Generation of a residual current by interaction between the coastal boundary layer and the Ekman layer in a tidal motion

Denis AELBRECHT a, Gabriel CHABERT D’HIERES a and Xiuzhang ZHANG b

a Laboratoire des Écoulements Géophysiques et Industriels, Institut de Mécanique de Grenoble, URA CNRS n° 1509, B.P. 53 X, 38041 Grenoble Cedex, France.

b Arizona State University, Department of Mechanics and Aerospace Engineering, Tempe, Arizona 85287-6106, USA.

ABSTRACT

Some experiments have been made with the large rotating tank at the Institut de Mécanique de Grenoble to reproduce a sinusoidal flow along a vertical wall, over a flat bottom, with or without slope, in a homogeneous fluid. The objective of these experiments was to simulate a tidal motion, restricted to its main harmonic component M2, parallel to a vertical coast; and to demonstrate a particular mode of generation of residual circulation near the coast as a result of the interaction between friction, background rotation of the system (Ekman pumping) and oscillatory motion (Stokes effects). Moreover, the stabilizing (or otherwise) effects of the Earth’s rotation on this shear flow may modify the rectification process. The configuration is in a good agreement with the general topographic and hydrographic situation along the French coast of the Pas-de-Calais, near Boulogne-sur-Mer.

In the Northern hemisphere, the bottom current is diverted to the left of the main current due to the balance between the Coriolis force and the pressure gradient. This imposes a dissymmetric behaviour of the flow between flux and ebb, as well as the generation of an alternative component perpendicular to the coastline, together with lateral transport (vertical close to the coasts) of horizontal momentum which modifies the coastal boundary layer. A rectified (residual) current is observed with the coast to its right. Experiments were systematically made on a flat bottom moving in the rotating tank by varying the main parameters of the flow; and we measured the velocity field with ultrasonic velocimetric probes and photographs of surface floats. The results (amplitude and width of the rectified current) show a good correlation with the few observations at sea and with the scaling analysis we provide.

Oceanologica Acta, 1993. 16, 5-6, 479-487.

RÉSUMÉ

Formation d’un courant résiduel par interaction entre une couche limite côtière et une couche d’Ekman dans un écoulement de marée

Des expériences ont été menées sur la grande plaque tournante de l’Institut de Mécanique de Grenoble afin de reproduire un écoulement alternatif sinusoidal en fluide homogène ou stratifié, sur un fond plat incliné ou non, le long d’un mur rectiligne vertical. L’objectif est de simuler un écoulement de marée parallèle à une côte, et de mettre en évidence un mode particulier de génération d’un courant
INTRODUCTION

The general objectives of this study are to describe the structure of the residual current resulting from an alternative flow, parallel to a rectilinear wall, over a flat bottom; and to understand the physical process which tends to generate this rectified current. The first experiments were performed on a horizontal bottom. A better similarity with nature was obtained with a sloping bottom. In order to realize such a flow, we used the large rotating tank of the Institut de Mécanique de Grenoble to create the oscillation of a flat plate in homogeneous water. We do not reproduce exactly the tide and all its effects, and neglect, for example, the variation of water depth which imposes a variation of vorticity that becomes dissymetric due to friction; possible gravity currents due to longitudinal slope of the free surface; the specific nonlinearity of the tidal wave, etc. Nor, of course, do we take into account the effects of wind-forcing, sand-banks, etc. The physical model provides some information concerning the particular process known as Ekman Pumping. Moreover, the dynamics in nature and on the model are different, especially in regard to the pressure gradient, due to the fact that in our experiments the bottom is moving and not the water. But the general process of Ekman pumping remains valid. In nature, and particularly close to the French coastline at Boulogne-sur-Mer in the Channel (see Fig. 1), the residual circulation reflects the combined effect of tide, wind, irregular bathymetry, and other factors. The regular shape of the coastal topography near Boulogne-sur-Mer allows us to extend our results to the particular case of this region: the French coast is oriented in a virtually rectilinear north-south direction, the average depth some ten kilometres offshore is about 30 m, and the tidal currents observed there appear to be quite parallel to the coast, with an average amplitude of about 1 m/s. For these reasons, our experiments can bear a good resemblance to nature. Because of the real impact of the fresh water contribution in the region studied, we carried out some specific experiments in stratified fluid by introducing fresh water into the salt water at the coast. Particular attention was paid to the evolution of the density front which occurs. But the observation results of this phenomenon lie outside the scope of the present article.

Numerous papers have been written on the subject of tidal motions and particularly on the generation of residual currents in such flows. Salomon and Breton (1991) computed a 2-D hydrodynamic model showing the general structure of residual currents throughout the Channel, according to the different weather conditions (notably of wind) which modify the residual circulation near the coast. Tee (1979 and 1980) analyzed the tidal motion in terms of oscillating currents on the one hand, and residual currents on the other. He first computed a depth-averaged velocity field which permitted determination of the vertical velocity gradient. But he failed to take into account the effects of vertical variations of tidal currents in the lateral boundary layer. Prandle (1982) studied the vertical structure of tidal currents in the case of an infinite flat bottom (merely in terms of oscillating currents) and proposed some analytical solutions for different forms of eddy viscosity (constant or linear depth dependence). Maas and Van Haren (1987) obtained a large number of observations at

Figure 1

a) general map of the channel; b) bathymetric map near Boulogne-sur-Mer (from "Carte SHOM 5107, September 1943").

a) carte générale de la Manche ; b) carte bathymétrique au large de Boulogne-sur-Mer (tirée de "Carte SHOM 5107, septembre 1943").
sea of tidal currents (in the middle of the North Sea) and compared their observations with the theoretical model of Prandle concerning the vertical structure of the currents (configuration without any coast).

EXPERIMENTS

Experimental setup (see Fig. 2)

We use Cartesian co-ordinates (Oxyz), rotating around the z-axis with angular velocity $\Omega$, the x-axis being parallel to the rectilinear coastline, so that "y" is the offshore distance. The relative movement of the water is produced by oscillating the flat plate. This plate has a rectangular shape, 8 m long and 2 m wide. The parameters of the flow are: the depth, $H(y)$ which can be constant or varying linearly with the offshore coordinate $y$: $H(y) = H$ or $H(y) = H_1 + y \tan \alpha$; the slope of the oscillating plate, $\alpha$; the Coriolis parameter, $f = 2\Omega \sin \lambda$ (period of rotation $T = 2\pi/\Omega$); the oscillation frequency of the plate $\alpha$ (period of oscillation $t = 2\pi/\alpha$); and the amplitude of the oscillating current, $U_0$. We also assume the flow to be independent of the "x" coordinate, and we study what happens through the $(x = 0, y, z)$-plane. Note that disturbance of frequency $\sigma$ could drive shallow-water gravity waves with a wavelength of $L_w = 2\pi/\sigma \sqrt{(gH)}$.

Typical values are: $H = 5 \text{ cm}$, $\tau = 60 \text{ s}$, so that $L_w = 42 \text{ m}$; this indicates that the effects of gravity waves are negligible and that the Froude number is not an important parameter of the flow.

We can show, by writing the equations governing the dynamics (conservation of momentum and mass), that the flow characteristics are governed by the following non-dimensional parameters:

\[ R_{e0} = \frac{\alpha/\Omega}{f} = \text{inertia}/\text{Coriolis} \quad \text{temporal Rossby number} \quad (1) \]
\[ E_k = \frac{v/\Omega^2}{H^2} = \text{friction}/\text{Coriolis} \quad \text{Ekman number} \quad (2) \]
\[ R_e = \frac{U_0}{H/v} = \text{inertia}/\text{friction} \quad \text{Reynolds number} \quad (3) \]

The respect of the Reynolds number is not systematic, but we chose values for $R_e$ in the range of fully turbulent flows. In the definition of the Rossby number, we selected a typical horizontal distance $L$ being the elongation of the tide (i.e. $L = U_0/\alpha$). Thus, $R_{e0} = U_0/\Omega L = U_0/ f(U_0/\alpha) = \sigma/\alpha = R_{e0}$: temporal Rossby number.

Similitude

We deal merely with the similitude of the flow in homogeneous fluid. The important point to consider here is the difficulty of realizing and measuring a significant eddy viscosity. Chabert d'Hières (1991) chose a typical value of 0.04 cm$^2$ s$^{-1}$ for the eddy viscosity. We carried out a large number of preliminary experiments (i.e. Ekman flow, uniform motion in a rotating frame; Stokes flow, oscillating motion in a fixed frame) in order to determine a significant value of the eddy viscosity. These gave a typical value of 0.03 cm$^2$ s$^{-1}$ (by assessing Ekman layer thickness; experiment using a Reynolds number of approximately 3000). Of course, this value should vary in time and space. For nature, Maas and Van Haren (1987) found a good representation with a constant eddy viscosity of $1.4 \times 10^{-3}$ m$^2$ s$^{-1}$. We retain a value of $10^{-3}$ m$^2$ s$^{-1}$ for the nature scale. In the following, the index "N" will specify natural values, the index "m" model values. $[X]$ will be the scale of quantity $X$, i.e. the ratio between model value of $X$ and natural value of $X$.

Then, the above considerations impose $[v] = 3 \times 10^{-3}$.

An expression of Ekman thickness may be (see Pedlosky, 1987):

\[ \delta_e = \pi (2v/\Omega)^{1/2} \quad \text{or} \quad \delta_e = \pi H (2E_k)^{1/2} \quad (6) \]

Since the nature values are: $H_N = 30 \text{ m}$, $U_{ON} = 1 \text{ m s}^{-1}$ (mean amplitude of tidal current), $f_N = 2\Omega \sin \lambda = 1.1 \times 10^{-4}$ s$^{-1}$, where $\Omega$ is the earth angular velocity and $\lambda$ the latitude ($\lambda = 51^o$ N), $T_N = 12 \text{ h 25 mn} = 44700 \text{ s}$ (tidal period), $v_N = 10^{-3}$ m$^2$ s$^{-1}$, so $(\delta_e)_N = 13 \text{ m}$. In nature (near Boulogne-sur-Mer), the thickness of the Ekman layer may be estimated to be half the total depth. This ratio will be conserved on the physical model: $(\delta_e)_m = H_N/2$. We have to select one scale to solve the problem.

We first choose: $H_m = 5 \text{ cm}$, then $[H] = 0.05/30 = 1.67 \times 10^{-3}$. We have consequently $(\delta_e)_m = 2.5 \text{ cm}$. Since $[v] = 3 \times 10^{-3}$, equation (2) gives $[T] = 1/1076$. Equation
We also used some floats to visualize Lagrangian trajectories (see Fig. 8).

The methods used to study the effect of the contribution of fresh water comprised investigation of the density field with conductivity probes and visualization of the front with a laser sheet and video tape recorder.

RESULTS

First harmonic components

In appendix A, we give the theoretical solution of the first-order harmonic components $u_1$ and $v_1$ [see equations (5) and (5')] for the case of an infinite horizontal bottom with: a) a no-slip boundary condition; or b) a stress condition on the bottom; no stress being the boundary condition at the surface. The problem involves the possible estimation of the temporal characteristics of the flow field by theories assuming an infinite horizontal plate. We consider this question in relation to the analyses of the vertical structure of tidal currents by Prandle (1982), and the observations of Maas and Van Haren (1987). Figure 3 plots the vertical structure of the normalized amplitude of the streamwise component $u_1/U_0$ at two different lateral locations (near the vertical wall and far from it). The symbols represent the measured data and the dashed line the theoretical predic-

We also used some floats to visualize Lagrangian trajectories (see Fig. 8).

The methods used to study the effect of the contribution of fresh water comprised investigation of the density field with conductivity probes and visualization of the front with a laser sheet and video tape recorder.

RESULTS

First harmonic components

In appendix A, we give the theoretical solution of the first-order harmonic components $u_1$ and $v_1$ [see equations (5) and (5')] for the case of an infinite horizontal bottom with: a) a no-slip boundary condition; or b) a stress condition on the bottom; no stress being the boundary condition at the surface. The problem involves the possible estimation of the temporal characteristics of the flow field by theories assuming an infinite horizontal plate. We consider this question in relation to the analyses of the vertical structure of tidal currents by Prandle (1982), and the observations of Maas and Van Haren (1987). Figure 3 plots the vertical structure of the normalized amplitude of the streamwise component $u_1/U_0$ at two different lateral locations (near the vertical wall and far from it). The symbols represent the measured data and the dashed line the theoretical predic-

We also used some floats to visualize Lagrangian trajectories (see Fig. 8).

The methods used to study the effect of the contribution of fresh water comprised investigation of the density field with conductivity probes and visualization of the front with a laser sheet and video tape recorder.

RESULTS

First harmonic components

In appendix A, we give the theoretical solution of the first-order harmonic components $u_1$ and $v_1$ [see equations (5) and (5')] for the case of an infinite horizontal bottom with: a) a no-slip boundary condition; or b) a stress condition on the bottom; no stress being the boundary condition at the surface. The problem involves the possible estimation of the temporal characteristics of the flow field by theories assuming an infinite horizontal plate. We consider this question in relation to the analyses of the vertical structure of tidal currents by Prandle (1982), and the observations of Maas and Van Haren (1987). Figure 3 plots the vertical structure of the normalized amplitude of the streamwise component $u_1/U_0$ at two different lateral locations (near the vertical wall and far from it). The symbols represent the measured data and the dashed line the theoretical predic-

We also used some floats to visualize Lagrangian trajectories (see Fig. 8).

The methods used to study the effect of the contribution of fresh water comprised investigation of the density field with conductivity probes and visualization of the front with a laser sheet and video tape recorder.

RESULTS

First harmonic components

In appendix A, we give the theoretical solution of the first-order harmonic components $u_1$ and $v_1$ [see equations (5) and (5')] for the case of an infinite horizontal bottom with: a) a no-slip boundary condition; or b) a stress condition on the bottom; no stress being the boundary condition at the surface. The problem involves the possible estimation of the temporal characteristics of the flow field by theories assuming an infinite horizontal plate. We consider this question in relation to the analyses of the vertical structure of tidal currents by Prandle (1982), and the observations of Maas and Van Haren (1987). Figure 3 plots the vertical structure of the normalized amplitude of the streamwise component $u_1/U_0$ at two different lateral locations (near the vertical wall and far from it). The symbols represent the measured data and the dashed line the theoretical predic-

We also used some floats to visualize Lagrangian trajectories (see Fig. 8).

The methods used to study the effect of the contribution of fresh water comprised investigation of the density field with conductivity probes and visualization of the front with a laser sheet and video tape recorder.

RESULTS

First harmonic components

In appendix A, we give the theoretical solution of the first-order harmonic components $u_1$ and $v_1$ [see equations (5) and (5')] for the case of an infinite horizontal bottom with: a) a no-slip boundary condition; or b) a stress condition on the bottom; no stress being the boundary condition at the surface. The problem involves the possible estimation of the temporal characteristics of the flow field by theories assuming an infinite horizontal plate. We consider this question in relation to the analyses of the vertical structure of tidal currents by Prandle (1982), and the observations of Maas and Van Haren (1987). Figure 3 plots the vertical structure of the normalized amplitude of the streamwise component $u_1/U_0$ at two different lateral locations (near the vertical wall and far from it). The symbols represent the measured data and the dashed line the theoretical predic-
THE EKMAN PUMPING

Figure 4

Schematic representation of Ekman pumping when: a) the wall is to the right of the flow; b) the wall is to the left of the flow.

Figure 5

3-D profile of the streamwise residual current \( u_0 \). For \( y \) and \( z \) coordinates, the laboratory and nature (using similitude) scales are represented: a) for experiment No. 1; b) for experiment No. 3.

Rectified current

The present physical system leads to a rectified current in the streamwise direction with the coastline on its right for the Northern hemisphere rotation. When the flow has the coastline to its right (see Fig. 4a), the parcels of fluid in the Ekman layer move away from the wall. This implies that the momentum in the upper layer is advected towards the coast and modifies (decreases) the lateral boundary layer. Conversely, when the streamwise flow has the coast to its left (see Fig. 4b), the parcels in the Ekman layer first head towards the coast and move close to the vertical wall [in the so-called Stewartson layer (see Pedlosky, 1987)]. Then, weaker momentum near the wall is advected away from the coastline, and modifies (increases) the lateral boundary layer. As a result, a rectified (residual) current is generated with the coast on its right-hand side. The vertical upward and downward transports occur in a very close lateral region, the Stewartson layer (see Pedlosky, 1987), whose thickness is \( \delta_r \). It should be noted that for the present experimental conditions, \( \delta_r/H = 0.2 \). During the half-period described on Figure 4b, fluid parcels are carried along the plate floor towards the wall in the Ekman layer at a characteristic lateral speed \( u = 2U_0/\pi \). By conservation of mass,
this transport must be balanced by an adjustment flow through an interior of thickness $H - (\delta e/\pi)$ which is about $H$, with a characteristic cross-stream speed $v$, so that: $vH = u(\delta e/\pi)$. For $v > 0$ (Fig. 4 b), low-momentum particles are transported into the fluid interior for a characteristic time $t = \pi/\sigma$ (half a cycle). The characteristic lateral distance $dy$ to which such parcels move during this half-cycle is taken as a typical width of the rectified current: $\delta y = v.t = (\delta e/\pi H)$. $u(\sigma/\pi)\delta y/H$ is scaled by:

$E_k^{1/2}R_\alpha R_\sigma^{-1}$, by writing $R_\sigma = U_j/(f H)$ (a typical Rossby number). This rectified aspect of the motion can be clearly demonstrated by tracking free-surface floats (Lagrangian point of view, see Fig. 8), as well as by analyzing the mean of the velocimetric signal over a cycle of motion (Eulerian point of view).

Analysis of the ultrasonic signals shows that for the standard experiment (No. 3, sloping bottom) in similarity with nature, the schematic structure of the depth-averaged mean current extrapolated to nature is (see Fig. 5 b):

* $u_j/U_0 = 20\%$ at a distance of about 1 km from the coast
* $u_j/U_0 = 12\%$ " 2 km " ;
* $u_j/U_0 = 7\%$ " 3 km " ;
* $u_j/U_0 = 0$ " 4 km " .

This structure has to be compared with the nature value of 6 cm/s (6 % of the tidal amplitude) for residual current measured at Approche Boulogne station in 1989 during one month (see point AB on Fig. 1). Figures 6 a and 6 b clearly show the presence of residual circulation because of the drift of the ellipse current centres near the coast. Moreover, Figure 6 b shows that the Ekman effect is completely modified near the coast (see the size of the minor axis on the ellipse current). Figures 7 a and 7 b depict the vertical structure of the current. We observe on Figure 7 b (far from the coast) the inversion of rotation sense with depth and the increasing of $v$ component towards the bottom. Figure 8 shows surface floats trajectories for experiment No. 1. This demonstrates the occurrence of a Lagrangian mean current, but Eulerian component and Stokes drift cannot be disconnected. We do not deal with this problem in the present paper.

A numerical study of the laboratory experiment is currently underway. It will provide some general information on the circulation in the cross-stream section, particularly concerning the presence of cells of circulation (see Zhang et al., submitted).
Effects of sloping bottom, roughness

The previous explanation of Ekman pumping is still valid, but the general effect of sloping bottom is an increase in the width of rectified current: the slope pushes the rectified current in the offshore direction. Beginning with No. 9, our experiments were done with bottom roughness. The impact of this roughness is a heightened degree of turbulence, without modifying strongly the typical value of eddy viscosity (according to our preliminary findings). Because of friction, the rectified current is found to be weaker than in the smooth case. Some experiments have been performed with a coastal input of fresh water to reproduce the impact of the contribution of a river (for example, the Somme along the French coastline). We observed the possible stabilization of a density front at a certain distance from the wall; and the variation of this front according to the tidal strength (i.e. according to the tidal velocity amplitude).

In standard conditions (experiment No. 11), we carried out some experiments involving the input of fresh water at the coast (simulation of a river discharge). Although this is not directly related to the process involved there, we clearly observed a density front whose the distance from the coast is estimated at 50 cm (about 5 km in nature). Moreover, the influence of the tidal motion is such that the front becomes vertical when the tidal currents increase. This is in a very good agreement with the observations of Brylinski and Lagadeuc (1990). During neap tide, the front becomes almost horizontal; and particularly during the reverses, some salt water may flow under the fresh water and move close to the coast. This thesis is confirmed by oceanographers in Boulogne-sur-Mer. It seems that the buoyancy force has no significant effect on the dynamics described before, whereas dynamics obviously modify the structure of the stratification (see remarks above).

CONCLUSION

Shallow coastal waters call for intensive study, because they constitute the location of a large number of interactions between ocean processes and human activities. The coastal process considered here leads to the generation of a residual circulation due to Ekman pumping along a vertical, rectilinear coastline. A coastal residual current is observed, with the coastline on its right-hand side (Northern hemisphere). The net flux of coastal water occurring in nature due to this process is estimated from our model to average 10000 m³/s. The intensity of this residual current accounts for some 10% of the mean tidal currents (20% very close to the coast). It generates movements of water masses and materials (sediments) and thus contributes to the dynamic local balance between offshore and coastal waters in the region (see point FX 6 on Fig. 1) where the variation in concentration of metals, sediments, etc, is important. The formation of a density front is also observed, whose evolution corresponds well with observations at sea. Other phenomena are observed, notably the instabilities of the flow in the Ekman layer (possible occurrence of Faller waves, see Faller, 1963). These waves appear to break down when the currents are reversing, perhaps generating powerful turbulence from bottom to surface at a particular moment during the tidal period. This matter is to be studied in the near future.

Acknowledgements

This research was carried out within the framework of the MAST-Fluxmanche research project supported by the EEC. Particular thanks are due to Messrs Carcel and Roche (CNRS) for their technical cooperation. Special thanks to Mr. Didelle (CNRS) who helped with the drafting of this paper.
APPENDIX A

Analytical solution in the case of an infinite horizontal plate (without any coastal boundary)

We consider a homogeneous, incompressible fluid limited below by an infinite horizontal plate, without any coastal boundary. The fluid is rotating about the z-axis with origin on the plate (Coriolis parameter $f$). The turbulence is represented by the assumption of a constant eddy viscosity $\nu$.

Near the surface, the fluid is assumed to have an oscillatory motion along the x-axis at velocity $u = U_0 \cos(\sigma t)$. The equations of motion in this system may be written:

1. $u_t - fv = v u_{zz} - g v_x$  \hspace{1cm} (A.1)
2. $v_t - fu = v v_{zz} - g v_y$  \hspace{1cm} (A.2)
3. $0 = -(1/p) p_z - g$  \hspace{1cm} (A.3)

(in A.1 to A.3, indexes represent partial derivative operator: for ex. $u_t = \partial u/\partial t$).

The continuity equation being automatically verified because of the independence in x and y (no coastal boundary), and negligible vertical velocity component. Following Maas and Van Haren (1987), we look for solutions of (A.1), (A.2) and (A.3) with the form of $u(z, t) = \text{Re} \{\exp(i\sigma t) f(z)\}$ and similarly for $v$, assuming two cases of boundary conditions.

REFERENCES


