Assimilation of Geosat data into a quasigeostrophic model of the North Atlantic between 20° and 50° N: preliminary results

Satellite altimetry
Geosat
Data assimilation
Quasigeostrophy
Numerical model

Altimétrie satellitaire
Geosat
Assimilation de données
Quasigéostrophie
Modèle numérique

Jacques VERRON, Jean-Marc MOLINES and Eric BLAYO

Laboratoire des Écoulements Géophysiques et Industriels, Institut de Mécanique de Grenoble, BP 53X, 38041 Grenoble Cedex, France.

ABSTRACT

Geosat altimeter residuals of the sea surface height (RSSH) are combined with the mean sea surface computed from the non-linear inverse model by Mercier *et al.* (1991) to provide a synthetic data set of the along-track sea-surface height over the North Atlantic during the period from 5 November 1986 to 2 December 1987. This data set is assimilated into a predictive numerical model of the North Atlantic circulations between 20° and 50°N. The model is quasigeostrophic, four-layered and has realistic coastline and topography. The ocean circulation is forced by a constant wind stress curl from Hellerman and Rosenstein's data (1983). Nudging is the assimilation technique used to insert sea-surface data information into the model and is carried out along the Geosat ground tracks sequentially as data appears.

The feasibility of the approach is initially demonstrated by comparing the model prediction along certain ground tracks with independent Geosat measurements obtained along these tracks and which have not been subjected to the assimilation process. Succinct comparisons are then performed to examine the validity of several model-predicted synoptic features of the circulation against other global observations over the North Atlantic. The usefulness of assimilating Geosat data has been clearly demonstrated since the model predictions, at least at the surface, seem more realistic than simulations realized without assimilation with this model or with other models, including much more sophisticated ones.

Oceanologica Acta, 1992. **15**, 5, 575-583.

RÉSUMÉ

Assimilation de données Geosat dans un modèle quasigéostrophique de l'Atlantique Nord entre 20° et 50° N : résultats préliminaires

Les mesures des résidus de la topographie dynamique obtenues par le satellite Geosat sont combinées avec la surface moyenne issue du modèle inverse non-linéaire de Mercier *et al.* (1991) pour fournir un ensemble de données composites de topographie dynamique sur l'Atlantique Nord durant la période du 5 novembre 1986 au 2 décembre 1987. Ces données sont assimilées dans un modèle numérique pronostique des circulations océaniques dans l'Atlantique Nord entre 20° et 50° N. Le modèle est basé sur la formulation quasigéostrophique et est configuré en quatre couches verticales avec des côtes et une topographie de fond réalistes. Les circulations sont forcées par une tension de vent constante, les données de vent utilisées étant celles de Hellerman et Rosenstein (1983). Pour assimiler les données de topographie dynamique, on utilise la technique de relaxation newtonienne qui est appliquée séquentiellement le long des traces au sol Geosat.

La faisabilité de l'approche est validée dans un premier temps en comparant les prédictions du modèle le long de certaines traces au sol avec des observations Geosat indépendantes obtenues sur ces traces et non utilisées pour l'assimilation. Des comparaisons préliminaires sont ensuite réalisées pour examiner certains caractères synoptiques des circulations prédites par le modèle par référence à d'autres observations globales dans l'Atlantique Nord. L'apport de l'assimilation des données Geosat est largement démontré puisque les prédictions du modèle, au moins en surface, sont sensiblement plus réalistes que les simulations obtenues sans assimilation avec ce modèle ou avec d'autres modèles, y compris des modèles beaucoup plus sophistiqués.

Oceanologica Acta, 1992. **15**, 5, 575-583.

INTRODUCTION

Recent research using Geosat products clearly demonstrates the feasibility of accessing the mid-latitude oceanic circulation from altimetric measurements, at least where the most energetic oceanic mesoscales of the surface circulation are concerned. However, besides posing precision and accuracy challenges, satellite altimeter measurements are limited by their surface nature and because the space and time coverage of satellite ground tracks is sequential. For these reasons, observations must be used in conjunction with ocean models in order to draw real benefit from their information, and in particular to extrapolate undersampled scales and to infer information for the deep ocean. The development of assimilation techniques to introduce such observational data into ocean circulation models is thus of crucial importance.

Ocean modellers have recently shown increasing interest in the development of such data assimilation techniques. Broad perspectives have opened up with the applications of techniques such as statistical estimation (Kalman filter and smoother) and variational methods [*see*, for example, the review by Ghil and Malanotte-Rizzoli (1991)]. For computational reasons, they are not yet tractable in their complete optimal derivation for realistic applications, especially at mid-latitudes. Marked linearity of low latitude dynamics makes progress and applications more favourable in such regions. At the same time, studies based on the more simple nudging technique designed specifically to examine the prospects of using altimeter data in ocean models have given encouraging results and suggest that such endeavours are well worth pursuing (Verron and Holland, 1989 and Holland and Malanotte-Rizzoli, 1989). This method acts as a Newtonian relaxation which pulls the model's predictive variables towards the observations, thereby nudging the assimilating model response towards the data. Nudging was first employed for similar purposes in a meteorological context (Anthes, 1974) and is still used successfully by meteorologists on the global scale (Krishnamurti *et al.*, 1991) and in a wide variety of research applications on mesoscale models (Ramamurthy and Carr, 1987; Wang and Warner, 1988; Stauffer and Seaman, 1990). Due to its simplicity, the nudging technique can be carried out at the scale of ocean basins with low impact on the cost of model computations.

In this note, we explore the feasibility of using the Newtonian relaxation method in a QG model over the full extent of the North Atlantic between 20° and 50°N. The Geosat satellite altimeter provides measurements of the sea-surface height (SSH) along the ground tracks, but the only residuals (RSSH) are sufficiently reliable for assimilation purposes. Approximately one year of Geosat data is used. This residual data is involved as it comes sequentially without further approximation or statistical information in areas devoid of data. To constitute the data to be actually assimilated, local (along-track) composite "total" sea-surface height data are constructed by adding to the previous RSSH the mean sea surface as computed from the inverse model by Mercier *et al.*(1991) based on multiple observational data sets including subsurface float information.

The goal of our research is to run prognostic simulations of the model in assimilative mode in order to reconstruct as many of the real features of the North Atlantic circulation as are compatible with the (restricted) model physics and the (inaccurate) satellite altimetric measurements. At the present stage, the objective is limited to assessing the feasibility of using the nudging technique in the local (in space) and sequential (in time) mode of data insertion for the purpose indicated above. The second section presents the model configuration over the North Atlantic being used. The third section discusses the Geosat data set used for assimilation. Implementation of the assimilation technique is presented in the fourth section. Finally, the fifth section presents and discusses the results of the numerical investigations.

THE MODEL

The model is an adaptation and an extension of the two-layer quasigeostrophic (QG) model presented by Holland (1978). Use is made of the capacitance matrix method to introduce realistic configuration for the coastlines (Blayo and Le Provost, 1992). Open boundary conditions are also introduced at the northern and southern model boundaries

(Verron, 1986). These are based on the Orlanski (1976) radiation boundary condition mixed with a relaxation towards the local climatology which is also deduced from the Mercier *et al.* (1991) data base. This type of QG models and in particular, their ability to adequately to simulate the mid-latitude eddy dynamics, has been extensively discussed in the literature (for example, Schmitz and Holland, 1982; 1986).

The familiar equations for the QG multilayer model can be written in a concise form as

$$\partial/\partial t\,(\Delta\psi_k) = F_k - J\,(\psi_k, \Delta\psi_k + f_k) - f_0/H_k\,(w_{k,k+1} - w_{k-1,k}) \qquad k \in [1, n]$$

$$w_{k,k+1} = f_0/g'_{k,k+1}\,\partial/\partial t\,(\psi_{k+1} - \psi_k) + J\,(\psi_{k,k+1}, \psi_{k+1} - \psi_k)] \qquad k \in [1, n-1]$$

with

$$\psi_{k,k+1} = \frac{H_k\psi_{k+1} + H_{k+1}\psi_k}{H_k + H_{k+1}} \qquad k \in [1, n-1]$$

The subscript k stands for the vertical layer going from the surface (k = 1) to the bottom (k = n). The variables are the streamfunction ψ_k and the relative vorticity $\Delta\psi_k$ in each layer. H_k is the layer thickness. The subscript k, k + 1 represents intermediate variables involving variables of both k and k + 1 layers. The Coriolis parameter is given by $f_k = f_0 + \beta y$ except in the bottom layer where the presence of bottom topography *h* introduces a new term $f_n = f_0 + \beta_y + f_0\, h/H_n$. The term F_k which includes forcing and dissipation is written as

$$F_k = - A_4\, \nabla^6\, \psi_k \qquad k \in [2, n-1]$$

except at the surface, which is directly affected by the wind stress

$$F_1 = - A_4\, \nabla^6\, \psi_1\, 1/H_1\, \mathrm{curl}\, \tau$$

and at the bottom, due to the bottom friction effect

$$F_n = - A_4\, \nabla^6\, \psi_n - \varepsilon\Delta\psi_n$$

As indicated above, biharmonic lateral friction has been selected throughout.

The system is written in the standard latitude/longitude spherical coordinate system but β-plane approximation for the Coriolis effect is still assumed. Resolution is 1/6° in both horizontal directions. The stratification profile is schematized by a four-layer representation: layer depths are 300, 350, 400 and 4 450 m from surface to bottom. The baroclinic Rossby radii are respectively 30.3, 12.2 and 8.9 km and computed from an average over the central North Atlantic of the data provided by Mercier *et al.* (1991). This representation with four layers is based on the conclusions of the work by Barnier *et al.* (1991) which emphasizes the role of vertical modes on the interactions between the mean circulation and the eddies, especially for representing the dynamical features of Gulf Stream-like inertial jets; they demonstrate that at least four modes are required in this regard. Refinement of the vertical discretization is planned in further simulations.

Realistic topography is built into the model using the Synbaps II bathymetric data base. A standard four-point Hannig filter has been applied once to the original 1/12° x 1/12° file to fit on the 1/6° model grid. In regions where the topographic amplitude is too large to satisfy the QG approximation, the flow is overcontrolled by the topographic gradients and streamlines will mainly follow the f/H contours. In this exploratory series of simulations, forcing by the wind stress curl is assumed to be constant and chosen as the yearly average of the Hellerman and Rosenstein (1983) wind stress data.

The satellite altimeter measurement is the sea-surface height *h* (or dynamic topography). In a QG layered model, *h* appears to be proportional to an explicit variable, the surface streamfunction ψ_1, and is written as

$$h = (f_0/g)\, \psi_1.$$

SEA-SURFACE HEIGHT DATA

Due to our poor knowledge of the geoid, actual satellite altimeter can provide only with a reliable measurement of the topographic height anomaly (*i. e.* RSSH) and not with the complete signal including the mean sea-surface. In the most turbulent regions of the world ocean, where mesoscale eddies actively contribute to driving the mean circulation, we may expect the assimilation of the residuals alone to be effective enough to control some part of the modelled large-scale ocean circulation (Kelly and Gille, 1990; Verron, 1992). However, this is unlikely to be the case over the bulk of the ocean and assimilation would require some knowledge of the mean sea-surface height (MSSH). (It should be noted that all these issues are still awaiting intensive investigation). One possible way would be to trust the model prediction for the MSSH. In practice, due to the deficiencies of model physics and forcings and to the inaccuracy of numerical implementation and parameterization, this prediction for the mean circulation is far from being satisfactory even with the most sophisticated numerical ocean models now available. A classical example is the incapacity of models faithfully to reproduce the Gulf Stream mean path in the western North Atlantic. One must then rely more or less directly on *in situ* observations from atlases [as demonstrated for example by Willebrand *et al.* (1990) and Schroeter *et al.* (1992) using Robinson *et al.* (1979) atlas] or on the use of inverse models (which is the case here).

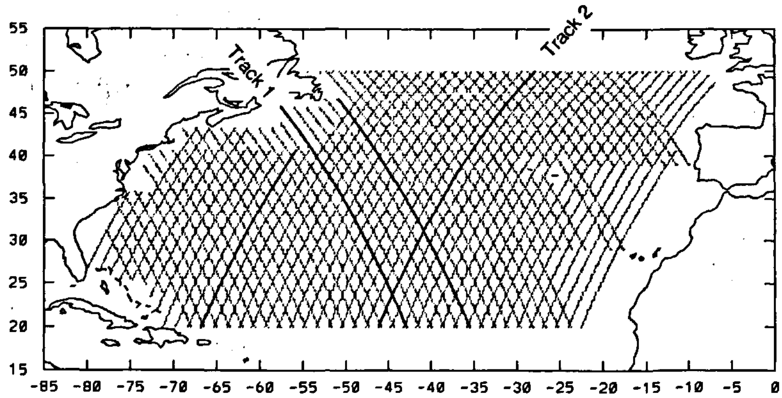
Geosat residuals

We used about one year of Geosat geophysical data records obtained from NOAA, spanning the period from 5 November 1986 to 2 December 1987, *i. e.* 23 cycles of 17.05 days. In these geophysical data records, the sea-surface height was corrected for the following effects: electromagnetic bias by adding 2 % of significant wave height to the sea-surface height; ocean tides using the Schwiderski model; terrestrial tides using the Melchior model; ionospheric effects using the GPS climatic model; and tropospheric corrections using FNOC data. No inverted barometer correction was applied. To eliminate remaining spurious measurements still present in the data, an iterative process is used as described by Le Traon *et al.* (1990).

Figure 1

Geosat coverage over the computational domain. Tracks indicated by bold line are those from which data has been kept for validation purposes and were not used for assimilation.

Couverture des traces au sol Geosat sur le domaine de calcul. Les traces indiquées par un trait épaissi sont celles dont les données ont été mises de côté pour les besoins de la validation, et n'ont pas été assimilées.



The computation of the residuals itself is performed using the method of polynomial adjustment implemented at GRGS/UMR39 in Toulouse. Final smoothing is performed to remove wavelengths smaller than 60 km (Le Traon *et al.*, 1990).

The tracking coverage over the region of interest is presented in Figure 1.

Mean sea surface

Mercier *et al.* (1991) developed a non-linear inverse model which is based on the use of hydrographic data, mean float velocities from surface and subsurface Lagrangian floats as well as geostrophy, Ekman pumping, mass, heat and salt conservations. The inverse model provides with a picture of the North Atlantic mean general circulation between 20° and 50°N; its features appear realistic in many respects, especially regarding the transport characteristics of the Gulf Stream. A by-product of particular interest is a 2° x 2° resolution map of the MSSH together with the associated error map [Fig. 2 (Le Traon and Mercier, 1992)].

Data to be assimilated is thus along-track data resulting from the addition of the Geosat RSSH and of the local value of the previously mentioned MSSH interpolated on the tracks from the 2° x 2° grid. To optimize data storage, this synthetic data set is built up on a daily basis, *i. e.* every occurrence of the RSSH measurements is gathered daily. Shorter time scales are considered to be irrelevant for assimilation purposes. Previous studies with simulated altimeter data have validated this approximation (Verron, 1990; 1992). However, it is planned to study the sensitivity to this time-filtering more intensively in further simulations using Geosat data.

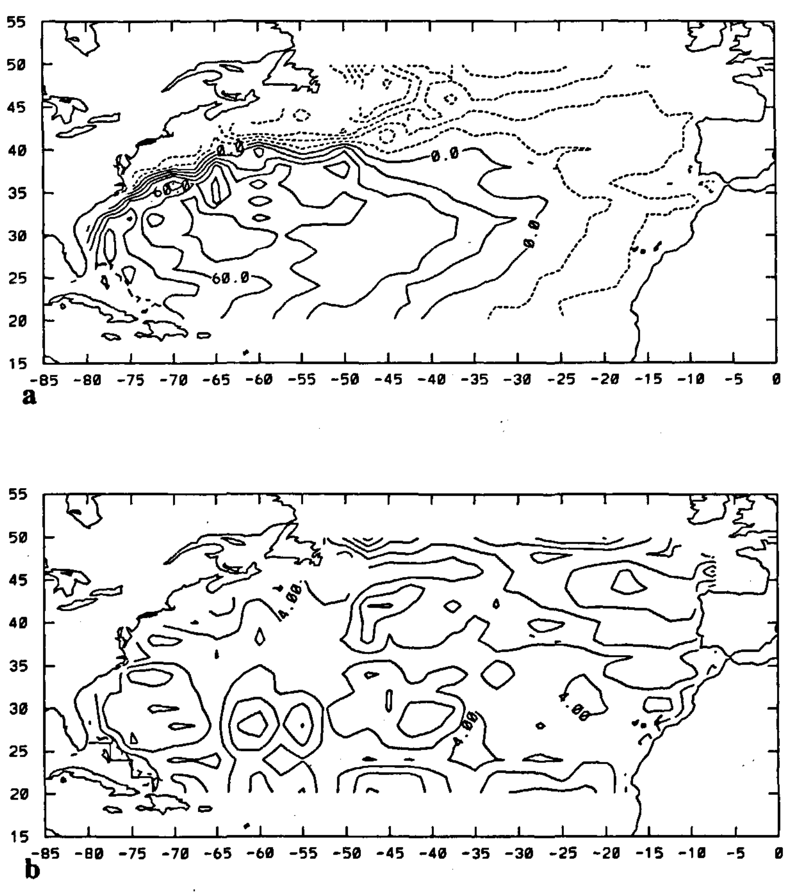


Figure 2

Mean sea surface (a) and Rms error map (b) as deduced from the non-linear inverse model by Le Traon and Mercier (1992). Contour interval is respectively 15 cm (a) and 1 cm (b).

Surface moyenne océanique (a) et carte d'erreur Rms (b) déduites du modèle inverse de Le Traon and Mercier (1992). Les intervalles entre les contours sont respectivement de 15 cm (a) et 1 cm (b).

THE ASSIMILATION TECHNIQUE

Interest focuses here on the nudging technique because it is considered to be a simple means (and one of the few feasible in the near future) of assimilating altimeter data into models of non-linear ocean dynamics. Moreover, nudging has been successfully used for testing the feasibility of altimeter data assimilation and answering basic questions on the effects of sampling in time and space (Verron and Holland, 1989; Holland and Malanotte-Rizzoli, 1989; Verron, 1990; 1992]. The nudging method is a flexible assimilation technique which is computationally much more economical than statistical estimation and variational methods. However, the results from nudging experiments might be quite sensitive to the specification, based on empirical grounds, of the nudging factor (although some parameter studies by Verron (1992) do not clearly reveal so much sensitivity). Obviously, the derivation of some optimality for the nudging coefficients would require the definition of some optimality criterion. The fully optimal derivation in the framework of linear statistical estimation will lead back to the complete derivation of the Kalman filter with the associated computational cost (the nudging procedure can indeed be seen as a degraded form of the Kalman filter.) But other "optimal nudging" could also be thought of in the context of variational approach (Zou *et al.*, 1991). The basic nudging technique, as used here, is unable to provide alone for an estimate of the error characteristics of the predicted fields. Therefore, it is necessary to rely on *a posteriori* validations based on independent datasets for assessing such errors.

In the QG layered-model formulation, predictive equations are written for the vorticity and the streamfunction in each layer as indicated above. Nudging involves modifying the upper layer equation by adding a blending term to the right-hand side:

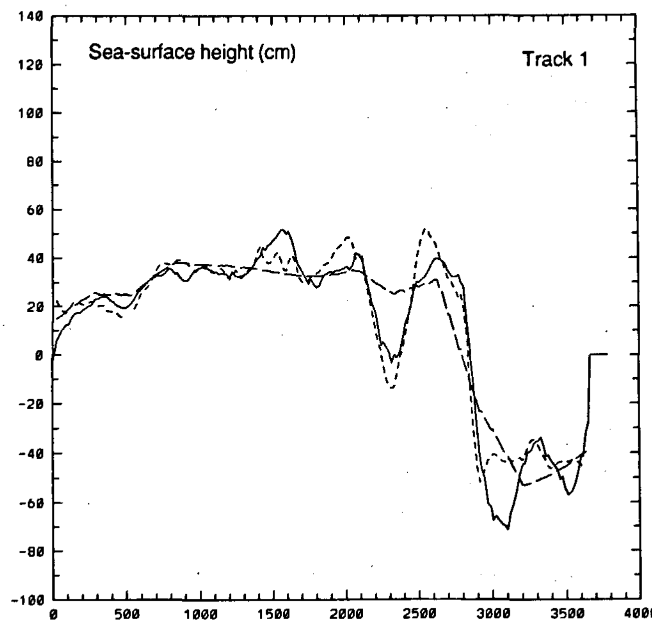
$$\partial/\partial t\,(\Delta\psi_1) = F_1 - J\,(\psi_1, \Delta\psi_1 + f_1) - (f_0/H_1)w_{1,2} - R\,(\psi_1 - \psi_{OBS})$$

The blending term nudges the prediction for the upper layer streamfunction ψ_1 towards the observed streamfunction ψ_{OBS} which is directly proportional to the sea-surface height (Verron, 1992). Equations for other layers are unchanged.

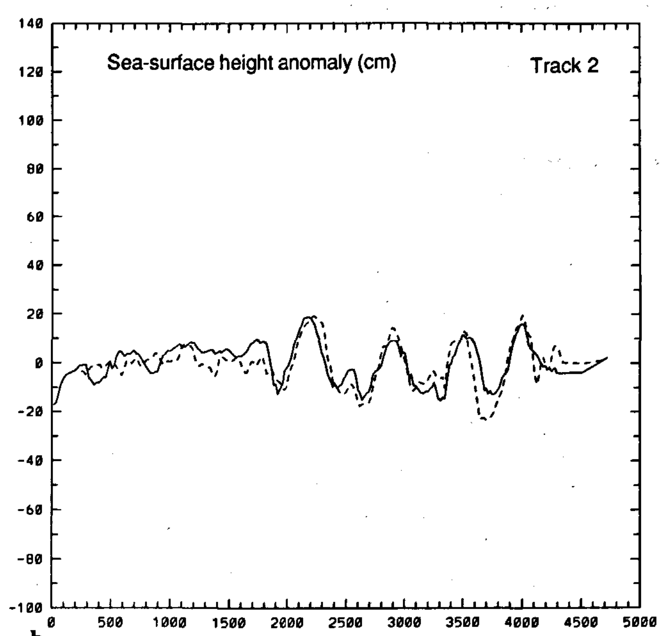
In the nudging equation (involving the only upper-layer equation), the surface information given by satellite data is transmitted to all the modelled flow, including deeper layers, since it is projected along normal modes in the process solving the system of QG equations (Verron and Holland, 1989; Holland and Malanotte-Rizzoli, 1989). This projection along the vertical direction is (numerically) instantaneous except for the fact that the nudging term must be lagged one time step for numerical stability reasons.

VALIDATION OF THE ASSIMILATION EXPERIMENTS

The model was first spun up from the rest situation in a "free mode", *i. e.* driven only by the wind stress curl applied to the surface without using external information for the sea-surface height. Once a statistical steady state has been reached, which requires about fifteen years of integration, several assimilation experiments were carried out. The exa-



a

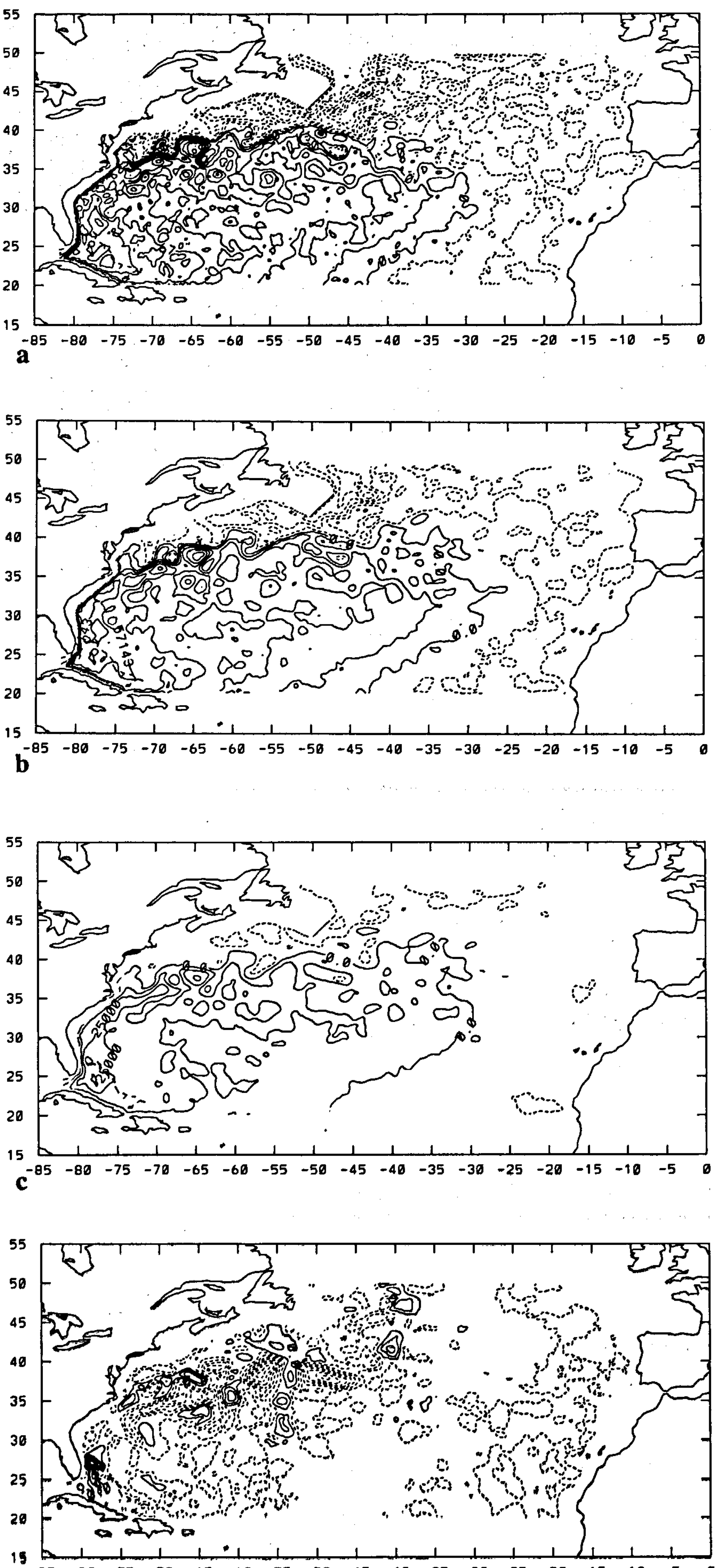


b

Figure 3

Along-track sea-surface height at the end of a one-year assimilation for tracks indicated on Figure 1: total sea-surface height along track 1 (a); sea-surface anomaly along track 2 (b). Along-track distance is measured in km starting from the 20°N latitude. Geosat independent measurements (light dashed line), model prediction (solid line), reference mean sea-surface (dark dashed line).

Topographie de la surface océanique le long des traces de référence indiquées sur la figure 1, prédite par le modèle après une année d'assimilation : topographie dynamique le long de la trace 1 (a) ; anomalie de la topograhie dynamique le long de la trace 2 (b). Les distances le long des traces au sol sont mesurées en kilomètres à partir de la latitude de 20°N. Les mesures Geosat de référence sont indiquées par une ligne pointillée simple, les prédictions du modèle après assimilation par une ligne continue, et la surface moyenne de référence par un trait pointillé épaissi.



mination of the model predictions in the free mode as well as the detailed analysis of the various assimilation experiments are still under way and will not be presented here. Interest is focused on presenting certain results of one typical assimilation experiment which suggests (as do the others) some degree of efficiency for the nudging approach and tends to demonstrate that the use of dynamical height data can substantially improve the realism of the model prediction. In the only assimilation experiment discussed here, the nudging factor R is constant and chosen as 4×10^{-14} m^{-2} s^{-1}. This value typically corresponds to a relaxation time-scale of five days for 100 km size eddies.

A first validation approach was to assess the deterministic model prediction against independent Geosat measurements. Altimeter data along some tracks (actually four tracks as shown in Fig. 1) were kept apart during the whole assimilation cycle and the assimilation experiment was carried out using only the other Geosat data available. Then the model prediction along the "spare" tracks can be compared with real observations of the sea-surface height at the same time and the same place. Two examples are shown on Figures 3 *a*, *b* for an ascending track which crosses over the central North Atlantic and the mid-Atlantic ridge and a descending track which crosses over the western part of the basin an the Gulf Stream system. The agreement between the two, especially visible at the mesoscales, is within the error bar of the Geosat data. Results agree more favourably for track 2 (Fig. 3 *b*), particularly for the waves with spatial scales of about 400 to 500 km. Along track 1, the assimilation reproduces the profile sharpness of the Gulf Stream frontal zone quite realistically. This profile is much oversmoothed in the Mercier *et al.* MSSH data because of their spatial averaging and/or insufficient data coverage. Comparing

Figure 4

Examples of the instantaneous streamfunction fields in the four layers of the model after a one-year assimilation. Contour intervals are respectively 5 Sv for layer 1 (a); 5 Sv for layer 2 (b); 5 Sv for layer 3 (c); and 15 Sv for layer 4 (d).

Exemples de champs de fonction de courant instantanés pour les quatre couches du modèle après une année d'expérience d'assimilation. Les intervalles entre les contours sont respectivement de 5 Sv pour la couche 1 (a), 5 Sv pour la couche 2 (b), 5 Sv pour la couche 3 (c) et 15 Sv pour la couche 4 (d).

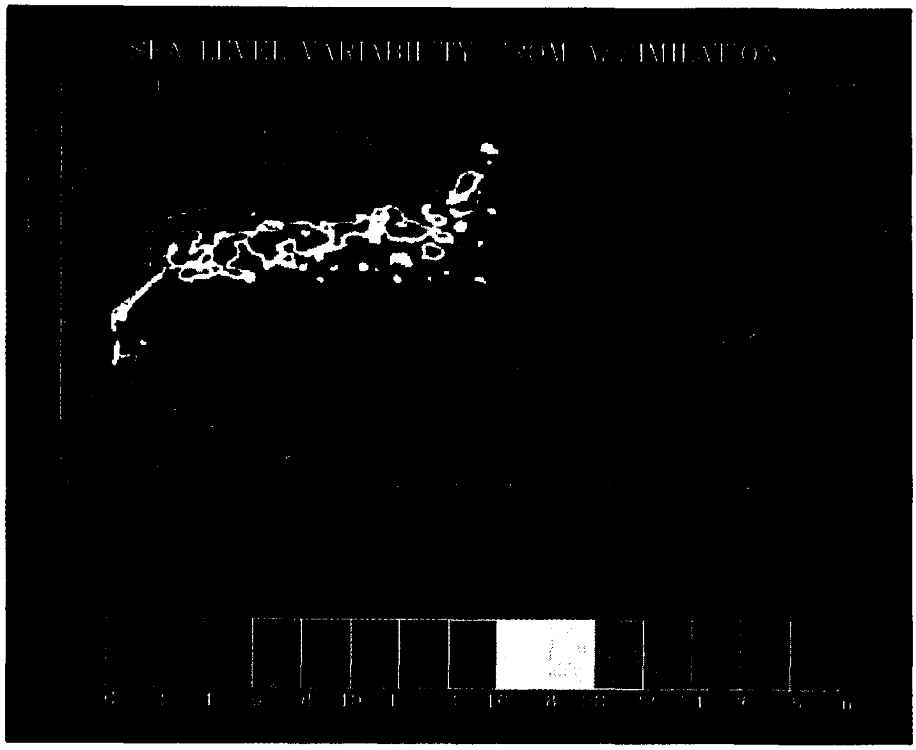


Figure 5

Maps of the Rms of sea-surface height residuals as predicted by the model after a one-year assimilation and computed over all the model grid-points. Colour coding has been chosen similar to Le Traon et al. *(1990) in order to allow direct comparisons.*

Carte de la Rms des résidus de la topographie dynamique telle que prédite par le modèle après une année d'expérience d'assimilation et calculée pour tous les points de grille du modèle. Le codage des couleurs est le même que Le Traon *et al.* (1990), afin de permettre des comparaisons directes.

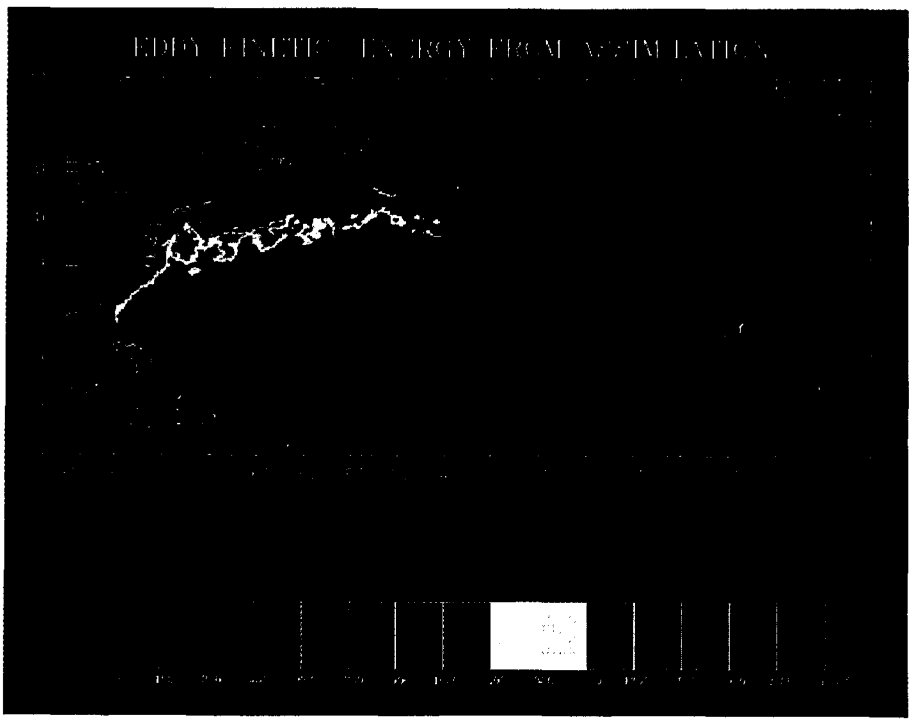


Figure 6

Maps of the surface kinetic energy fields provided by the model after one year's assimilation. Colour coding has been chosen similar to Le Traon et al. *(1990) in order to allow direct comparisons.*

Carte du champ d'énergie cinétique tourbillonnaire superficiel ou de surface tel que prédit par le modèle après une année d'expérience d'assimilation. Le codage des couleurs est le même que Le Traon *et al.* (1990), afin de permettre des comparaisons directes.

the along-track spatial spectrum (not shown) of the RSSH at various instants in the model and in the original data, shows, as expected from a QG model, that the highest wave numbers of Geosat data are damped during the assimilation. Most mesoscales are seen however, in both Geosat data along-track spectra and model along-track spectra with similar intensities which demonstrates the effectiveness of nudging at such scales. Depending on the specific scale, noticeable attenuation or amplification is still present, the interpretation of which will require further investigation.

Figure 4 presents the instantaneous streamfunction fields at the end of the assimilation experiment for the four layers of the model. Some large scale features of the main currents seem to be reproduced correctly, but others have dubious features especially in the bottom flow. Mean circulation drawings (not shown) indicate that the maximum Gulf Stream transport is about 130 Sv and that the two branches of the North Atlantic current and the Azores current have about the right order of magnitude. In the free mode, the model circulation does not exhibit some of the realistic features that the assimilative mode is partly able to reproduce: the Gulf Stream penetration is far from being sufficient due to the inadequate departure from Cape Hatteras as it continues to flow along the continental shelf.

The Rms map of the sea-surface height anomaly (RSSH) is shown in Figure 5 and the map of the surface eddy kinetic energy (EKE) field is presented in Figure 6. The maximum values are present in the western part of the Gulf Stream region but also along the Gulf Stream path. In the core of the open ocean jet stream, surface EKE levels have maximum peak values of about 5 000 cm^2/s^2 but at the mesoscales, the level is about 2 000 cm^2/s^2 which is comparable to EKE levels deduced from Seasat (Ménard, 1983), Geosat (Willebrand *et al.*, 1990; Le Traon *et al.*, 1990; Stammer and Böning, 1992) or drifters (Richardson, 1983). Both fields of Figures 5 and 6 reveal finer scale structure than similar maps obtained from the Lagrangian floats or the satellite evaluations referred to above. Time averaging is also performed over a year or so but spatial smoothing in the above studies filters out the fine scales of the dynamics. Note that in the western boundary current close to the coast, Geosat data (this is partly true for Lagrangian floats also) are unable to provide reliable information. Therefore, measured variability is greatly underestimated or unknown in these regions. Early estimations of EKE in the North Atlantic by Wyrtki *et al.* (1976) probably underestimate eddy activity within the core of the Gulf Stream system at longitudes ranging from 50° to 70° W, but they point to significant levels of EKE within the western boundary current, especially upstream from Cape Hatteras, which are likely to be more realistic and more consistent with the present model results.

CONCLUSIONS

Although preliminary in several respects, particularly with regard to the model configuration and forcing, these first simulations of altimeter data assimilation at the scale of the North Atlantic lead to the following important conclusions:

Firstly, they demonstrate the feasibility of the nudging technique, applied sequentially in the local along-track insertion mode, in terms of assimilating real satellite altimeter data into ocean models.

Secondly, they confirm that the use of a composite data set is feasible by constructing along-track altimeter data consisting of the sum of the remotely-sensed sea-surface height anomaly and mean sea-surface height estimated from inverse models using an adequate *in situ* data base.

Further improvements of the assimilation performances are expected. Obviously there is room for improving the model configuration, while further evolution of the assimilation technique may also be expected from optimizing the nudging factor, notably by exploiting measurement error information. The usefulness of assimilating Geosat data has been clearly demonstrated since the model predictions, at least at the surface, seem more realistic than simulations realised without assimilation with this model or with other models, including much more sophisticated ones.

Acknowledgements

Geosat data were processed by GRGS/UMR39 and CLS-Argos. We are indebted to H. Mercier who provided us with the results of his inverse model. The calculations were made possible by the computer facilities of the Centre Grenoblois de Calcul Vectoriel (CGCV-CEA). This research was supported by the Direction des Recherches et Études Techniques under contract 91/140.

REFERENCES

Anthes R. (1974). Data assimilation and initialization of hurricane prediction models. *J. atmos. Sci.*, **31**, 702-718,

Barnier B, L. Hua and C. Le Provost (1991). On the catalytic role of high baroclinic modes in eddy-driven large scale circulations. *J. phys. Oceanogr.*, **21**, 7, 976-997,

Blayo E. and C. Le Provost (1992). Performance of the Capacitance matrix method for solving Helmholtz type equations in ocean modelling. to appear in *J. comp. Phys.*

Ghil M. and P. Malanotte-Rizzoli (1991) Data assimilation in meteorology and oceanography. *Adv. Geophys.*, **33**, 141-266.

Hellerman S. and M. Rosenstein (1983). Normal monthly wind stress over the world ocean with error estimates. *J. phys. Oceanogr.*, **13**, 1093-1104.

Holland W. R. (1978). The role of mesoscale eddies in the general circulation of the ocean. Numerical experiments using a wind-driven quasigeostrophic model. *J. phys. Oceanogr.*, **8**, 363-392.

Holland W.R. and P. Malanotte-Rizzoli (1989). Along-track assimilation of altimeter data into an ocean circulation model: space *versus* time resolution studies. *J. phys. Oceanogr.*, **19**, 1507-1534,

Kelly K.A. and S.T. Gille (1990). Gulf Stream surface transport and statistics at 69° W from the Geosat altimeter. *J. geophys. Res.*, **95**, C3, 3149-3161.

Krishnamurti T.N., X. Jishan, H.S. Bedi, K. Ingles and D. Oosterhof (1991). Physical initialization for numerical weather prediction over the tropics. *Tellus*, **43** A, 53-81.

Le Traon P.-Y. and H. Mercier (1992). Estimating the North Atlantic mean surface topography by inversion of hydrographic and Lagrangian data. *Oceanologica Acta*, **15**, 5, 563-566 (this issue).

Le Traon P.-Y., M.-C. Rouquet and C. Boissier (1990). Spatial scales of mesoscale variability in the North Atlantic as deduced from Geosat data. *J. geophys. Res.*, **95**, C11, 20267-20285.

Ménard Y. (1983). Observations of eddy fields in the Northwest Atlantic and Northwest Pacific by Seasat altimeter data. *J. geophys. Res.*, **88**, C3, 1853-1866.

Mercier H., M. Ollitrault and P.-Y. Le Traon (1991). An inverse model of the North Atlantic circulation using Lagrangian float data. submitted to *J. phys. Oceanogr.*

Orlanski I. (1976). A simple boundary condition for unbounded hyperbolic flows. *J. comp. Phys.*, **21**, 251-269.

Ramamurthy M.K. and F.H. Car (1987). Four-dimensional data assimilation in the monsoon region. Part I: Experiments with wind data. *Mon. Weath. Rev.*, **115**, 1678-1706.

Richardson P.L. (1983). Eddy kinetic energy in the North Atlantic from surface drifters. *J. geophys. Res.*, **88**, C7, 4355-4367.

Robinson M., R. Bauer and E. Schroeder (1979). Atlas of North Atlantic-Indian Ocean monthly mean temperatures and mean salinities of the surface layer, Department of the Navy, Washington, D.C., USA.

Schmitz W.J. and W.R. Holland (1982). A preliminary comparison of selected numerical eddy-resolving general circulation experiments with observations. *J. mar. Res.*, **40**, 75-117.

Schmitz W.J. and W.R. Holland (1986). Observed and modelled mesoscale variability near the Gulf Stream and KuroShio extension. *J. geophys. Res.*, **91**, C8, 9624-9638.

Schroeter J., U. Seiler and M. Wenzel (1992), Variational assimilation of Geosat data in an eddy-resolving model of the Gulf-Stream extension area. submitted to *J. phys. Oceanogr.*

Stammer D. and C.W. Böning (1992). Mesoscale variability in the Atlantic Ocean from Geosat altimetry and Woce high resolution numerical modelling. *J. phys. Oceanogr.*, **22**, 7, 732-752.

Stauffer D.R. and N.L. Seaman (1990). Use of four-dimensional data assimilation in a limited area mesoscale model. Part I: Experiments with synoptic-scale data. *Mon. Weath. Rev.*, **118**, 1250-1277.

Verron J. (1986). Utilisation de frontières ouvertes pour la simulation numérique d'un pavé d'océan. *Oceanologica Acta*, **9**, 4, 415-424.

Verron J. (1990). Altimeter data assimilation into an ocean model: sensitivity to orbital parameters. *J. geophys. Res.*, **95**, C7, 11443-11459.

Verron J. (1992). Nudging altimeter data into quasigeostrophic ocean models. *J. geophys. Res.*, **97**, C5, 7479-7491.

Verron J. and W.R. Holland (1989). Impacts de données d'altimétrie satellitaire sur les simulations numériques des circulations générales océaniques aux latitudes moyennes, *Annls Geophys.*, **7**, 1, 31-46.

Wang W. and T.T. Warner (1988). Use of four-dimensional data assimilation by Newtonian relaxation and latent heat forcing to improve a mesoscale model precipitation forecast: a case study. *Mon. Weath. Rev.*, **116**, 2593-2613.

Willebrand J., R.H. Kase, D. Stammer, H.H. Hinrinchsen and W. Krauss (1990). Verification of Geosat sea-surface topography in the Gulf Stream extension with surface drifting buoys and hydrographic measurements. *J. geophys. Res.*, **95**, C3, 3007-3014.

Wyrtki K., L. Magaard and J. Hager (1976). Eddy energy in the oceans. *J. geophys. Res.*, **81**, 2641-2646.

Zou X., I.M. Navon and F.X. Le Dimet (1991). An optimal nudging data assimilation scheme using parameter estimation, to appear in *Q. Jl R. met. Soc.*