OCEANOLOGICA ACTA 1980 - VOL. 3 - N° 3

Residual circulation, long waves and mesoscale eddies in the North Sea

Tide
Eddy
Residual circulation
Turbulence
North Sea
Marée
Tourbillon
Circulation résiduelle
Turbulence
Mer du Nord

J. C. J. Nihoul
University of Liège and University of Louvain, B 6 Sart Tilman, B-4000 Liège, Belgium.

Received 23/11/79, in revised form 25/2/80, accepted 6/3/80.

ABSTRACT

A three-dimensional model is applied to the study of long waves (tides and storm surges) and residual circulation in the North Sea.
It is shown that the non-linear interactions of tides and storm surges result in a residual stress which may foster the mean flow in some regions, and extract energy from it in other regions. This effect is associated with the existence of positive viscosity and negative viscosity eddy-like structures, which may play an important rôle in the transport and the distribution of marine properties.

Oceanol. Acta, 1980, **3**, 3, 309-316.

RÉSUMÉ

Circulation résiduelle, ondes longues et tourbillons à la méso-échelle en Mer du Nord

Un modèle tri-dimensionnel est appliqué à l'étude des ondes longues (marées, tempêtes) et de la circulation résiduelle en Mer du Nord.
On montre que les interactions non linéaires des marées et tempêtes résultent en une tension résiduelle qui peut fournir de l'énergie aux courants moyens dans certaines régions et extraire de l'énergie dans d'autres régions. Cet effet est associé à l'existence de structures tourbillonnaires à viscosité positive ou négative, susceptibles de jouer un rôle important dans le transport et la distribution des propriétés marines.

Oceanol. Acta, 1980, **3**, 3, 309-316.

INTRODUCTION

The North Sea is characterized by strong currents produced by tides and storm surges (of the order of $1\ \mathrm{m.s^{-1}}$), superimposed on small (one or two orders of magnitude smaller) slowly varying "residual" currents created by the in- and out-flows of two branches of the North Atlantic current and by non-linear long wave interactions.

As the residual circulation pattern evolves very slowly with time, it can be regarded as a steady flow or at most, as a succession of – say seasonal – steady states. This can be expressed in mathematical form by considering a time T sufficiently large to cover at least one or two tidal periods – to a large extent tidal oscillations and transitory wind currents thus cancel over T – but smaller than the characteristic time of residual changes (T can go from a few weeks to a few months). The residual circulations is then defined as the mean motion over T.

The residual models of the North Sea can be grouped in four categories (Nihoul, Ronday, 1976 *a*):

Semi-empirical models

Here the model is used only to compute the tidal current which is then subtracted from the *observed* current. The difference is taken as the residual current at the point of observation.

Regarding the residual flow pattern which is obtained in this way as a steady picture, streamlines are drawn by classical interpolation techniques, and the tracery of streamlines is taken as figuring the residual circulation (e.g. Otto, 1970).

The residual currents obtained by such semi-empirical models, are not entirely conform to the classical definition (i.e. mean currents over a chosen characteristic time T). In particular, they still contain components related to transitory wind effects, and their steady character is questionable.

Long waves residue models

These models undertake to solve the time-dependent hydrodynamic equations, and deduce the residual currents by averaging the solution over the chosen time T.

However, because the contribution of tides and storm surges are one or two orders of magnitude larger than the residual part, the solution obtained in the first place is essentially the long-wave contribution; the residual part being of the same order as the error which results from the imprecision of open-sea boundary conditions and the approximation of the numerical scheme. What such models provide is probably not entirely meaningless as non-linear interactions ensure that after averaging there is indeed a residue, but in non-linear problems, the errors do not cancel out in the averaging process and the residue can only represent the true residual motion with a 100 % error (Nihoul, Ronday, 1976 *a*, *b*).

Inflow-outflow models

These models seek a steady state solution of the hydrodynamic equations which satisfies appropriate inflow and outflow conditions at the open-sea boundaries. They are, in a sense, complementary to the models described in the precedent section as they incorporate what they ignore and ignore what they take solely into account, i.e. the non-linear residue of long wave motions.

These models are found to reproduce the broad trends of the residual circulation, but they fail to uncover local secondary flows which have been observed and may influence appreciably the residence time of nutrients, pollutants, etc. in certain areas.

"Tidal stress" models

Here, the residual flow is obtained by solving, with appropriate inflow-outflow boundary conditions, averaged *equations* where the stress exerted by the long wave motions is explicitly taken into account.

The importance of the residual stress produced by the non-linear interactions of long-waves (tides and storm surges) was discovered, first, by numerical models of tidal and residual currents in shelf sea areas (Nihoul, 1974; Ronday, 1974; Nihoul, Ronday, 1975), as well as in an inland sea (Tee, 1976).

The name "tidal stress" appeared at that time. One realizes however that the mean circulation over a given period of time is determined by the total stress due to long wave motions, storm surges as well as tides, during that period. More appropriate appelations like "long wave radiation stress" (Zimmerman, 1978 *a*), somehow never caught on and "tidal stress", will be used in the following with the proviso that one is not necessarily concerned with tides only.

Tidal stress models have been shown to be the only ones to reproduce in details the observations and, in the case of the North Sea, in particular (e.g. Nihoul, Ronday, 1976 *a*, *b*), they reveal residual gyres which the classical models had failed to uncover.

These gyres and other residual vortices of similar origin have been reproduced by means of hydraulic models in the laboratory (Sugimoto, 1975; Yanagi, 1976), and their existence has been traced in the field by analysis of current meter measurements (Zimmerman, 1976; Riepma, 1977; Tee, 1977) and chemical characteristics of water masses (Becker *et al.*, 1976).

Zimmerman (1978 *a*) showed how an irregular bottom topography can supply the necessary inhomogeneity of the tidal stress to produce a non-linear transfer of vorticity, and he calculated the effect of residual vortices on dispersion in a tidal channel (Zimmerman, 1978 *b*).

Although the term "eddy" has been used occasionally to refer to residual secondary flows, eddies are better defined on the basis of a residual energy balance equation and of the mean work done by the tidal stress. In the North Sea, it has been shown that this work may result in a transfer of energy, either from the mean flow to the long wave motion (positive "eddy viscosity") or, reversely, from the wave motion to the mean flow (negative "eddy viscosity") (Nihoul, Ronday, 1976 *b*).

All the studies mentioned above were based on depth-averaged models. The associated energy balance equations were thus inevitably approximate, and the analysis can be refined by the use of a three-dimensional model.

A three-dimensional model of tides and storm surges was developed by Nihoul (1977), and applied to the North Sea by Nihoul *et al.* (1979).

In the following, the model is extended to include the residual circulation, and to determine the energy transfers between the residual flow and the long waves and from the long waves to the turbulence.

THREE-DIMENSIONAL MODEL OF LONG WAVES AND RESIDUAL CIRCULATION

The three-dimensional hydrodynamic equations applicable to a well-mixed continental sea, like the North Sea can be written (e.g. Nihoul, 1975):

$$\nabla . \mathbf{v} = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla . (\mathbf{v}\mathbf{v}) + 2\,\boldsymbol{\Omega} \wedge \mathbf{v} = -\nabla q + \nabla . \mathfrak{R}, \tag{2}$$

where $\mathbf{\Omega}$ is the Earth's rotation vector, $q=(p/\rho)+gx_3$, p is the pressure, ρ the specific mass of sea water, x_3 the vertical coordinate and $\mathfrak{R}$ the Reynolds stress tensor (the stress is here per unit mass of sea water) resulting from the non-linear interactions of three-dimensional turbulent fluctuations.

The velocity $\mathbf{v}$ includes the residual flow $\mathbf{v}_0$ and the time-dependent mesoscale flow due to tides and storm surges $\mathbf{v}_1$, i.e.

$$\mathbf{v}=\mathbf{v}_0+\mathbf{v}_1. \tag{3}$$

The Reynolds stress can be parameterized in terms of eddy viscosity coefficients. In microscale three-dimensional turbulence, these coefficients are of the same order of magnitude in the horizontal and vertical directions. Then, horizontal length scales being much larger than the depth, the last term in the right hand side of equation (2) can be written simply, with a very good approximation

$$\nabla.\mathfrak{R}=\frac{\partial \tau}{\partial x_3}=\frac{\partial}{\partial x_3}\left(\tilde{\nu}\frac{\partial \mathbf{v}}{\partial x_3}\right), \tag{4}$$

where $\tilde{\nu}$ is the vertical eddy viscosity and τ the Reynolds stress.

The scalar product of equation (2) by $\mathbf{v}$ gives the equation for the kinetic energy

$$\frac{\partial}{\partial t}\left(\frac{1}{2}v^2\right)+\nabla.\left(\mathbf{v}\left(\frac{1}{2}v^2+q\right)\right)=\frac{\partial}{\partial x_3}(\mathbf{v}.\tau)-\varepsilon, \tag{5}$$

where ε is the rate of turbulent energy dissipation

$$\varepsilon=\tau.\frac{\partial \mathbf{v}}{\partial x_3}=\tilde{\nu}\left\|\frac{\partial \mathbf{v}}{\partial x_3}\right\|^2. \tag{6}$$

The residual velocity $\mathbf{v}_0$ has been defined as the mean velocity over a time T sufficiently large to cancel long wave oscillations and transitory wind-induced currents. One might thus think of solving equations (1) and (2), with appropriate boundary conditions, and then average the solution to obtain $\mathbf{v}_0$.

This cannot be done for the North Sea, where $\mathbf{v}_1$ is at least one order of magnitude larger than $\mathbf{v}_0$, and where – unless one uses, to solve equations (1) and (2), a prohibitively sophisticated and expensive model – the error $\delta\mathbf{v}$ made on $\mathbf{v}$ is likely to be of the same order as $\mathbf{v}_0$. Solving equations (1) and (2), one gets, in practice,

$$\mathbf{v}=\mathbf{v}_0+\mathbf{v}_1+\delta\mathbf{v}, \tag{7}$$

and, by averaging the *solution*

$$(\mathbf{v})_0=\mathbf{v}_0+(\delta\mathbf{v})_0, \tag{8}$$

the index "0" denoting an average over T.

Although $\delta\mathbf{v}$ is small compared to $\mathbf{v}$, it would be wrong to assume that $(\delta\mathbf{v})_0$ (to be distinguished from $\delta\mathbf{v}_0$ used in the following to denote the error made on $\mathbf{v}_0$ by solving averaged *equations*), is also small compared to $\mathbf{v}_0$. One may expect on the contrary, because of the non-linearities, that $(\delta\mathbf{v})_0$ remains of the same order as $\delta\mathbf{v}$.

One notes indeed that if $\mathbf{v}_0$ is produced to a large extent by the rectification of $\mathbf{v}_1$, i.e. arises from quadratic terms of the form $\mathbf{v}_1\mathbf{v}_1$ (there is also of course a contribution from the boundary in- and out-flows), $(\delta\mathbf{v})_0$ is not, for the main part, produced by the rectification of $\delta\mathbf{v}_1$, it arises essentially from quadratic terms of the form $\mathbf{v}_1\,\delta\mathbf{v}_1$ much larger than $\delta\mathbf{v}_1\,\delta\mathbf{v}_1$ [this can be seen directly by writing equation (2) for $\mathbf{v}+\delta\mathbf{v}$, substracting the same equation for $\mathbf{v}$ and comparing the orders of magnitude of the different terms in the equations for $\delta\mathbf{v}$ and its mean value $(\delta\mathbf{v})_0$].

In the case of the North Sea, taking $\mathbf{v}_1\sim 0(1)$, this implies $(\delta\mathbf{v})_0\sim 0(\delta\mathbf{v}_1)\sim 0(\delta\mathbf{v})\sim 0(\mathbf{v}_0)$, assuming an error on $\delta\mathbf{v}$ of the order of, say, 10 %.

Thus, by averaging the *solution* of equations (1) and (2), one obtains the residual flow with typically a 100 % error.

The situation is different if one averages first the equations and then solve the averaged equations for $\mathbf{v}_0$.

Taking the mean of equations (1) and (2) over T, one gets

$$\nabla.\mathbf{v}_0=0, \tag{9}$$

$$\frac{\partial \mathbf{v}_0}{\partial t}+2\mathbf{\Omega}\wedge\mathbf{v}_0=-\nabla q_0+\frac{\partial \tau_0}{\partial x_3}+\nabla.(-\mathbf{v}\mathbf{v})_0. \tag{10}$$

In the last term in the right-hand side of equation (10), $\mathbf{v}$ can be practically replaced by $\mathbf{v}_1$. This term represents an additional forcing due to the non-linear interactions of the long wave motions

$$\mathcal{N}=(-\mathbf{v}\mathbf{v})_0\sim(-\mathbf{v}_1\mathbf{v}_1)_0, \tag{11}$$

is the tidal stress tensor.

The time derivative in equation (10) is of course negligible, but it is convenient to keep it to derive the energy equation where it is useful to determine the signs of energy transfers.

It is easy to show that, if the solution of equations (1) and (2) is used to compute the tidal stress and the latter is substituted in equation (10), then, the solution of equations (9) and (10) yields $\mathbf{v}_0$ with good accuracy.

Typical values for the North Sea show that, in general, the two terms $2\mathbf{\Omega}\wedge\mathbf{v}_0$ and $\nabla.(-\mathbf{v}\mathbf{v})_0$ are of same order of magnitude.

If δv is, as before, the error on v, one has

$$\delta(\nabla.(-\mathbf{v}\mathbf{v})_0)\sim(\nabla.(-\mathbf{v}\mathbf{v})_0)\frac{\delta v}{v}\sim 0\left(2\Omega v_0\frac{\delta v}{v}\right).$$

This error induces an error $\delta\mathbf{v}_0$ on $\mathbf{v}_0$ given by

$$2\mathbf{\Omega}\wedge\delta\mathbf{v}_0\sim\delta(\nabla.(-\mathbf{v}\mathbf{v})_0)\sim 0\left(2\Omega v_0\frac{\delta v}{v}\right),$$

i.e.

$$\frac{\delta v_0}{v_0} \sim \frac{\delta v}{v}.$$

Hence, the *relative* error is the same on $\mathbf{v}_0$ and on $\mathbf{v}$ while, by the method of equation (8), the *absolute* error is the same. Thus, if $\mathbf{v}$ can be computed with, say, a 90% precision, the solution of the averaged equation will give the residual circulation with the same 90% precision.

An energy balance equation can be derived form equation (10) by scalar multiplication by $\mathbf{v}_0$, one obtains

$$\frac{\partial}{\partial t}\left(\frac{1}{2} v_0^2\right) + \nabla.(\mathbf{v}_0 q_0 - \mathcal{N}.\mathbf{v}_0)$$

$$= \mathbf{v}_0 . \frac{\partial \boldsymbol{\tau}_0}{\partial x_3} - \mathcal{N} : \nabla \mathbf{v}_0. \tag{12}$$

In the left-hand side of equation (12), the first term is negligibly small (the slowly-varying residual circulation can be approximated by a steady flow), but it is useful to keep it formally to determine the signs of energy transfers : a positive contribution in the right-hand side being associated to a transfer of energy to the residual flow (tending to increase its kinetic energy), a negative contribution denoting a transfer from the mean flow to smaller scale motions and ultimate dissipation.

The second term in the left-hand side is of the divergence form, and corresponds to a redistribution of energy in physical space. This term, which may influence locally the rectification process, is not the immediate concern of the present analysis which addresses the transfer of energy *in Fourier space* between motions of different scales.

One is mainly interested here in the exchanges of energy between macroscale residual (mean) flow and mesoscale waves.

For this purpose, one would like to discuss the signs of the terms in the right-hand side of equation (12). In interpreting the discussion, one shall use a terminology borrowed from studies of macroscale and mesoscale ocean hydrodynamics, where non-linear interactions of Rossby waves and rectification are often described in terms of two-dimensional turbulence, and where a transfer of energy from "eddies" at the synoptic scale to the residual gyres is summarized by the concept of "negative eddy viscosity".

The 1979 SCOR-IAPSO Symposium on Turbulence in the Ocean has shown that much insight can be gained into this type of interaction by looking at it from the point of view of turbulence theory (e.g. Nihoul, 1980).

This is however nothing but a convenient way of phrasing energy transfers in Fourier space, and it may be regarded as defining the concept of eddy viscosity used in the following. Different definitions can of course be proposed in different contexts and for instance, positive or negative eddy viscosity could be associated with *momentum* transfers down or up the mean velocity gradient.

In this paper, attention is restricted to *energy*—and possibly enstrophy—transfers between different scales of motions and positive or negative viscosity refers essentially to the signs of these transfers.

The three-dimensional model which is used to determine the velocity field $\mathbf{v}$, has been described in previous publications (Nihoul, 1977; Nihoul *et al.*, 1979), and will not be reproduced here. Basically, the model computes the horizontal velocity vector $\mathbf{u} = \mathbf{v} - w\,\mathbf{e}_3$, the vertical velocity w is then calculated using the continuity equation (1).

Two important results of the model can be summarized as follows:

- The Reynolds stress can be written

$$\boldsymbol{\tau} \sim \tilde{\nu} \frac{\partial \mathbf{u}}{\partial x_3} = \boldsymbol{\tau}_s \xi + \boldsymbol{\tau}_b (1 - \xi)$$

$$+ \varkappa \|\boldsymbol{\tau}_b\|^{1/2} \mathrm{H} \sum_1^{\infty} \mathrm{A}_n \lambda(\xi) \frac{df_n}{d\xi}, \tag{13}$$

where $\boldsymbol{\tau}_s$ and $\boldsymbol{\tau}_b$ are respectively the surface stress and the bottom stress (per unit mass of sea water), $\xi = \mathrm{H}^{-1}(x_3 + h)$, $\mathrm{H} = h + \zeta$, h is the depth and ζ the surface elevation, the A_n's are functions of t, x_1 and x_2 involving $\boldsymbol{\tau}_s$, $\boldsymbol{\tau}_b$ and their time derivatives

$$\lambda(\xi) = \frac{\tilde{\nu}}{\varkappa \|\boldsymbol{\tau}_b\|^{1/2} \mathrm{H}},$$

and the functions $f_n(\xi)$ are the eigenfunctions of the problem

$$\frac{d}{d\xi}\left(\lambda \frac{df_n}{d\xi}\right) = -\alpha_n f_n, \tag{14}$$

$$\lambda \frac{df_n}{d\xi} = 0 \quad \text{at} \quad \xi = 0 \quad \text{and} \quad \xi = 1, \tag{15}$$

α_n being the corresponding eigenvalue.

The last term in the right-hand side of equation (13) plays an important role in the determination of the velocity field $\mathbf{v}$, but its effect is limited to a relatively short period of time at tide reversal (Nihoul, 1977; Nihoul *et al.*, 1979), and it contributes very little to the residual Reynolds stress obtained by averaging over a time T covering several tidal periods.

Hence, differentiating equation (13) with respect to x_3, one can write, with a good approximation

$$\frac{\partial \boldsymbol{\tau}_0}{\partial x_3} \sim \left(\frac{\boldsymbol{\tau}_s - \boldsymbol{\tau}_b}{\mathrm{H}}\right)_0, \tag{16}$$

- The bottom stress $\boldsymbol{\tau}_b$ is a function of $\boldsymbol{\tau}_s$, the depth-averaged velocity $\bar{\mathbf{u}}$ and the time derivatives of $\bar{\mathbf{u}}$.

If one excepts, again, short periods of weak currents (at tide reversal), $\boldsymbol{\tau}_b$ can be approximated by the classical "quadratic bottom friction law"

$$\boldsymbol{\tau}_b = \mathrm{D} \|\bar{\mathbf{u}}\| \bar{\mathbf{u}}, \tag{17}$$

where D is the drag coefficient.

Averaging over a time T as before, one obtains, then

$$\left(\frac{\tau_b}{H}\right)_0 \sim \left(\frac{D\|\bar{\mathbf{u}}_1\|}{H}\right)_0 \bar{\mathbf{u}}_0 + \left(\frac{D\|\bar{\mathbf{u}}_1\|\bar{\mathbf{u}}_1}{H}\right)_0 . \qquad (18)$$

Other contributions like $(D\|\bar{\mathbf{u}}_0\|\bar{\mathbf{u}}_0)/H_0$ are neglibly small.

One notes that while the time-dependent Reynolds stress $\boldsymbol{\tau}$ [to be substituted in equations (4) and (2)] is a quadratic function of the velocity, the residual Reynolds stress is a linear function of the residual velocity; the residual drag coefficient

$$K = \left(\frac{D\|\bar{\mathbf{u}}\|}{H}\right)_0 , \qquad (19)$$

being a function of position to be determined using, at each grid point, the solution of equation (2).

The second term in the right-hand side of equation (18) represents another residual effect of the non-linear interactions of long waves. It will be referred to as the "friction stress". The friction stress can easily be computed and combined with the tidal stress. Its contribution, however, turns out to be relatively small, except in a few localized places, such as very shallow areas like the Southern Bight of the Nord Sea (in fairly deep waters $H \sim h$, the contribution from a velocity $\mathbf{u}_1$ at a given time tends to be cancelled by an opposite contribution of a velocity $-\mathbf{u}_1$, about one half tidal period later; this is not true in shallow waters, where the time dependence of $H = h + \zeta$ plays a more important rôle and where the smallness of the denominator increases the order of magnitude of the term).

TIDAL EDDIES IN THE NORTH SEA

The role of the wind stress in transferring momentum and energy to the sea is well-known, and it is not surprising that $\boldsymbol{\tau}_s$ appears in the parameterization of the Reynolds stress. In the North Sea, however, if one excepts periods of severe storms, the direct effect of the wind is small compared to bottom friction. For instance, for a wind of $10\ \mathrm{m.s^{-1}}$, $\boldsymbol{\tau}_s$ is one order of magnitude smaller than the maximum bottom stress $\boldsymbol{\tau}_b$, and it takes a wind of some $25\ \mathrm{m.s^{-1}}$ to have comparable values of $\boldsymbol{\tau}_s$ and $\boldsymbol{\tau}_b$ (e.g. Nihoul, Ronday, 1976*a*).

The superposition of different situations, corresponding to different mean wind conditions, reveals unmistakable trends and, in particular, the analysis of energy transfers shows distinct regions where the tides definitely extract energy from the residual circulation (positive viscosity) and distinct regions where, on the contrary, energy is transferred from the tides to the residual flow (negative viscosity).

Such regions can be regarded as positive viscosity eddies and negative viscosity eddies, respectively.

To illustrate the presence of these eddies, the tidal and residual circulations have been computed in negligible wind conditions.

Energy transfers between the different scales of motion in the North Sea can be characterized by:

- the rate of turbulent energy dissipation [eq. (5)]:

$$\varepsilon = \boldsymbol{\tau} . \frac{\partial \mathbf{v}}{\partial x_3} ; \qquad (20)$$

- the rate of exchange of energy between long waves and residual circulation by the action of the tidal stress on the residual flow [eq. (12)]:

$$\mathcal{N} : \nabla \mathbf{v}_0 = (-\mathbf{v}\mathbf{v})_0 : \nabla \mathbf{v}_0 ; \qquad (21)$$

- the rate of work of the friction stress on the mean flow [eq. (18)]:

$$\left(\frac{D\|\bar{\mathbf{u}}_1\|\bar{\mathbf{u}}_1}{H}\right)_0 . \mathbf{v}_0 = \left(\frac{D\|\bar{\mathbf{u}}_1\|\bar{\mathbf{u}}_1}{H}\right)_0 . \mathbf{u}_0 . \qquad (22)$$

The three quantities (20), (21) and (22) are computed by the three-dimensional model and, then, averaged to give the "transfer functions"

$$\varepsilon_R = (\bar{\varepsilon})_0 = \left(\frac{1}{H} \int_{-h}^{\zeta} \varepsilon \, dx_3\right)_0 , \qquad (23)$$

$$\varepsilon_N = \frac{1}{H_0} \int_{-h}^{\zeta_0} (\mathcal{N} : \nabla \mathbf{v}_0) \, dx_3 , \qquad (24)$$

where $H_0 = h + \zeta_0$ and ζ_0 is the residual surface elevation

$$\varepsilon_F = \frac{1}{H_0} \int_{-h}^{\zeta_0} \left(\frac{D\|\bar{\mathbf{u}}_1\|\bar{\mathbf{u}}_1}{H}\right)_0 . \mathbf{v}_0 \, dx_3 = \left(\frac{D\|\bar{\mathbf{u}}_1\|\bar{\mathbf{u}}_1}{H}\right)_0 . \bar{\mathbf{u}}_0 . \qquad (25)$$

The results are illustrated in Figures 1, 2 and 3:

— Figure 1 shows the regions where

$\varepsilon_N > 5 \times 10^{-9}\ \mathrm{m^2.s^{-3}}$,

figuring the positive viscosity eddies (ε_N is typically of the order $10^{-8} - 10^{-7}$ in the North Sea eddies and about one order of magnitude larger in the Southern Bight);

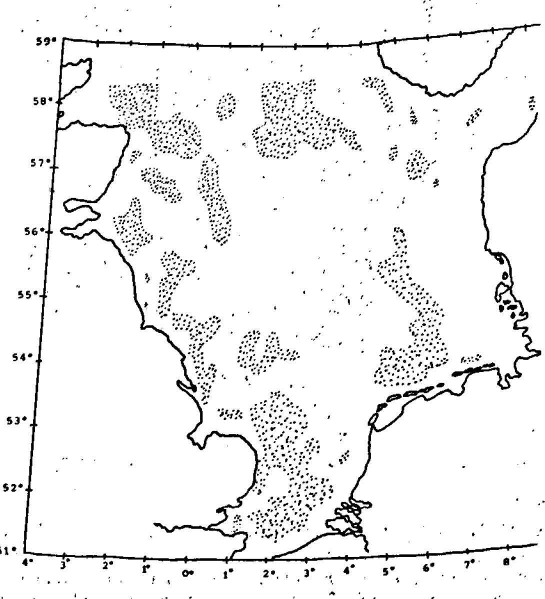

Figure 1
Positive viscosity eddies in the North Sea.

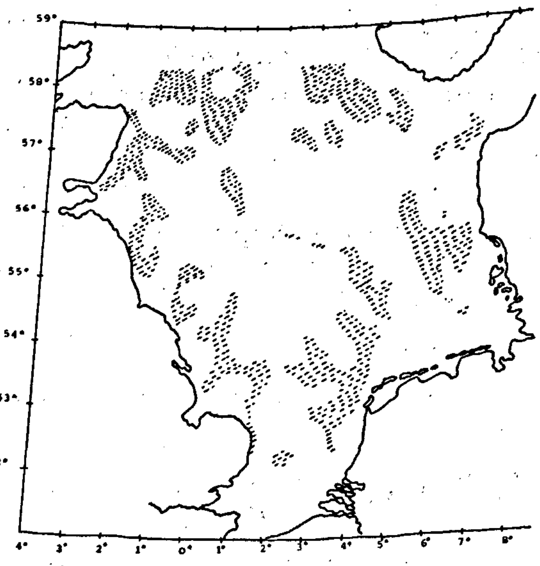

Figure 2
Negative viscosity eddies in the North Sea.

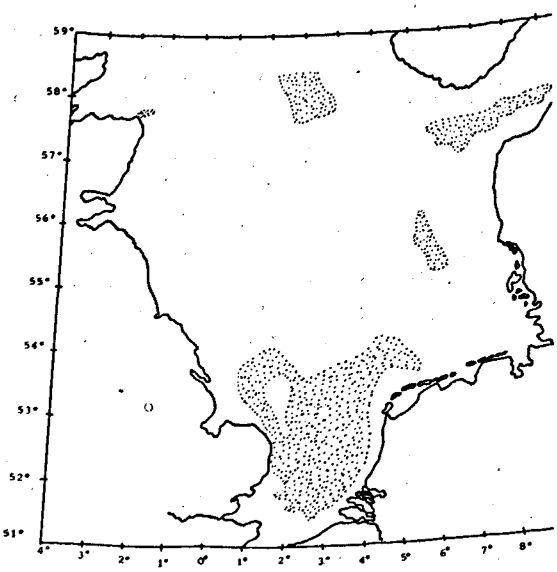

Figure 3
Regions where the friction stress extracts energy from the mean flow ($\varepsilon_F > 5 \times 10^{-9}\, m.s^{-3}$).

– Figure 2 shows the regions where

$$\varepsilon_N < -5 \times 10^{-9}\,\mathrm{m}^2.\mathrm{s}^{-3},$$

figuring the negative viscosity eddies;
– Figure 3 shows the regions where

$$\varepsilon_F > 5 \times 10^{-9}\,\mathrm{m}^2.\mathrm{s}^{-3}$$

(ε_F is typically of the order $10^{-8} - 10^{-7}$ in the Southern Bight). Outside those regions ε_F is small and almost nowhere significantly negative.

An interesting interpretation of these results is obtained by superposing Figures 1 and 3:
– Figure 4 shows the regions where the tidal stress and the friction stress combine to extract energy from the residual circulation.

The arrows indicate the residual flow pattern which one typically expects in weak wind conditions (Böhnecke,

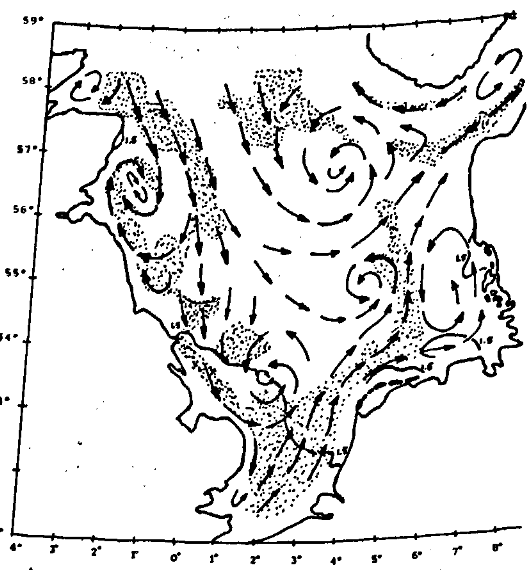

Figure 4
Regions (indicated by dots) where the tidal stress and the friction stress combine to extract energy from the residual circulation, showing the domains (without dots) where negative viscosity eddies appear. The arrows indicate the residual flow pattern which one typically expects in weak wind conditions (from Böhnecke, 1922; Hill, 1974). The curves marked 1.5 indicate the possible positions of fronts.

1922; Hill, 1974) (the demarcation between positive and negative viscosity in the South of the Southern Bight is only qualitative as the corresponding grid points are close to the boundary of the numerical model. A careful examination of the data confirms however the existence of a region of negative viscosity, extending out of the Thames Estuary – corresponding to the gyre indicated by Böhnecke – and another region off the Northern Belgian coast associated with a long-observed gyre revealed by fine-grid numerical models (e.g. Nihoul, Ronday, 1975) and confirmed by experimental surveys (Beckers *et al.*, 1976), as well as by remote sensing observations (Fig. 5) (Courtesy University of Dundee). A preliminary investigation (Nihoul, Ronday, 1976*b*) showed a slightly different picture but this was based on a depth integrated model – giving less accurate results – with a finer grid – giving more details – and with a non zero wind stress.

Figure 4 shows a rather striking correspondence between zone of negative viscosity and residual gyres.

It is tempting to draw here a comparison with large scale oceanic motions, and to interpret the formation of residual gyres as a transfer of vorticity from the macroscale residual circulation to smaller scales, associated with the opposite transfer of energy from the mesoscale (non-linear interacting long waves) to the mean flow (e.g. Nihoul, 1980). Although such a comparison may be rather far-fetched, it is interesting to put it to the test and look for other phenomena which seem to play an important rôle in energy and enstrophy transfers at ocean mesoscales, and which might also be observed in the North Sea.

One important example is the formation of fronts and frontal eddies:

Pingree and Griffiths (1978), using a two-dimensional depth-averaged model of tides, estimated the rate of turbulent energy dissipation ε_R, and argued that fronts would appear in the summer in specific areas where a critical value of ε_R is reached.

Figure 5
IR satellite picture of the Southern Bight, May 1978 (Courtesy University of Dundee).

The parameter used by Pingree and Griffiths was actually

$$S = \log_{10}(10^{-4}\varepsilon_R^{-1}), \tag{26}$$

and the critical value of S, derived from arguments of Simpson and Hunter (1974), was taken as 1.5 [this value is obviously only an indication. In deriving equation (26), several assumptions have been made on solar heating and energy exchange with the atmosphere. Moreover, ε_R is computed with a barotropic model and can only be an estimate of its value in partially stratified conditions when fronts occur. Pingree and Griffiths (1978) have shown however that S was nevertheless a good indicator of the regions where fronts may form. The model cannot of course go much further and investigate, for instance, problems of frontal instabilities and cyclogenesis. These aspects will only be mentioned later with reference to Pingree's observations (Pingree, 1978)].

The parameter S was recalculated using equation (23) of the three-dimensional model and the results are plotted on Figure 6. The critical line $S = 1.5$ is, in general, in agreement with Pingree and Griffiths' two-dimensional prediction in the Eastern North Sea, off the English coast, and in the German Bight. The two curves diverge off the Dutch coast where the line drawn by Pingree and Griffiths curves up and links on to the German Bight's segments. The critical curve $S = 1.5$ is reproduced for convenience on Figure 4.

Pingree and Griffiths (Pingree, 1978; Pingree, Griffiths, 1978) gave evidence from ship surveys and IR satellite images of frontal structures in coastal areas. In particular the Flamborough Head front off the coast of England, and the German Bight fronts seem to agree fairly well with the critical line $S = 1.5$ in these regions (there is also some indication on Figure 6 of frontal structures in the region where the critical line reaches the Dutch coast). There is however no real evidence that fronts can exist further off the coast and, in the central part of the critical line $S = 1.5$, which corresponds to a region of strong positive viscosity effect if one refers to Figure 4, no front has actually been traced.

Now the idea that fronts may develop, in the summer, only in regions of negative eddy viscosity, is very interesting in the light of the comparison made above with large scale oceanic motions.

The suggestion has indeed been made that, in negative eddy viscosity turbulence, a fraction of the kinetic energy may still be transferred to smaller scales, and that the mechanism for such a transfer could be the formation of fronts and their subsequent instabilities leading to intrusive layers and frontal eddies (e.g. Nihoul, 1980).

The observation of cyclonic eddies presumably of frontal origin was reported by Pingree (1978), who argued that they constituted an essential feature of cross-frontal mixing and phytoplankton dynamics in these areas (Pingree *et al.*, 1979).

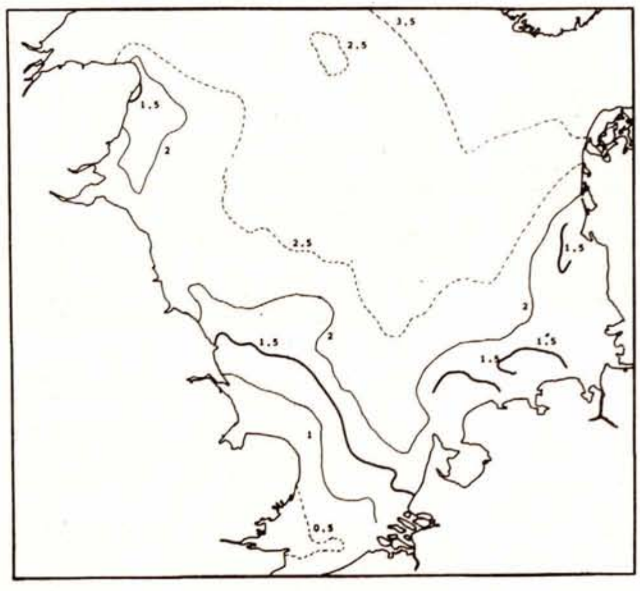

Figure 6
Curves of equal values of the Simpson-Hunter parameter $S = \log_{10}(10^{-4}\varepsilon_R^{-1})$ *in the North Sea.*

CONCLUSIONS

Although long-waves, tides and storm surges, dominate the hydrodynamics of the North Sea, the understanding of the residual circulation is of the greatest importance as it affects the drift and distribution of nutrients and biological populations and determines the clearing out routes and residence times of pollutants.

The mathematical modelling of the residual circulation deserves the greatest attention and models must be designed to predict with accuracy small residual currents in a background of large oscillatory and transitory currents which exert, as a result of non-linear interactions, an additional forcing on the mean flow.

The model can be used to explore in details the nature of this forcing, and it is found that it may foster the mean flow in some regions while it extracts energy from it in other regions.

This effect is associated with the existence of positive viscosity and negative viscosity eddy-like structures, which are reminiscent of essential features of large scale ocean dynamics.

A complete study of the residual circulation must include the determination of these structures and the related residual gyres, which are probably as important to understand marine chemistry and marine ecology as the classical drift pattern of water masses.

Acknowledgments

The author is indebted to Y. Runfola, P. Closset, Ch. Demez and F. Hanesse for their help in the numerical application of the model and the technical preparation of the manuscript.

He wishes to express his gratitude to the Belgian Ministry for Science Policy for the financial support of his research.

REFERENCES

Beckers O., Wollast R., Nihoul J. C. J., 1976. La circulation résiduelle et la caractérisation des masses d'eau dans la zône côtière belge, in: *Modélisation des systèmes marins*, edited by J. C. J. Nihoul, Projet Mer Rapport Final, Vol. I, Services du Premier Ministre, Programmation de la Politique Scientifique, Bruxelles 1976, 95-130.

Böhnecke G., 1922. Salzgehalt und Strömungen der Nordsee, Inst. Meeresk. Univ. Berlin, Neue Folge A, *Geogr. Naturwiss.*, Reihe **10**, 1-34.

Hill H. W., 1974. Currents and water masses, in: *North Sea Science*, edited by E. D. Goldberg, M.I.T. Univ. Press, Cambridge, Mass., 17-42.

Nihoul J. C. J., 1974. Mesoscale secondary flows and the dynamics of ecosystems in the Southern Bight of the North Sea, *Mémoires Soc. Sci. Lg.*, **7**, 83-91.

Nihoul J. C. J., 1975. *Modelling of marine systems*, Elsevier, Amsterdam, 272 p.

Nihoul J. C. J., 1977. Three-dimensional model of tides and storm surges in a shallow well-mixed continental sea, *Dyn. Atmos. Oceans*, **2**, 29-47.

Nihoul J. C. J., 1980. The turbulent ocean, in: *Marine Turbulence*, edited by J. C. J. Nihoul, Elsevier, Amsterdam, 1-19.

Nihoul J. C. J., Ronday F. C., 1975. The influence of the tidal stress on the residual circulation, *Tellus*, **29**, 484-490.

Nihoul J. C. J., Ronday F. C., 1976*a*. Hydrodynamic models of the North Sea, *Mémoires Soc. Sci. Lg.*, **10**, 61-96.

Nihoul J. C. J., Ronday F. C., 1976*b*. Coherent structures and negative viscosity in marine turbulence, *J. Méc.*, **15**, 119-132.

Nihoul J. C. J., Runfola Y., Roisin B., 1979. Non-linear three-dimensional modelling of mesoscale circulation in seas and lakes, in: *Marine Forecasting*, edited by J. C. J. Nihoul, Elsevier, Amsterdam, 1979, 235-259.

Otto L., 1970. The mean residual transport pattern in the Southern North Sea, *ICES Hydrography Committee*, CM 1970/C, 21, 12 p.

Pingree R. D., 1978. Cyclonic eddies and cross-frontal mixing, *J. Mar. Biol. Assoc. UK*, **58**, 955-963.

Pingree R. D., Griffiths D. K., 1978. Tidal fronts on the shelf seas around the British Isles, *J. Geophys. Res.*, **83**, 4615-4622.

Pingree R. D., Holligan P. M., Mardell G. T., 1979. Phytoplankton growth and cyclonic eddies, *Nature*, **278**, 245-247.

Riepma H., 1977. Spatial variability of residual currents in an area of the Southern North Sea. *ICES Hydrography Committee* CM 1977/C, 43, 7 p.

Ronday F. C., 1974. Mesoscale effects of the tidal stress on the residual circulation of the North Sea, *Mémoires Soc. Sci. Lg.*, **7**, 273-287.

Simpson J. H., Hunter J. R., 1974. Fronts in the Irish Sea, *Nature (London)*, **1250**, 404-406.

Sugimoto T., 1975. Effect of boundary geometries on tidal currents and tidal mixing, *J. Oceanogr. Soc. Jpn.*, **31**, 1-14.

Tee K. T., 1976. Tide-induced residual current, a 2-*d* numerical nonlinear tidal model, *J. Mar. Res.*, **34**, 603-628.

Tee K. T., 1977. Tide-induced residual current, verification of a numerical model, *J. Phys. Oceanogr.*, **7**, 396-402.

Yanagi T., 1976. Fundamental study of the tidal residual circulation, *J. Oceanogr. Soc. Jpn.*, **32**, 199-208.

Zimmerman J. T. F., 1976. Mixing and flushing of tidal embayments in the Western Dutch Wadden Sea-II, *Neth. J. Sea Res.*, **10**, 397-439.

Zimmerman J. T. F., 1978*a*. Topographic generation of residual circulation by oscillatory (tidal) currents, *Geophys. Astrophys. Fluid Dyn.*, **11**, 35-47.

Zimmerman J. T. F., 1978*b*. Dispersion by tide-induced residual current vortices, in: *Hydrodynamics of Estuaries and Fjords*, edited by J. C. J. Nihoul, Elsevier, Amsterdam, 1978, 207-216.