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The two-dimensional seiches of the Baltic Sea

Two-dimensional seiches
Numerical solution
Baltic Sea
Oscillations bidimensionnelles
Solution numérique
Mer Baltique

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ABSTRACT

Periods and structures of the gravitational free oscillations in the Baltic Sea are determined from theoretical calculations. The calculations take into account the bathymetry and shape of the Baltic and the earth's rotation. The method for determining the free oscillations is based on a difference approximation of the inviscid quasistatic shallow water equations. The seiches are produced by a forcing function which acts on the water body during an initial time. The resulting oscillations after the force is switched off are analyzed at each grid point and displayed in form of co-range and co-tidal lines. The calculations were carried out for the entire Baltic Sea and for the Baltic proper. A large variety of oscillations is possible. The results are compared with those of the channel theory. It is shown that the eigenoscillations of the Baltic Sea are strongly modified by the Coriolis force. Rotation converts all modes into positive amphidromic waves; only the interior parts of the Gulfs and of the Western Baltic exhibit standing waves. The periods of the oscillations are reduced by earth rotation, if they are longer than the inertial period, and prolongated, in case they are shorter. The Gulf of Riga, neglected in computations based on channel theory, is of considerable importance for the seiches of the Baltic Sea.

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RÉSUMÉ

Oscillations bidimensionnelles dans la mer Baltique.

On détermine à partir de calculs théoriques les périodes et les structures des oscillations libres de gravité dans la Mer Baltique. Ces calculs prennent en compte la bathymétrie et la géographie de la Baltique et la rotation de la terre. La méthode utilisée pour déterminer les oscillations libres est basée sur une approximation en différences finies des équations non visqueuses des ondes longues. Les seiches sont produites par une force qui agit sur la masse liquide durant un temps donné. Les oscillations obtenues, après avoir enlevé cette force, sont analysées à chaque nœud de la grille de calcul et présentées sous la forme de cartes d'égales amplitudes et d'égales phases. Les calculs sont réalisés pour toute la Mer Baltique et pour la Baltique proprement dite. Une grande variété d'oscillations est possible. Les résultats sont comparés avec ceux donnés par la théorie du canal. On montre que les oscillations propres de la Mer Baltique sont fortement modifiées par la force de Coriolis. La rotation transforme tous les modes en ondes amphidromiques positives; seule la partie intérieure des golfes et la partie ouest de la Baltique présentent des ondes stationnaires. Les périodes des oscillations sont diminuées par la rotation de la terre, si elles sont plus grandes que la période d'inertie, et augmentées dans le cas où elles sont plus courtes. Le Golfe de Riga, négligé dans les calculs basés sur la théorie du canal, est d'importance considérable pour les seiches de la Mer Baltique.

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INTRODUCTION

Owing to the very strong response of the Baltic Sea to wind action, longitudinal oscillations are a dominant feature in all tide gauge records (Lisitzin, 1959), especially at the Finnish coast. The oscillations, however, turn out to be heavily damped. Typically, we find only two or three oscillations. This may be due to the complicated

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configuration of the Baltic Sea (Fig. 1 and 2), to the division into several sub-basins (Fig. 2) and to the open connections to the North Sea which allow water exchange up to 10^5 $m^3 . s^{-1}$ through the Belts (Hardtke, 1978). Furthermore, the periods are by no means well separated. A wide variety of oscillations seems to belong to the eigenfunctions. The tide gauge records of the Baltic Sea have been studied by several authors during the past decades, notably by Neumann, 1941; Magaard and Krauss, 1966; Lisitzin, 1974. Observations indicate that the dominant oscillation Western Baltic-Gulf of Finland has a period of about 26-27 hours. This, however, may vary between 26 and 32 hours. The second mode of this system seems to be centered around 17-19 hours, but several 20 hours have also been observed. Neumann (1941) also detected a period of about 39 hours by careful visual inspection of the records; it was interpreted as the basic mode for the system Western Baltic-Gulf of Bothnia. This oscillation, however, was strongly damped and transformed into shorter ones. Spectral analysis of one year records of tide gauges around the Baltic did not shed any light into the problem (Magaard, Krauss, 1966).

The traditional theory of free oscillations of elongated basins is based on the "channel hypothesis" which approximates the difficult two-dimensional problem with a simple one-dimensional problem by integrating over cross sections and assuming that no cross oscillations occur. The method was first applied to the Baltic Sea by Neumann (1941) and further used by Krauss and Magaard (1962). With respect to the Baltic Sea the method has several shortcomings:

- the Baltic proper forks into two basins, the Gulf of Finland and the Gulf of Bothnia, which are of different length and depth. Consequently, a simultaneous reflection at both ends is impossible. One, therefore, needs to distinguish between oscillations of the system Western Baltic-Gulf of Finland and Western Baltic-Gulf of Bothnia;

- the main axis of the Baltic is S-shaped which leaves some arbitrariness to select the cross sections;

- the separation into two systems, mentioned above, is an artifice on a rotating earth. The Coriolis force always tends to push water into that system which is not under consideration. Only the system Western Baltic-Gulf of Finland may allow separate eigenfunctions because of the narrow entrance to the Gulf of Bothnia which is very shallow in most parts of the Aland Sea. Only a trench (Alands deep), 40 km wide, is a real opening to the Baltic proper.

Nevertheless, the periods computed by the channel approximation are in the range of observed ones. These periods are listed in Table 1 according to Krauss and Magaard (1962) for a Baltic Sea closed at Fehmarn Belt.

In contrast to previous computations, the present study aims to determine the two-dimensional seiches of the Baltic Sea. The free oscillations of a two-dimensional water body on a rotating plane consist of two distinct types; the gravitational modes (oscillations of the first class) and the rotational modes (oscillations of the second class). This paper is concerned with the gravitational modes only (seiches).

Figure 1
Stereographical projection of the Baltic Sea and numerical grid. Centre of projection: 60°N, 20°E. Grid scale 10 km.

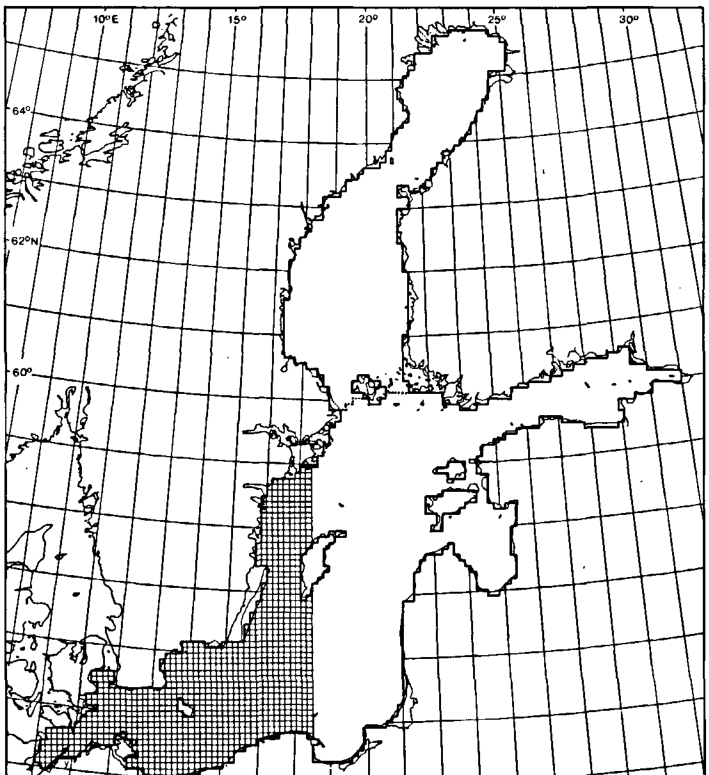


Figure 2
Depth contours of the Baltic Sea. Depth contour interval is 40 m. Units indicated in 10 m.

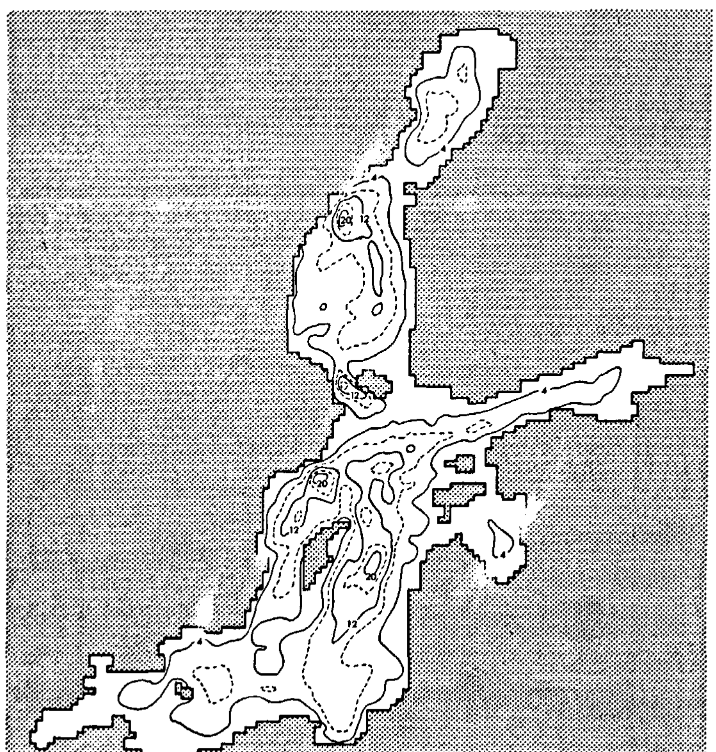


Table 1
Periods of the Baltic Sea based on one-dimensional seiches theory (hours).

System	Mode 1	2	3	4	5	6
Western Baltic-Gulf of Finland	27,4	19,1	13,0	9,6	8,1	6,9
Western Baltic-Gulf of Bothnia	39,4	22,5	17,9	12,9	9,4	7,3

General methods to compute the eigenoscillations of a natural basin of some arbitrary shape on a rotating earth have been developed by Rao and Schwab (1976) and Platzman (1972). Platzman's method is based on resonance iteration whereas the method of Rao and Schwab requires the numerical construction of two sets of orthogonal functions. Both methods are rather complicated. For an oblong basin like the Baltic Sea where the basic structure of the modes must be similar to those on a non-rotating earth, it seems to be much more efficient to use a simpler approach. In the present case the procedure is as follows: the linearized momentum equations and the equation of continuity are solved numerically as an initial value problem subject to a forcing function. As forcing function we prescribe an air pressure field whose space dependency is a crude approximation of the expected basic modes. The forcing function is switched off after some hours and, consequently, the set-up decomposes into a sum of eigenoscillations. Their amplitudes and phases are analysed at each grid point by spectral methods. The grid system used is shown in Figure 1 (10 km grid distance). A similar approach is due to Radach (1971) and Papa (1977). The present study, however, avoids frictional damping.

THE NUMERICAL MODEL

The model is based on standard approximations, i.e. we assume quasistatic dynamics of an incompressible homogeneous fluid on a rotating earth, where the Coriolis parameter f varies with latitude. U and V are the horizontal mass transport components in x- and y-direction, $H(x, y)$ is the equilibrium depth and ζ the departure of the sea level from the mean. Letting P_0 denote the air pressure the linearized shallow water equations are

$$\frac{\partial U}{\partial t}=f\,V-g\,H\frac{\partial \zeta}{\partial x}-\frac{H}{\rho}\frac{\partial P_0}{\partial x}, \quad (1)$$

$$\frac{\partial V}{\partial t}=-f\,U-g\,H\frac{\partial \zeta}{\partial y}-\frac{H}{\rho}\frac{\partial P_0}{\partial y}, \quad (2)$$

$$\frac{\partial \zeta}{\partial t}=-\frac{\partial U}{\partial x}-\frac{\partial V}{\partial y}, \quad (3)$$

subject to the boundary condition

$$(U, V).n=0 \quad (4)$$

along the coast, where n is the unit normal vector to the coast line.

Note that the equations contain no frictional terms which is essential for an accurate spectral determination of the eigenfunctions.

The problem (1)-(4) is solved as initial value problem. The initial values are given by the state of rest

$$\zeta=0, \quad U=0, \quad V=0. \quad (5)$$

The forcing function P_0 is prescribed as

$$P_0(x, y, t)=\begin{cases}\left(1-\cos\frac{\pi}{T_0}t\right)F(x, y) \\ \text{for} \\ 0\leqq t\leqq T_0, \\ 0 \text{ else}\end{cases} \quad (6)$$

where $F(x, y)$ is chosen in such a way as to force distinct modes (typically, we use cosine functions in length direction of the Baltic). After time T_0 (typically 30 hours) the pressure field is switched off. During $0\leqq t\leqq T_0$ the sea level changes from 0 to $\zeta(x, y, T_0)$ and may be described at time $t=T_0$ by a sum of eigenfunctions

$$\zeta(x, y, T_0)=\sum_n A_n(x, y)\cos(\omega_n T_0-\theta_n(x, y)). \quad (7)$$

Because the model is inviscid and $P_0=0$ for $t>T_0$, these oscillations are undamped. We continue to compute $\zeta(x, y, t)$ numerically until $t=T_1$, where

$T_1-T_0=1\,024$ hours.

Then the time series, based on hourly values, are Fourier-analyzed at each grid point.

THE NUMERICAL SCHEME

A numerically stable difference scheme of the inviscid equations (1)-(3) has been given by Sielecki (1967). It is based on central differences for space derivatives and forward differences for time derivatives. Its explicit form is

$$\zeta_{i,j}^{n+1}=\zeta_{i,j}^{n}-\frac{\Delta t}{2\Delta s}(U_{i+1,j}^{n}-U_{i-1,j}^{n}+V_{i,j+1}^{n}-V_{i,j-1}^{n}), \quad (8)$$

$$U_{i,j}^{n+1}=U_{i,j}^{n}+f\,\Delta t\,V_{i,j}^{n}-\frac{g\,\Delta t}{2\Delta s}H_{i,j}(\zeta_{i+1,j}^{n+1}-\zeta_{i-1,j}^{n+1})-\frac{\Delta t\,H_{i,j}}{\rho}\left(\frac{\partial P_0}{\partial x}\right)_{i,j}^{n+1}, \quad (9)$$

$$V_{i,j}^{n+1}=V_{i,j}^{n}-f\,\Delta t\,U_{i,j}^{n+1}-\frac{g\,\Delta t}{2\Delta s}H_{i,j}(\zeta_{i,j+1}^{n+1}-\zeta_{i,j-1}^{n+1})-\frac{\Delta t\,H_{i,j}}{\rho}\left(\frac{\partial P_0}{\partial y}\right)_{i,j}^{n+1} \quad (10)$$

for all interior points.

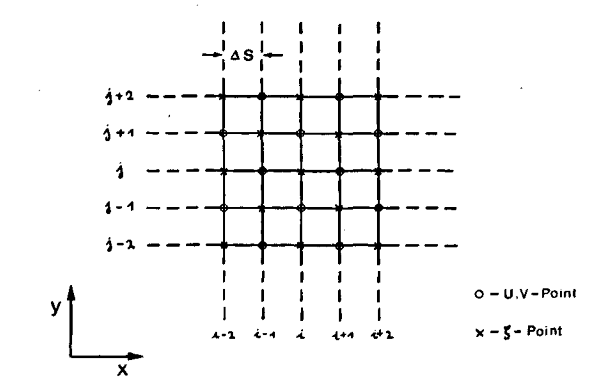


Figure 3
Distribution of variables on the numerical grid.

The equations refer to a grid for U, V and ζ as shown by Figure 3. The x, y, and t coordinates are replaced by discrete points $x_i = i\,\Delta s$, $y_j = j\,\Delta s$ and $t = n\,\Delta t$. U, V and ζ are defined at all points such that $i+j=$even for U, V and $i+j=$odd for ζ.

The system (8)-(10) is stable if

$$g\,\mathrm{H}_{i,j}\frac{\Delta t^2}{\Delta s^2} \leqq g\,\mathrm{H}_{\max}\frac{\Delta t^2}{\Delta s^2} \leqq 2 - f\,\Delta t \tag{11}$$

holds where $\mathrm{H}_{\max}$ is the maximum depth. With $f=0$ we obtain the well-known Courant-Friedrichs-Lewy criterion. The phase error of the numerical solutions compared to analytical ones can be derived in case of $f=0$, H=Const. and $\mathrm{P}_0=0$ according to the procedure described by O'Brien and Grotjahn (1976). The dispersion relation for progressive waves according to (1)-(3) is in the present case

$$\omega_a = \pm\sqrt{g\,\mathrm{H}\,(\varkappa^2+\eta^2)} \tag{12}$$

where $\varkappa$ and η are wave numbers in x and y direction. The numerical dispersion relation, obtained by inserting U, V, $\zeta \sim \exp\,[i\,(\omega_a t - \varkappa x - \eta y)]$, $t=n\,\Delta t$, $x=l\,\Delta s$, $y=k\,\Delta s$ into (1)-(3) is

$$\omega_n = \sqrt{\frac{g\,\mathrm{H}}{2\,\Delta s^2}}\,\mathrm{arc\,cos}\,[1-(\sin^2\varkappa\,\Delta s + \sin^2\eta\,\Delta s)] \tag{13}$$

if Δt is chosen according to (11). For progressive waves in x direction having a wave length of 500 km we obtain $\omega_n/\omega_a = 0.997$ for $\Delta s = 10$ km. Thus, the numerically obtained frequency is slightly less than the correct analytical value. The error, however, is negligible.

THE BOUNDARY CONDITIONS

A major source of errors may result from the boundary conditions. As well known (Harris and Jelesnianski, 1964) an approximation of space derivatives by central differences may yield grid dispersion if the boundary is parallel to the coordinate axis. To avoid this we rotate the coordinate system clockwise by 45° (Fig. 4*a*). Boundaries are chosen such that they pass through U, V-points. We then have to specify the boundary condition (4) for the cases (*b*)-(*f*) in Figure 4.

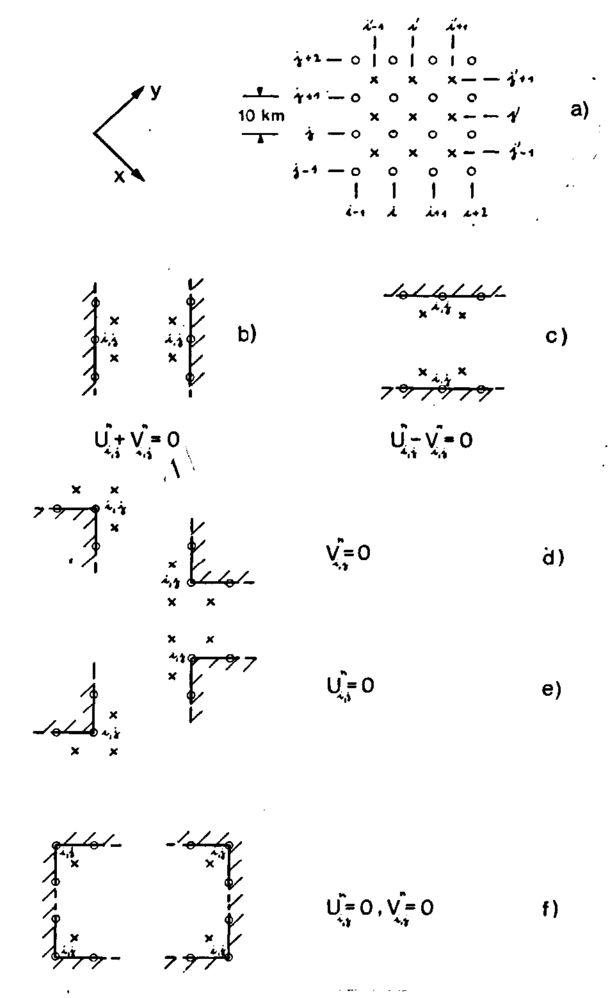


Figure 4
Rotation of the coordinate system by 45° and boundary conditions used in the numerical model: (a), rotated coordinate system and numerical grid (grid points as in Figure 3); (b)-(c), boundary conditions at straight coast lines; (d)-(e), boundary conditions at outer corner points; (f), boundary conditions at inner corner points.

If the coast line is straight (Fig. 4 *b, c*) we require either $\mathrm{U}_{i,j}^n + \mathrm{V}_{i,j}^n = 0$ or $\mathrm{U}_{i,j}^n - \mathrm{V}_{i,j}^n = 0$ in order to obtain a vanishing normal component for the currents in the rotated system. If a point i, j is an outer corner point the normal direction is defined as either the x or y coordinate and, consequently $\mathrm{U}_{i,j}^n$ or $\mathrm{V}_{i,j}^n$ must vanish (Fig. 4 *d, e*). In case of an inner corner point we require both $\mathrm{U}_{i,j}^n$ and $\mathrm{V}_{i,j}^n$ to vanish. In case of Figure 4 *d, e*, the remaining component $\mathrm{U}_{i,j}^{n+1}$ or $\mathrm{V}_{i,j}^{n+1}$ can be computed directly from (9) and (10) respectively.

In case of Figure 4 *b, c* we proceed as follows: if we assume an additional ζ-value at i, j, denoted by $\underline{\zeta}$, the values $\zeta_{i-1,j}^{n+1}$ in (9) and $\zeta_{i,j+1}^{n+1}$ in (10) may be replaced by $\underline{\zeta}_{i,j}$, which corresponds to a replacement of the central differences by foreward or backward differences, respectively. Together with $U^n_{i,j}=V^n_{i,j}$ (in case *c*) we then obtain

$$U^{n+1}_{i,j}=U^n_{i,j}-\frac{H_{i,j}}{2+f\,\Delta t}\times\left[g\frac{\Delta t}{\Delta s}(\zeta^{n+1}_{i,j-1}-\zeta^{n+1}_{i-1,j-1})+\frac{\Delta t}{\rho}\left(\frac{\partial P_0}{\partial x}+\frac{\partial P_0}{\partial y}\right)^{n+1}_{i,j}\right], \quad (14)$$

$$V^{n+1}_{i,j}=U^{n+1}_{i,j} \quad (15)$$

and similar relations in case of Figure 4 *b* (for details we refer to Wübber, 1979).

TEST OF THE NUMERICAL SCHEME

The numerical procedure described in the preceding section has been tested against some simple cases for which results are either known or determined by symmetry. The first test was carried out for a symmetric basin of constant depth (60 m) as shown by Figure 5. The dimensions of the basin correspond to the system Western Baltic-Gulf of Finland. The Coriolis parameter was chosen to be $10^{-4}.s^{-1}$, the central axis has a length of 1 162 km. The aspect ratio length/width is 5.8. Furthermore, $\Delta t=240$ s, $\Delta s=10$ km, $T_0=500\,\Delta t$ have been used. The results for mode 1-3 are displayed in Figure 5. The periods are 26, 8, 13, 1 and 8,9 hours, respectively and the amphidromic systems appear to be symmetric. If f is chosen to be zero, we obtain nodal lines in cross direction through the amphidromic points. The periods are slightly shorter in this case (26,4, 13,0 and 8,8 hours) and could be also obtained from a one-dimensional model.

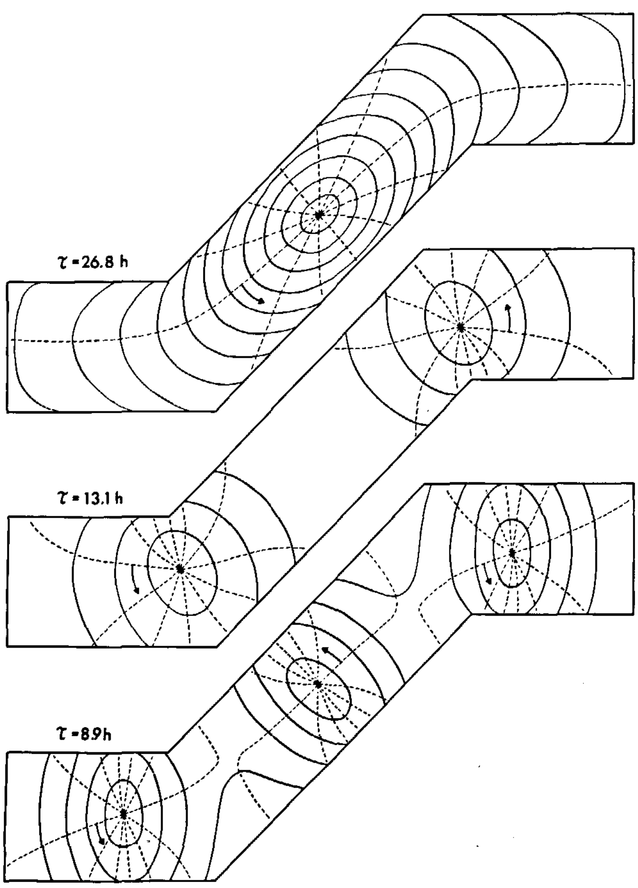


Figure 5
The first, second, and third normal modes for a symmetric S-shaped test basin of constant depth ($f=10^{-4}$ s^{-1}).

The next test was performed on a rotating rectangular basin of uniform depth for which results are available from Rao (1965). This test confirmed the amplitude and phase distributions displayed by Rao. The periods agree with those computed by Rao within 0,6% as seen from Table 2.

We are, therefore, confident that the present method allows to compute the seiches of the Baltic Sea with the same accuracy as other methods.

APPLICATION TO THE BALTIC SEA

A grid of 10 km is used as shown in Figure 1. This gives a total of 3 570 ζ-points and 3965 U, V-points. 798 of these are boundary values. The coordinate system is rotated clockwise by 45° as mentioned previously. The effective grid scale in the stability criterion is 7.07 km. The bottom topography as displayed in Figure 2 is taken from Kielmann (1976). The maximum depth is 232 m. The Baltic is closed at the Fehmarn Belt.

For the basic modes the variation of the Coriolis parameter is taken into account. For higher order modes a constant $f_0=1.26\times10^{-4}$ s^{-1} turned out to be sufficient to correspond to the mean value of f between 54°N and 66°N. In order to further elucidate the influence of earth rotation on the seiches in Baltic Sea case studies with distinct values of f between $f=0$ and $f=1.26\times10^{-4}$ s^{-1} have been performed. Furthermore, we display also cases for a Baltic closed at the Aland Sea. The forcing function for oscillations of the entire Baltic was chosen to be

$$F(x,y)=\cos\left[\frac{\pi\sqrt{0.5}}{L}(y-x+42.5)\right]+\cos\left[\frac{2\pi\sqrt{0.5}}{L}(y-x+42.5)\right], \quad (16)$$

where L = 1 320 km. This represents a variation of the forcing function in North-South direction only with a

Table 2
Periods of oscillations in Rao's rectangular basin.

Order	Period according to present model (hours)	Period by Rao (1965) (hours)
(1.0)	12.87	12.83
(2.0)	6.58	6.56
(0.1)	4.65	4.64
(1.1)	3.81	3.79
(2.1)	3.65	3.67
(3.0)	3.11	3.09
(3.1)	3.04	3.03
(0.2)	2.59	2.58
(1.2)	2.54	2.54
(4.0)	2.40	2.41

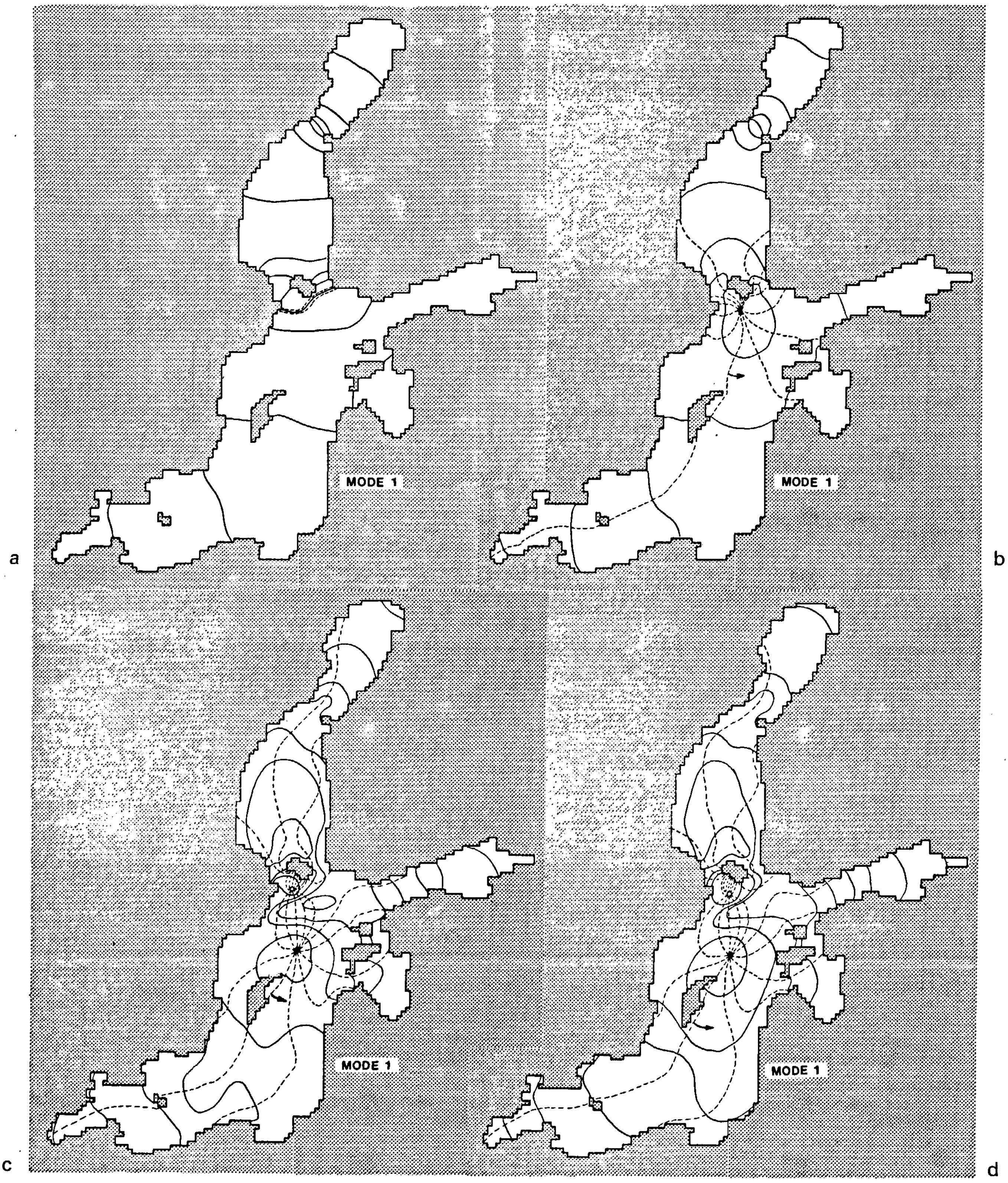


Figure 6

The influence of earth rotation on the first normal mode of the entire Baltic Sea: (a), the non-rotating case: $\tau = 40.55$ hours; (b), the slowly rotating case $f = 0.6 \times 10^{-4}\ s^{-1}$: $\tau = 37.58$ hours; (c), the first rotating normal mode with $f = 1.26 \times 10^{-4}\ s^{-1}$ and (d), f variable: $\tau = 31.03$ hours. Co-tidal line $\theta = 0°$ is indicated by an arrow. Distance of co-range lines is 2.5 cm, of co-tidal lines 30°.

nodal line due to the first term at 58°N. The simple function creates all modes desired due to the complicated configuration and bathymetry of the Baltic. Other test functions have been used without yielding more information.

For oscillations of the system Western Baltic-Gulf of Finland the forcing function was

$$F(x, y) = \cos\frac{\pi}{L}y + \cos\frac{4\pi}{L}y, \qquad L = 1\,650 \text{ km} \qquad (17)$$

which varies along a line running from SW-NE. The nodal line is at 57°30'N (Gotland).

The remaining parameters are: $\Delta t = 180$ s, $\Delta s = 7{,}07$ km, $T_0 = 600$, $\Delta t = 30$ hours. The computations have been performed for 1 024 hours, and the hourly values of ζ have been Fourier analysed at each of the 3 570 ζ-points.

To determine the location of each peak in the spectrum more accurately than given by the resolution 9.7656×10^{-4} h^{-1}, we supplemented the time series by zeros to 4096 data points. This yields a "spectral resolution" of 2.44×10^{-4} h^{-1} which allows to determine a period of 30 hours within $\pm$ 0.11 hour, of 20 hours within $\pm$ 0.05 hour and of 10 hours within $\pm$ 0.01 hour.

The determination of the eigenoscillations in the spectra was no problem because the amplitude peaks are well pronounced and the phase changes by 180°.

The spectral coefficients, i. e. the amplitudes A_n and the phases θ_n according to (7), have been plotted as co-range (full lines) and co-tidal lines (dashed lines). The distance between the co-range lines is 2,5 cm, between co-range lines 30°. The reference phase 0° has been arbitrarily chosen to run through the western most point at Fehmarn. Zero-phase is indicated by an arrow which points into the direction of rotation. In case of $f = 0$ the nodal lines which correspond to a phase jump of 180°, are indicated by a dashed line at each side of a full line.

Based on the channel theory the common notation of the seiches of the Baltic Sea refers to the systems Western Baltic-Gulf of Finland and Western Baltic-Gulf of Bothnia. The modes are ordered according to the periods of each separate system.

In the present study we consider the entire Baltic Sea and, therefore, denote the modes according to the entire system.

THE FIRST MODE OF THE BALTIC SEA

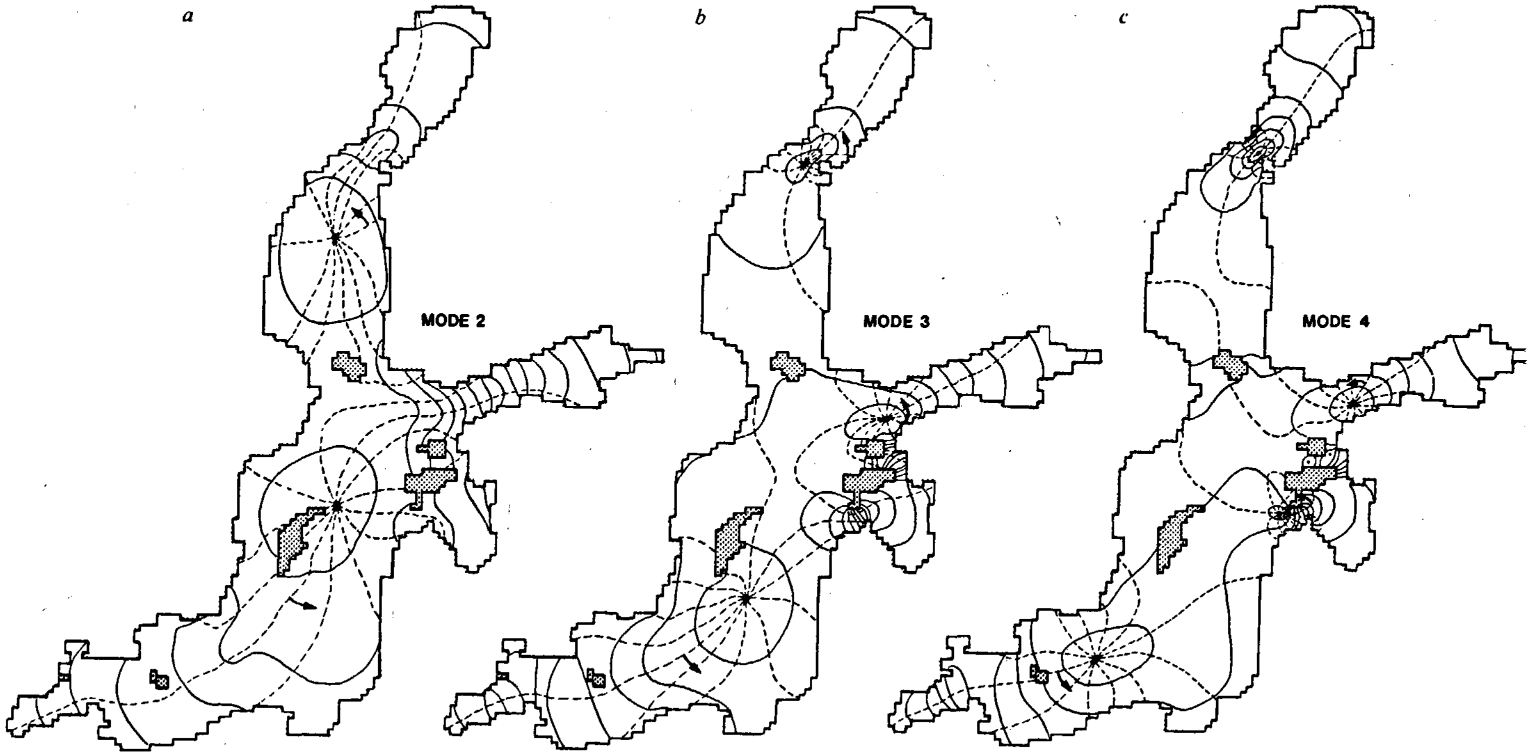
The present study yields considerable modifications with respect to the basic mode of the Baltic Sea due to the rotations of the earth. From channel theory the 1st mode of the system Western Baltic-Gulf of Bothnia (where the Gulf of Finland is supposed to be closed) has a period of 39.4 hours. The results of the two-dimensional model which includes the Gulf of Finland gives a basic mode in the non-rotating case as shown by Figure 6 *a*. The structure is the same as in the channel model; the nodal line is just south of the Aland Islands, the period is 40.55 hours. The sea level height in the Gulf of Finland is less than 5 cm, otherwise the results agree with those of the channel theory (the height in the Gulf of Bothnia is slightly reduced due to the water which enters the Gulf of Finland).

The effect of earth rotation on this oscillation is best shown by increasing the Coriolis parameter from zero to the real values. The computations have been performed for 5 cases: $f = 0.3 \times 10^{-4}$ s^{-1}, $f = 0.6 \times 10^{-4}$ s^{-1}, $f = 0.9 \times 10^{-4}$ s^{-1}, $f = 1.26 \times 10^{-4}$ s^{-1} and f variable according to the real distribution. We display only three figures out of this series which serve to interpret the influence of the earth's rotation.

For $f = 0.6 \times 10^{-4}$ d^{-1} (Fig. 6 *b*) the basic structure of the first non-rotating mode is still retained with respect to the co-range lines. The nodal line South of the Aland Islands is transformed into an amphidromic point and the wave rotates in anticlockwise direction around this area. Consequently, water must enter the Gulf of Finland. According to this the sea level rises to more than 5 cm in the interior of this Gulf. This amount of water (which is lost in the longitudinal oscillation) influences the entire oscillation in such a way as to reduce the period to 37.58 hours. A further increase of the Coriolis parameter to 0.9×10^{-4} s^{-1} rises the sea level in the interior of the Gulf of Finland to more than 15 cm and reduces the period of the entire oscillation to 34.42 hours. At this stage (not shown in Fig. 6) three amphidromic systems occur: a first one between Gotland and the Aland Island and the second one in the Aland deep. They rotate counterclockwise. The third system is centered between these two and rotates clockwise. If the Coriolis parameter is further increased to its mean value of 1.26×10^{-4} s^{-1}, the northern amphidromic system moves further towards north (disappearing at the Aland Islands) and the southern one travels further towards Gotland. The clockwise system remains in the Aland Sea. The period is reduced to 31.03 hours. This final result for f = Const. together with that for variable f is shown in Figures 6 *c* and 6 *d*. The major amphidromic system (north of Gotland) is the center of an anticlockwise rotation which steers the wave in the Baltic proper and the Gulf of Finland. The minor one is clockwise and guides the wave out of the Gulf of Bothnia into the Baltic proper. The sea level at the end of both Gulfs varies by about $\pm$ 20 cm. Thus, the basic mode of the system Western Baltic-Gulf of Bothnia as obtained from channel models on a non-rotating earth cannot exist under real conditions. In the two-dimensional case we obtain a period of 31.03 hours for the basic mode.

It is obvious from Figures 6 *c* and 6 *d* that the narrow and shallow parts of the Aland Sea play a crucial role for this mode. Frictional effects and the detailed bottom topography of this area may have further influence on this oscillation. We assume that the real period is slightly prolongated by these effects to values between 31 and 33 hours.

a b c



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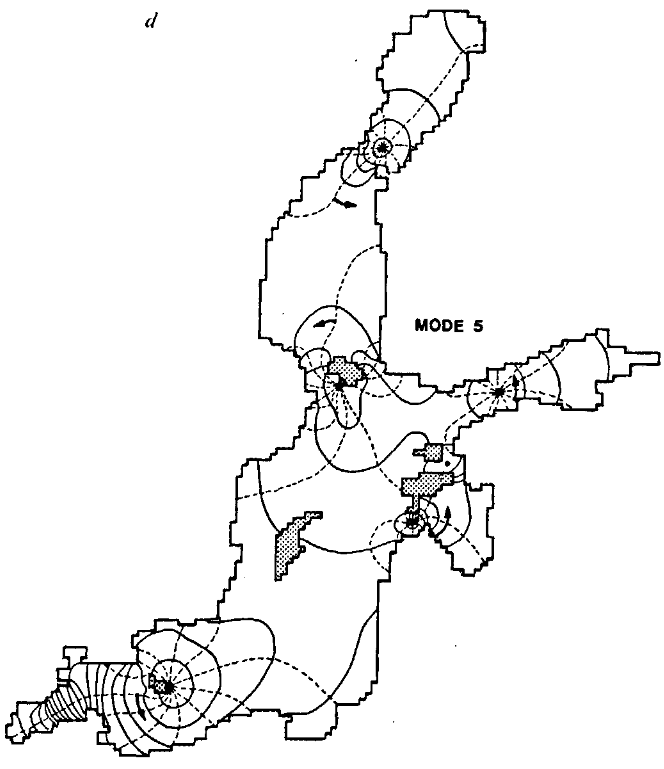


Figure 7 *a-d*

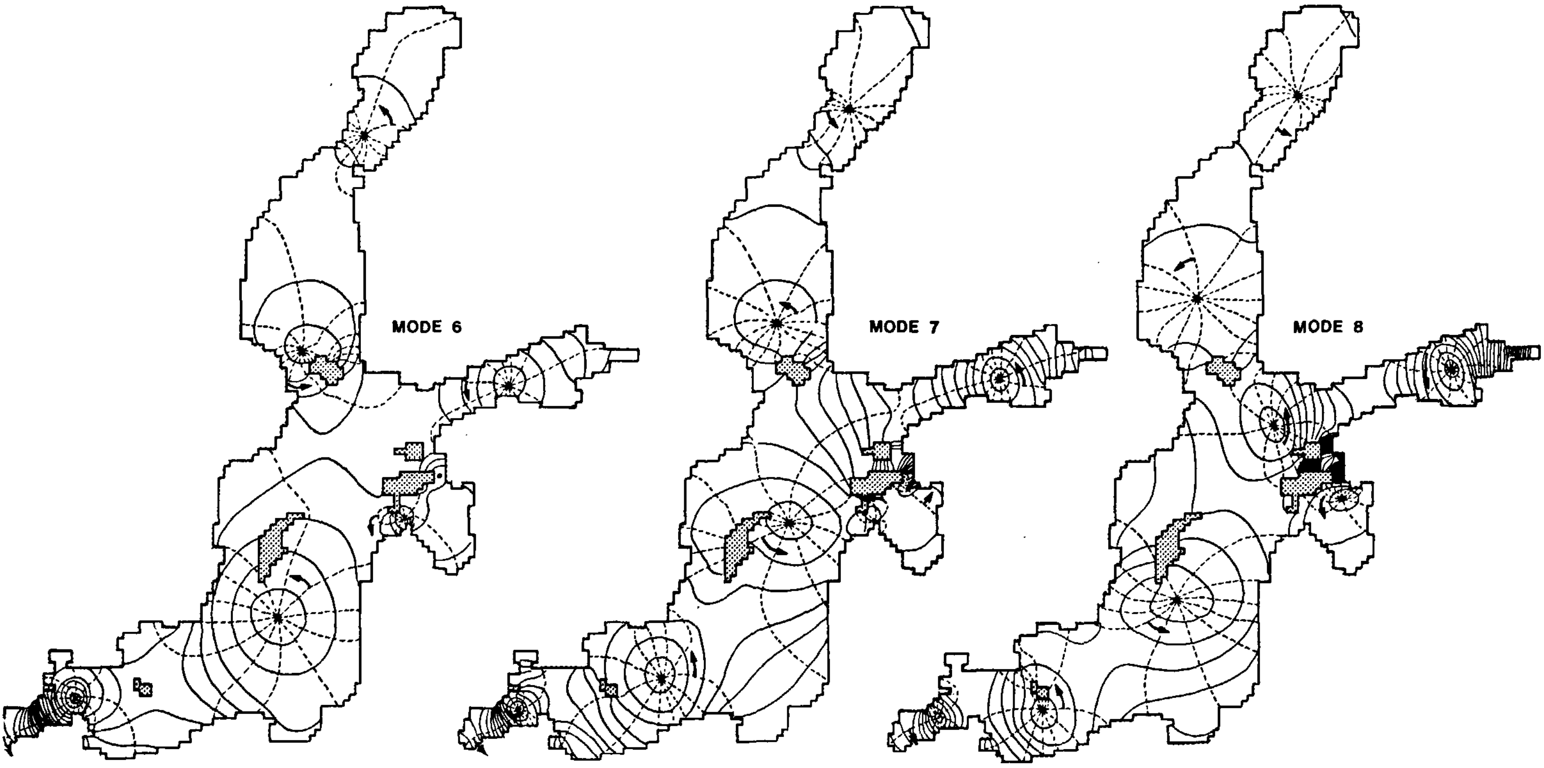
The second, third, fourth and fifth normal modes for the entire Baltic Sea (7 a: f variable, 7 b-7 d: $f=1.26\times10^{-4}$ s^{-1}). The periods are 26.42, 22.38, 19.79 *and* 17.07 *hours. Co-tidal lines $\theta=0°$ are indicated by arrows. Distance of co-range lines is 2.5 cm, of co-tidal lines 30°.*

THE SECOND TO TENTH MODE OF THE BALTIC SEA

Similar calculations with and without earth rotation have been carried out for the higher order modes. In Figures 7 *a*-7 *i* we display only the final results for these modes with f variable for the second and $f_0=1.26\times10^{-4}$ s^{-1} for the higher modes. Computations have shown that the variability of the Coriolis parameter does neither influence the period nor the structure of these gravitational modes.

The second mode (Fig. 7 *a*) consists of two amphidromic systems located north of Gotland and in the southern part of the Gulf of Bothnia, respectively. The period of this oscillation is 26.42 hours. This oscillation exhibits maximum sea level heights in the Gulf of Finland (35 cm) and only 15 cm in the Western Baltic. The maximum elevation in the Gulf of Bothnia is less than 15 cm. When high water occurs in the Western Baltic, low water occurs in the Gulf of Finland (with slight delay) and high water in the interior of the Gulf of Bothnia (preceeding the high water in the Western Baltic by 60°-90°). It is obvious that this oscillation resembles the main features of the first mode obtained by channel theory for the system Western Baltic-Gulf of Finland. The present results confirm the observations that the amplitudes of this oscillation have minimum values along the line Landsort-Libau and that the phase difference between Helsinki and Tallin is about 30° (Neumann, 1941). Furthermore, the period of 26.42 hours fits well into the observational results obtained by Laska (1969) and Lisitzin (1974). They obtained 26.6 and 26.2 hours, respectively, from tide gauge analysis.

For the third mode (Fig. 7 *b*) we obtain a period of 22.38 hours with three amphidromic systems southeast of Gotland, at the Kvarks and at the entrance to the Gulf of Finland. Sea level variations reach maximum values in the Western Baltic and in the Gulf of Finland and only half of these values at the end of the Gulf of Bothnia. Similar to the first and second mode the sea level variations in the Western Baltic



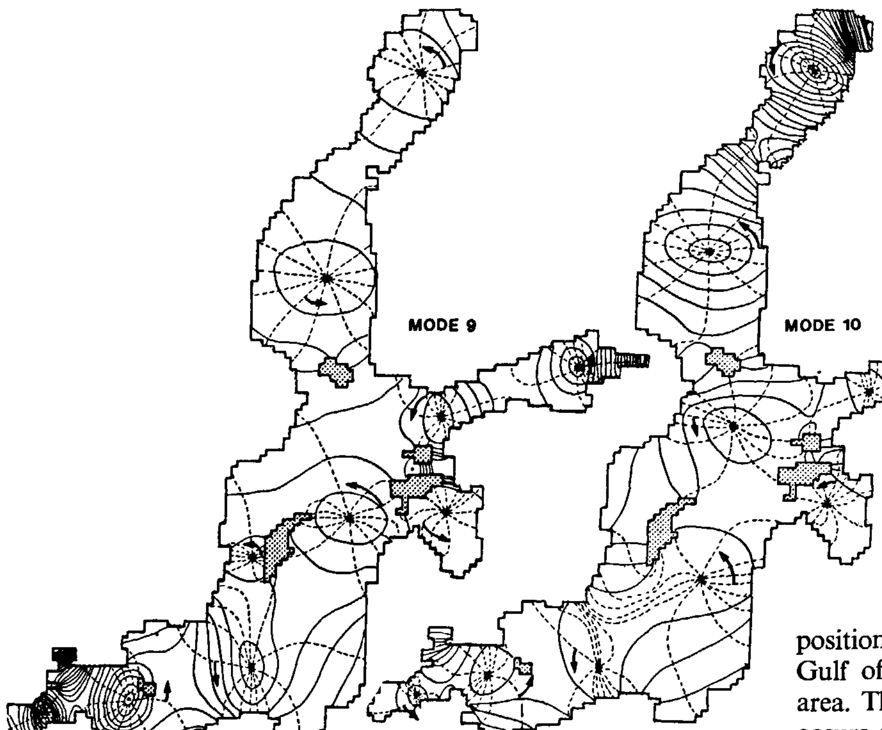


Figure 7 *e-h*

The sixth, seventh, eighth and ninth normal modes for the entire Baltic Sea with $f = 1.26 \times 10^{-4}\ s^{-1}$. *The periods are* 13.04, 10.45, 8.75, *and* 7.82 *hours. Co-range and co-tidal lines as in the previous figures.*

Figure 7 *i*

The tenth normal mode for the entire Baltic Sea with $f = 1.26 \times 10^{-4}\ s^{-1}$. *The period is* 7.31 *hours. Co-range and co-tidal lines as in the previous figures.*

and in the interior areas of the Gulfs are approximately in phase, i. e. they behave like standing waves in these areas. The same holds for the Gulf of Riga which exhibits the highest amplitudes. This seems to be due to a cooscillating mode of this basin which opens through two major channels towards north and west. A similar effect has been reported by Platzman (1972) for the Gulf of Mexico on a different scale. The 4th and 5th mode of the Baltic have very much in common (Fig. 7 *c*, *d*). Their periods are very close, 19.79 and 17.07 hours and the amphidromic points are located at similar positions: the Bornholm basin, the entrance to the Gulf of Riga, the Gulf of Finland and the Kvarken area. The fifth amphidromic system for the 5th mode occurs near Aland island. The distribution of the 4th mode is supported by Neumann (1941) who analysed records of Ystad and Primorsk which showed oscillations of 19.5 hours and no phase difference.

The higher modes (6-10) are mainly of local interest. It is unlikely that large scale forcing functions produce these systems with considerable amplitudes simultaneously in the entire Baltic. Local forcing by wind or air pressure, however, especially in the Gulfs and in the Western Baltic, could result in oscillations of this type. A wind, for instance, in the Western Baltic which piles up water in that area and then ceases could produce the amphidromic systems in the Western Baltic resembled by the 6th-9th mode. Similarly, the 10th mode seems to be of considerable importance for the Gulf of Bothnia, whereas the 7th-9th mode are important for the Gulf of Finland. The respective periods of these modes are 13.04, 10.45, 8.75, 7.82 and 7.31 hours. The structure of these modes is shown in Figures 7 *e*-7 *i*.

COMPARISON WITH THE RESULTS FROM CHANNEL MODELS

From the preceding results it is clear that a satisfactory treatment of the seiches of the Baltic Sea requires a two-dimensional consideration. The two systems Western Baltic-Gulf of Finland and Western Baltic-Gulf of Bothnia are crude approximations only. The influence of earth rotation is strongest in the wide area of the Baltic proper and, due to bottom topography, the Gulf of Finland takes part in all oscillations. There is, however, some preference for oscillations in which one of the Gulfs plays the dominant role. It is in these cases where the one-dimensional seiches theory gives remarkably good results, especially with respect to period. The results of the preceding section are summarized below. Periods of the 10 lowest modes of oscillations are given in Table 3, along with those obtained by channel theory. The first three columns refer to two-dimensional theory and compare the periods of a non-rotating Baltic with the rotational case. Examination of these periods indicate that the lowest given modes decrease in period due to earth rotation and the other five modes increase in period. The separation is at the inertial period which is 13.9 hours for $f_0 = 1.26 \times 10^{-4}$. The effect is more pronounced for long periods. The remaining four columns summarize the results from channel theory according to Krauss and Magaard (1962). The modes of channel theory appear in those lines where the period and the longitudinal distribution of ζ best fits into the two-dimensional oscillations. As mentioned already the basic mode (39.4 hours) of the one-dimensional system Western Baltic-Gulf of Bothnia cannot exist. The higher modes, however, fit reasonably well into the picture of the two-dimensional modes.

Table 3
Periods of the lowest 10 modes of the Baltic Sea as compared to the results of the channel theory.

Two-dimensional theory Entire Baltic			Channel theory (Krauss, Magaard, 1962)			
			W. Baltic-Gulf of Finland		W. Baltic-Gulf of Bothnia	
	Periods (hours)					
Mode	$f=0$	$f \neq 0$	Mode	Period (hours)	Mode	Period (hours)
1	40.5	31.0	–	–	1	39.4
2	27.7	26.4	1	27.4	–	–
3	23.7	22.4	–	–	2	22.5
4	21.4	19.8	2	19.1	–	–
5	17.9	17.1	–	–	3	17.9
6	13.01	13.04	3	13.0	4	12.9
7	10.45	10.45	4	9.6	5	9.4
8	8.68	8.75	5	8.1	–	–
9	7.74	7.82	6	6.9	–	–
10	7.22	7.31	–	–	6	7.3

This does not only hold for the periods but also for the height distribution of these modes as compared to the structures of the two-dimensional oscillations averaged over the cross sections.

THE MODES OF THE BALTIC PROPER

The development of seiches of the entire Baltic Sea requires large-scale meteorological forcing. The Baltic extends over more than 11 latitudinal circles. It often happens that the forcing is limited either to the Baltic proper (with little influence on the Gulf of Bothnia) or the Gulf of Bothnia only. In these cases the Aland area with its thousands of islands may behave like a solid border and seiches may develop which are nearly completely determined by the system Western Baltic-Gulf of Finland or by the configuration of the Gulf of Bothnia. We confine on the Baltic proper.

The channel theory for the Baltic proper is insufficient in so far as (i) the Coriolis effects are largest in the open Baltic and (ii) the Gulf of Riga is not taken into account. The area is about half that of the Gulf of Finland. The strong influence of this bay on the seiches of the Baltic become evident already from Figure 7.

It, therefore, seems worthwhile to consider the two-dimensional seiches of the Baltic proper as an independent phenomena. Considerable deviations from the channel theory are to be expected due to the deficiencies mentioned above.

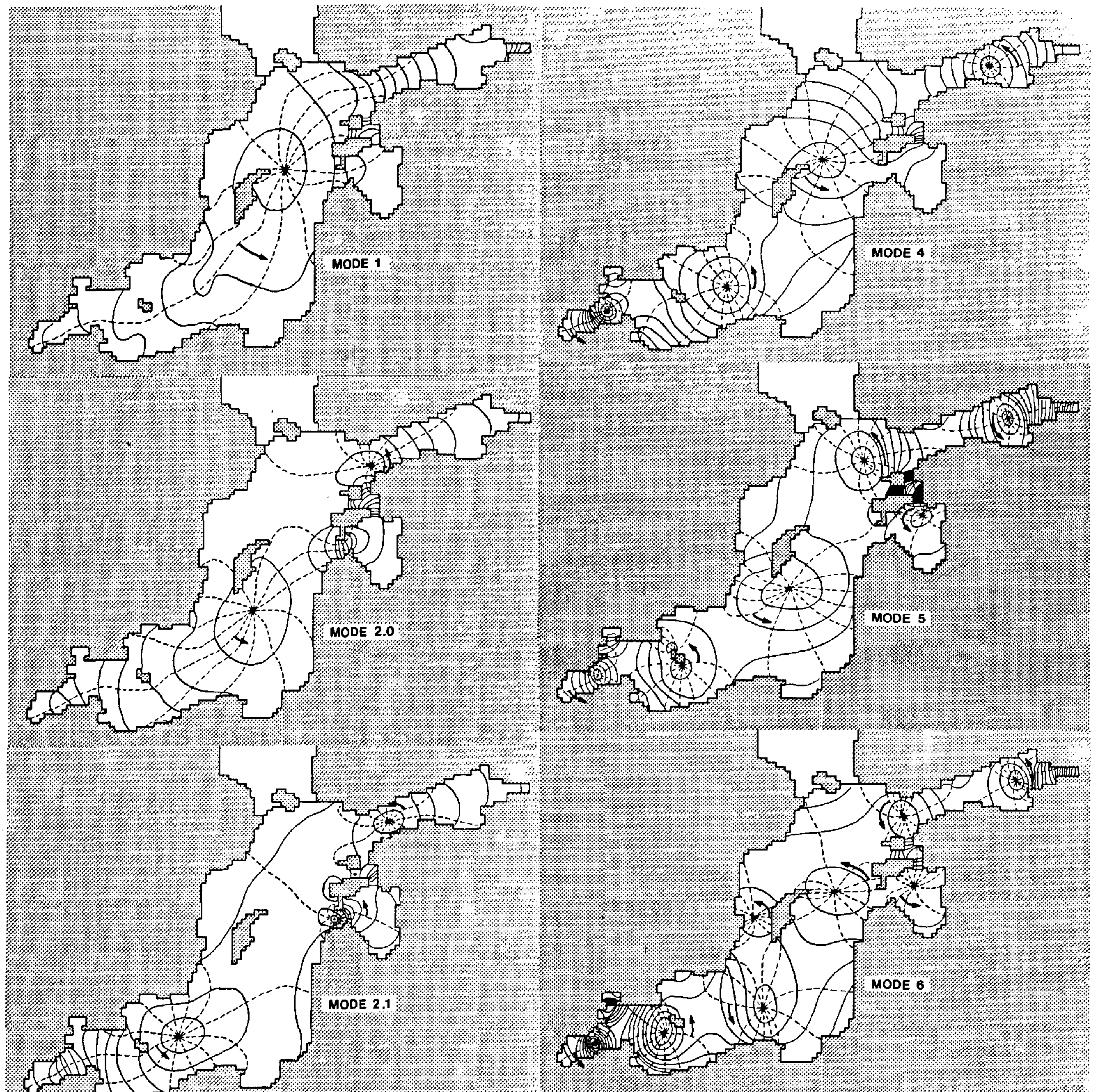
The results are displayed in Figures 8 *a*-8 *g* for the seven gravest modes. The periods are listed in Table 4 as compared to the seiches of the entire Baltic Sea.

From this it becomes evident that the periods of the oscillations of the Baltic proper are very close to those of corresponding modes of the entire Baltic. The correspondence is shown in Table 4. It becomes even more obvious if one compares Figures 8 *a*-8 *c* with 7 *a*-7 *c* and 8 *d*-8 *g* with 7 *e*-7 *h*. The structures are nearly identical.

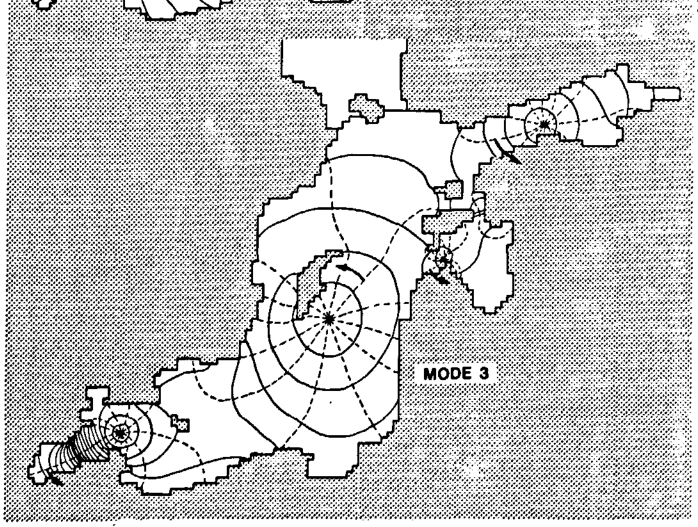
Some remarks are necessary with respect to the second mode of the Baltic proper (Fig. 8 *b*, 8 *c*). According to our computations there exist two second order modes depending on the Gulf of Riga. The notation "second order mode" refers to the two amphidromic systems in the open Baltic Sea. To be more distinct, we are dealing with modes 2.0 and mode 2.1, referred to the longitudinal axis of the Baltic. Mode 2.0 is the second order longitudinal one, mode 2.1 contains a cross oscillation Gulf of Riga-Baltic proper. Compared to a second order longitudinal oscillation on a nonrotating earth,

Figure 8 *a-d* ▶
The first, second and third normal modes for the Baltic proper (closed at the Aland Islands). $f = 1.26 \times 10^{-4}\ s^{-1}$. *The periods are 27.68, 23.81, 18.62 and 13.43 hours. Co-range and co-tidal lines as in the previous figures.*

Figure 8 *e-g* ▶
The fourth, fifth and sixth normal modes for the Baltic proper (closed at the Alands Island). $f = 1.26 \times 10^{-4}\ s^{-1}$. *The periods are 10.72, 8.83, 7.88 hours. Co-range and co-tidal lines as in the previous figures.*



a b c e f g



d

Table 4
Periods of the seiches of the Baltic proper as compared to the entire Baltic and the channel theory.

Entire Baltic		Baltic proper			
Mode	Period (hours)	Mode	Period (hours)	Channel theory	
1	31.0	–	–	–	–
2	26.4	1	27.7	1	27.4
3	22.4	2 *a*	23.8	2	19.1
4	19.8	2 *b*	18.6	–	–
5	17.1	–	–	–	–
6	13.0	3	13.4	3	13.0
7	10.0	4	10.7	4	9.6
8	8.8	5	8.8	–	8.1
9	7.8	6	7.9	–	6.9

where the oscillations in the Western Baltic and in the Gulf of Finland lacks behind by about 75° in case of mode 2.0 and leads by 15° in case 2.1. Corres-mode 2.0 and leads by 15° degrees in case 2.1. Correspondingly, the oscillation in the Gulf of Riga is either in phase with the water in the area north of Gotland (mode 2.0) or out of phase by 180° (mode 2.1). The latter requires the amphidromic system at the western entrance to the Gulf. The periods of these two oscillations differ by more than 5 hours and the amphidromic systems in the open Baltic Sea are more separated in the latter case. Both modes are characterized by high amplitudes in the Gulf of Riga.

CONCLUSIONS

It is evident from tide gauge records all along the Baltic coast that eigenoscillations play a major role in the Baltic Sea. On the other hand, spectral analysis does not show any preference for spectral peaks. A wide variety of periods seems to be possible.

The present study on the two-dimensional seiches sheds some light into this problem. Depending on the meteorological conditions which create the seiches, either the entire Baltic (including the Gulf of Bothnia) or the Baltic proper may be regarded as the oscillating system. Then, the total of periods which is possible in the Baltic proper due to the first ten modes, is as follows: 31.0, 27.7, 26.4, 23.8, 22.4, 19.8, 18.6, 17.1, 13.4, 13.0, 10.7, 10.0, 8.8, 7.9. Spectral analysis based on long records will smear out the energy of these damped oscillations over the entire period range considered. We can discriminate between these periods only by studying distinct cases. But even then the real periods may be masked by superposition of several eigenfunctions.

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