^{3/2}/l$; (ii) all these quantities can be considered as scalar ones. They are transported by both the advection, due to the mean motion, and the diffusion related to the turbulent motion. Hence, the relevant equations, for a so-called $k-\varepsilon$ model, after lateral integration, are as follows:

Turbulent energy equation:

$$\frac{\partial (bk)}{\partial t} + \frac{\partial (buk)}{\partial x} + \frac{\partial (bwk)}{\partial z} = bK_a \left( \frac{\partial u}{\partial z} \right)^2 + \frac{\partial }{\partial x} \left( bK_a \frac{\partial k}{\partial x} \right) + \frac{\partial }{\partial x} \left( bK_a \frac{\partial k}{\partial x} \right) + \frac{c_e bK_m}{\rho_o} \frac{\partial \rho}{\partial z} - b\varepsilon$$

Energy dissipation equation:

$$\frac{\partial (b\varepsilon)}{\partial t} + \frac{\partial (bue)}{\partial x} + \frac{\partial (bwe)}{\partial z} = c_e bK_m \frac{\partial e}{\partial x} + \frac{c_e bK_m}{\rho_o} \frac{\partial \rho}{\partial z} - c_e b\varepsilon$$

The energy dissipation process is associated with small-scale turbulence, so that it does not seem appropriate to use the dissipation equation here to determine the required turbulent length-scale $l$. Thus, Mellor and Yamada (1982) have proposed a turbulence model using directly the parameter pair $k-l$. Moreover, in equation (19), several coefficients have to be determined, and this is one of the main difficulties of the $k-\varepsilon$ model. Therefore, the one-equation model ($k$ equation) has been adopted here. But the rate of dissipation of $k$, in equation (19), must be expressed in terms of the mean properties of the turbulence field. Nihoul et al., (1989) and Beckers (1991) define this expression as follows:

$$\varepsilon = \alpha_\varepsilon \frac{k^2}{16K_s} \quad \alpha_\varepsilon = 1.0$$

with $K_s = \frac{1}{2} \alpha^4 \sqrt{K \cdot \gamma} \cdot I_w = (1 - R_t) I_w (Z)$

The boundary condition at the bottom, for equation (18) is as follows:

$$k_{z=1} = \frac{1}{\sqrt{C}} u_0^2$$

$u_0$ is the friction velocity associated with the bottom friction stress, $u_0 = (\gamma \rho)^{1/2}$. $C$ is a constant ($=0.09$, Rodi, 1980).

At the water surface, the turbulent energy is zero (no energy exchange between the water and the air). At the downstream and upstream boundaries, the diffusive energy flux is equal to zero:

$$\frac{\partial k}{\partial x} = 0$$

Coefficients of dispersion

Turbulent diffusion coefficients

$K_{zz}$, $K_{zc}$ are calculated from $K_z$ (Nihoul et al., 1989):

$$K_n \sim \gamma_s \sqrt{1 - R_t} K_s$$

$$K_n \sim \gamma_c \sqrt{1 - R_t} K_s$$

$\gamma_s$ and $\gamma_c$ are constant ($=1.1$).

Horizontal diffusion coefficients

Boerick and Hogan (1977) proposed an approximate relationship to calculate the momentum dispersion coefficient:

$$K_x = C_d \cdot u$$
where \( b \) is the width of the estuary and \( C_d \) a friction coefficient (= 0.0025).

The horizontal diffusion coefficients \( K_x \) and \( K_z \) are used as follows:

\[
K_x = K_x, \quad K_z = K_x
\]  

(27)

**Coordinate transformation and numerical method**

To take into account the real topography, a sigma transformation is adopted. Then, an arbitrary variable \( \Phi \) is transformed as follows:

\[
\Phi(x, z, t) \rightarrow \Phi(x^*, \sigma, t^*)
\]

with \( x^* = x, \ t^* = t, \ \sigma = \frac{z + h}{\eta + h} = \frac{z + h}{H} \)  

(28)

where \( h \) is the depth of the mean water level, \( \eta \) is the water elevation. \( \sigma \) ranges from zero at \( z = -h \) to one at \( z = \eta \) (Fig. 2). With this transformation, equations (1)-(4) and (18) become:

\[
\frac{\partial (bu)}{\partial x} + \frac{\partial (bw')}{\partial \sigma} = 0
\]  

(29)

\[
\frac{\partial (bu)}{\partial t} + \frac{\partial (bu')}{\partial x^*} + \frac{\partial (bw')}{\partial \sigma} = -bg \frac{\partial \eta}{\partial x} - bgh(1 - \sigma) \frac{\partial S}{\partial x} - \frac{\partial u_j}{\partial \tau} \left[ \frac{\partial \eta}{\partial \tau} + \frac{\partial \xi}{\partial \tau} \right] + \frac{\partial}{\partial x^*} \left( K_b \frac{\partial u}{\partial \sigma} \right) + \frac{\partial}{\partial \sigma} \left( K_b \frac{\partial u}{\partial \sigma} \right)
\]  

(30)

\[
\frac{\partial (bS)}{\partial t} + \frac{\partial (buS)}{\partial x^*} + \frac{\partial (bw'S)}{\partial \sigma} = \frac{\partial}{\partial H^2} \left( K_{uS} \frac{\partial S}{\partial \sigma} \right) + \frac{\partial}{\partial \sigma} \left( K_{uS} \frac{\partial S}{\partial \sigma} \right)
\]  

(31)

\[
\frac{\partial (bC)}{\partial t} + \frac{\partial (buC)}{\partial x^*} + \frac{\partial (bw'C)}{\partial \sigma} = \frac{\partial}{\partial H^2} \left( K_{uC} \frac{\partial C}{\partial \sigma} \right) + \frac{\partial}{\partial \sigma} \left( K_{uC} \frac{\partial C}{\partial \sigma} \right)
\]  

(32)

\[
\frac{\partial (bk)}{\partial t} + \frac{\partial (buk)}{\partial x^*} + \frac{\partial (bw'k)}{\partial \sigma} = bK \frac{\partial \eta}{\partial \sigma} \frac{\partial \eta}{\partial \tau} - \frac{\partial}{\partial \sigma} \left( bK \frac{\partial \eta}{\partial \sigma} \right) + \frac{\partial}{\partial x^*} \left( bK \frac{\partial u}{\partial \sigma} \right) + \frac{\partial}{\partial \sigma} \left( bK \frac{\partial u}{\partial \sigma} \right) - bgK \frac{\partial \rho}{\partial \sigma} - \frac{b\sigma}{\partial \sigma}
\]  

(33)

The vertical “velocity” in the transformed system is:

\[
w' = -\frac{1}{H} \left[ \frac{\partial \eta}{\partial \tau} + w \right]
\]  

(34)

The settling velocity is then:

\[
w_i = \frac{w_i}{H}
\]  

(35)

The governing equations are solved by a splitting technique. The advection terms and the diffusion terms (including the wave propagation terms) are computed separately at each half-time step. The advection term is computed by the characteristic method and the diffusion term is computed with the finite difference method (Nguyen and Martin, 1988). Then, the second-order accurate central differences in space and the first-order forward differences in time are adopted. The scheme of the diffusion term is explicit regarding \( x \) and implicit regarding \( z \), and is easily shown to be stable in this condition of calculation.

**APPLICATION IN ACTUAL CONDITIONS**

**Physical parameters**

The computation domain consists of three parts which correspond to the three branches of the estuary. The first branch is discretized in 35 sections with the longitudinal space step \( \Delta x = 2 \) km. The Garonne and Dordogne rivers, with two upstream ends: La Réole and Pessac, are respectively discretized in 24 and 25 sections, where \( \Delta x = 5 \) km. The vertical direction is discretized with 15 points which are refined near the bottom and the surface. For each point, the width \( b \) of the estuary is calculated by taking into account the water elevation (Fig. 2). The time step is \( \Delta t = 80 \) s.

The model operates with the tidal amplitude and river discharge values corresponding to the period of measurements made by the IGBA (19-25 May, 1975). The measurements of current velocity, salinity and turbidity were obtained simultaneously, a fact which confers a major interest on this data set obtained 18 years ago. The tidal amplitude ranged between 2.8 and 3.6 m (tidal coefficient from 58 to 79) and the river discharges were about 450-480 m³.s⁻¹ in the Garonne and 205-230 m³.s⁻¹ in the Dordogne rivers respectively. The wind field during this period is unknown, so the shear stress \( \tau_w \) at the water surface is set to zero in eq. 6. The numerical results are compared with those provided by IGBA at six stations located along six cross sections, from PK 20 to PK 75, close to the navigation channel. When matching calculated and measured values, the comparison is thus made between integrated values along the cross-section (calculated) and punctual values (measured). This may have no effect concerning the elevation values, because the water-surface slope regarding \( y \) can be neglected, but for the other parameters and mainly for the turbidity, comparison must be done with great care.

**Sedimentological parameters**

In this two-dimensional approach, the \( \tau_e, \tau_{cd} \) and \( M \) parameters must take a unique mean value along each
cross-section. Thus, the values of these parameters, used by the model, are determined along the longitudinal axis (Fig. 6), taking into account a rough evaluation of the bed properties. For $\tau_{ce}$:

- Regarding the sand bottom, extending from the estuary mouth to about 15 km upstream from Royan, and at the upstream boundaries, based on values reported by Hug (1975), $\tau_{ce}$ is proposed to be 2.0 – 3.0 N m$^{-2}$.

- Regarding the areas covered by the cohesive sediment, the $\tau_{ce}$ values are assumed to range from 0.5 to 1.7 N m$^{-2}$ (Migniot, 1968). The $\tau_{ce}$ value is set to 0.65 in the area where the fluid mud was located during the period of measurements. This area is established for the entire simulation.

The erosion constant M is also a knotty parameter to be determined. Cormault (1971) has proposed M to be equal to 2 $\times 10^{-3}$ kg m$^{-2}$.s$^{-1}$, based on laboratory measurements using a composite mud of the Gironde estuary. In the model, M varies from 2 $\times 10^{-3}$ to 3 $\times 10^{-2}$ kg m$^{-2}$.s$^{-1}$. The highest value ascribed to M corresponds to the softest mud having the lowest $\tau_{ce}$ values, i.e. where the fluid mud is located.

The state of consolidation of the sediment along the estuary varies with time, strongly with seasonal river flow and probably with tidal amplitude. These variations may induce some fluctuations of the bed shear strength values and thus of the $\tau_{ce}$ and M values. But, in this first approach, for a simulation of a few days period, these parameters are considered constants. This also means that the freshly-deposited matter obtains instantaneously the $\tau_{ce}$ and M values assumed to be in its sedimentation location (Le Hir and Karlikow, 1991). The quantities of sediment and fluid mud available for the erosion process are considered limitless.

The $\tau_{cd}$ value, below which deposition occurs, is principally characterized by the nature and the concentration of the suspended matter and has been measured in many laboratory experiments: for the San Francisco bay, with sediment having uniform properties, Krone (1962) found $\tau_{cd} = 0.078$ N m$^{-2}$ when the initial TSM ranged from 0.3 to 10 g.L$^{-1}$; for sediment having a broad size distribution, Mehta and Partheniades (1975) found that $\tau_{cd}$ varies from 0.18 N m$^{-2}$ to 1.1 N m$^{-2}$. However, there are few laboratory studies using the mud of the Gironde. Based on the variation of the minimum value of the calculated bottom shear stress calculated by the model, $\tau_{cd}$ is set to 0.3 – 0.5 N m$^{-2}$.

Upstream from PK 20, there is no turbidity maximum influence during the considered period of measurements; the TSM concentration is very low. Thus, the figures showing the calculated results cover the area from PK 20 to the mouth of the estuary.

**Initial conditions**

The computational time needed to obtain a stable periodic solution can be reduced by using some selected initial values.

The computation starts at a time corresponding to slack water. The initial values for the surface elevation are set to 1.4 m at each section. The initial velocity distribution is $u = 0.1$ m.s$^{-1}$ and $w = 0.0015$ m.s$^{-1}$. The initial salinity is obtained by a linear interpolation between the values at the top and the bottom of the water column. The concentration of TSM is set to 0.1 g.L$^{-1}$ for the Gironde estuary and to 0.05 g.L$^{-1}$ for the rivers. With these initial conditions, after two tidal periods of calculation, the stable periodic regime is reached.

**RESULTS AND DISCUSSION**

The tidal time indicated in the figures is referred to that at the mouth (Pointe de Grave). The origin time in the figures representing the results is delayed by three tidal periods from the starting time of the simulation.

**Estuarine circulation**

The surface elevation is continuously recorded along the Gironde estuary: this surface is very well described by the model (Fig. 7). The differences between the corresponding computed and measured values are less than 2% in amplitude and less than 10 minutes in phase. Moreover, the
numerical results show clearly that the tidal amplitude increases landward from the mouth of the estuary. This is in agreement with the fact that the Gironde is a hypersynchronous estuary (Salomon and Allen, 1983).

Longitudinal current velocities one metre below the surface and one metre above the bottom are shown in Figure 8. The phase difference between the numerical results and the observed ones is less than 5 minutes. The computed values of the current velocity are fairly in agreement with the observed ones (differences smaller than 15%). However, at Richard, during the tidal flow, the numerical value of the maximum velocity is higher than the observed ones; but Richard is located 20 km upstream, from the mouth, where the depth of the estuary varies rapidly and where the transverse component of the horizontal circulation exists. The instantaneous velocity field every 2 hours during one tidal cycle (tidal coefficient 75 ~ 79) is illustrated in Figure 9.

Figure 7
Comparison of observed (cross) and computed (continuous lines) elevations at 6 locations (tidal amplitude from 2.8 to 3.6 m, mean river discharge = 700 m³.s⁻¹).
The turbulent energy field permits a detailed illustration of the influence of tidal effects. Turbulent energy distribution every 2 hours during one tidal cycle is shown in Figure 10. The energy is always higher close to the bottom than to the surface (the wind effect is ignored). The field reaches its peak between high tide plus 4 hours and low tide, and the maximum value at the bottom can exceed \(5 \times 10^{-3} \text{ m}^2\text{s}^{-2}\) (Fig.10f, a). It presents a uniform state with the lowest values at high tide plus 2 hours and at low tide plus 2 hours (Fig.10b,c). A simple relationship between the turbulent energy field and the current velocity field is not obvious (Fig. 8 and 9); except when the current velocity is very low, the turbulent energy is also very weak.

Calculated salinity values versus the measured ones are shown (Fig.11). The lag time between them is less than
Figure 9

Instantaneous velocity field during one tidal cycle (tidal amplitude = 3.5 m, mean river discharge = 700 m$^3$.s$^{-1}$).

20 minutes. The decrease in mean salinity from 30 g.l$^{-1}$ at the mouth to 1 g.l$^{-1}$ at Marquis is well shown. The calculated salinity values fit well the measured data, regarding the tide phase, upstream from Lamena (differences < 20%). Downstream from Lamena (Richard and PK 68), salinity values at the water surface are smaller than the observed ones. This could indicate that the computed vertical salinity mixing is not strong enough despite the noticeable advantage induced by the turbulence closure sub-model used here; or it could be due to the limitations of the two-dimensional vertical model in this area, where a transverse horizontal circulation exists. The process of the salinity intrusion is also shown by the salinity distribution (Fig.12). Two hours after low tide, the isohaline 1 g.l$^{-1}$ reaches PK 40 (Fig.12 b). Two hours after high tide, it is located upstream of PK 30 (Fig.12 e). This means that a displacement of more than 10 km in the salinity pattern occurs during one semi-tidal cycle.
Turbidity

The complexity of the physics of the natural Gironde estuarine environment, principally the variability of the bed properties ($\tau_{ce}$ and $M$) along a cross-section and the limitation of a two-dimensional vertical model, do not allow a full calibration of the model. For each section of the estuary, a mean value, for $\tau_{ce}$ and for the $M$ parameters, is considered, taking into account our knowledge of these parameters in the natural environment (see section Sedimentological Parameters). Thus, two tests were performed.

- First, a unique value for $\tau_{ce}$ (1.2 N.m$^{-2}$) and $M$ (6 $10^{-3}$ kg.m$^{-2}$.s$^{-1}$) was set from PK 10 to PK 80, in order to verify whether the simulated turbidity maximum location does not depend solely on the $\tau_{ce}$ and $M$ values assumed in the area where the fluid mud exists. For these
conditions, the location of the turbidity maximum is the same as that described hereafter, but its intensity is much less than the observed one.

- Second, a sensibility test was performed to show how the simulation of the turbidity maximum is affected by the change of the assumed $\tau_{ce}$ and $M$ values, in the area where the fluid mud occurs. The test is based on the effects of these changes on the calculation of the total mass of the turbidity maximum, by reference to the mass obtained with the assumed parameters. This, for a 5-day period simulation, with a constant tidal amplitude of 3.35 metres (tidal coefficient of 70). When the assumed value of $M$ is multiplied by 10, the total mass of the turbidity maximum increases by 30%; when this value is divided by 10, the mass decreases by 13%. When the assumed value of $\tau_{ce}$ is increased by 30%, the total mass of the turbidity maximum decreases by 13%; when this value is decreased by 30%, the total mass increases by 43%. This test shows that the model is not too greatly affected by some uncertainties concerning the $\tau_{ce}$ and $M$ parameter values.

The main characteristics of the turbidity maximum, its formation and decay and its displacement during one tidal
cycle for the tidal amplitudes and the river discharge corresponding to the period of measurements are shown.

The time evolution of the calculated turbidity (Fig.13) shows that the TSM concentration increases during the portion of the 5 days corresponding to the increase of the tidal coefficient greater than 60. The computed TSM values are in good agreement with the observed data. The observed maximum of the TSM concentration, at Lame-na, is also shown by the model, despite the fact that the calculated bottom turbidity value is less than the observed one.

Turbidity one metre above the bottom shows (clearly from Pauillac to the estuary mouth) two maximum values during one tidal cycle. This is related to the module of bottom current velocity which displays two maxima during one tidal cycle.

At Richard, the TSM concentration is greater during the ebb than during the flood, while the bottom current velocity is symmetrical during one tidal cycle. Richard is located at the seaward end of the turbidity maximum zone; consequently, due to the dilution resulting from the sea water intrusion during the flood, the TSM concentration is lower during this period. On the contrary, at Marquis located upstream the turbidity maximum, the greatest TSM concentration appears during the flood.

Figure 12
Longitudinal distribution of salinity (‰) during one tidal cycle (tidal amplitude = 3.5 m, mean river discharge = 700 m³ s⁻¹).
The evolution of the turbidity maximum every 2 hours during one tidal cycle is represented Figure 14. The turbidity maximum zone with a concentration $> 1 \, \text{g} \cdot \text{l}^{-1}$ spreads over about 40 km; its core ($> 5 \, \text{g} \cdot \text{l}^{-1}$) is situated at PK 65 at low tide (Fig. 14 a). It has a longitudinal oscillation of 15 km during one tidal cycle: at high tide, the seaward boundary of the maximum is displaced upstream, and its core is situated at PK 50 (Fig. 14 d). The size and the intensity of the maximum strongly depends on the phase of the tide cycle: after low and high tide, its concentration decreases (Fig. 14 b, e). Conversely, with the increase of current velocity, the maximum arises again (Fig. 14 c, f).
The use of a turbulence closure sub-model induces several effects on the turbulent viscosity and diffusion coefficient values. Thus, compared with the use of the mixing-length method, the calculated turbidity maximum seems to match better the observed structure (Fig.15).

The simulated results are also compared with the scheme of the longitudinal turbidity distribution given by Castaing (1981) (Fig.16). The observed values were collected in the navigation channel on 22 May, 1974 with a spring tidal amplitude of 4.1 m (tidal coefficient of 91-93) and a mean river discharge of 600 m$^3$s$^{-1}$. The simulated results are in
good agreement with the turbidity pattern reconstructed by Castaing, especially the decay of the TSM intensity two hours after high tide (Fig. 16d).

The calculated total mass of the TSM in the estuary has its maximum value \(2 \times 10^6 \, T\) at high tide and its minimum value \(1.3 \times 10^6 \, T\) at low tide. The mean total mass of TSM deduced from the in situ results during a spring tide (Allen, 1973) falls close to \(2.5 \times 10^6 \, T\).

The tidal amplitudes 2.5 m and 4.1 m (tidal coefficient of 50 and 90) which are outside the observed ones are also used to examine the response of the model, with a constant water discharge of 700 m\(^3\) s\(^{-1}\). The tidal coefficient 90 induces an increase of the intensity of the turbidity maximum, the total mass of the TSM is \(2.2 \times 10^6 \, T\) and a fraction of the maximum is flushed towards the sea (Fig. 17) at low tide. For a tidal coefficient of 50, there is a large decrease of the size and of the intensity of the maximum (Fig. 18). The total mass of the TSM is \(0.45 \times 10^6 \, T\) and there is practically no flux of TSM towards the sea. These effects of low and high tidal amplitude have indeed been observed (Allen, 1973 and Castaing, 1981).

In this model, the TSM settling velocity is only a function of the TSM concentration, following the Thorn (1981) results. This means that particle size distribution is not taken into account, in spite of its dramatic change in the low salinity zone, depending on the flocculation process. An improvement of this model implies that account must be taken of particle size distribution, bed shear strength which depends of the state of the sediment consolidation, and finally of the history, on short and long time scales, of the estuary.
Comparaison of the numerical turbidity distribution (g.l\(^{-1}\)) (A) and the observed distribution (B) (Castaing, 1981) during a spring tidal cycle (tidal amplitude ~ 4.1 m, mean river discharge ~ 700 m\(^3\)s\(^{-1}\)).
CONCLUSION

A vertical two-dimensional width-integrated model, coupled with a turbulence closure sub-model, has been used to simulate the estuarine circulation, the salinity intrusion and the total suspended matter transport process in the Gironde estuary. The detailed time evolutions of the elevation, the velocity, the salinity and the turbidity during five days with a tidal amplitude varying from 2.8 to 3.6 m, have been calculated and compared with the observed data at six stations along the estuary axis. The longitudinal distributions of salinity, turbidity and turbulent energy during one tidal cycle have been described to illustrate the semidiurnal tidal effects on these processes in this macrotidal estuary.

The results indicate that:

a) The computed hydrodynamical variables are in good agreement with the observed data.

Figure 17

Displacement of the "turbidity maximum" (g.l⁻¹) during a spring tidal cycle (tidal amplitude ≈ 4.1 m, mean river discharge ≈ 700 m³.s⁻¹)
b) The tidal oscillation induces a 15 km displacement of the turbidity maximum during one tidal cycle. The formation and the decay of this maximum have been simulated, and the total mass of the maximum obtained by the model is in good agreement with the observed one. Turbidity maximum behaviour is strongly controlled by the turbulent energy field, connected with the macrotidal mechanisms.

c) An increase of the tidal amplitude up to 4.1 m, provokes the increase of the size and the intensity of the turbidity maximum and its partial flushing to the sea.

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