The transit time distribution for the waters along the Netherlands coast

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ABSTRACT
Salinity observations made between 1964 and 1969 have been used, together with run-off data for the Rhine and Meuse, to estimate the transit time distribution for the waters along the Netherlands coast. The results do fit well with other estimates. Some possibilities for further improvement are indicated.


INTRODUCTION
The concept “turn-over time” of the water in a sea area is often used in studies concerning the pollution or, more general, the chemistry or such an area (assuming a similar behaviour of the water and of the substances dissolved in it). It is defined as

\[ \tau_0 = \frac{M_0}{F_0} \]  

(Bolin and Rodhe, 1973).

Where \( M_0 \) is the total mass of the water in the sea area (or, more general, of the reservoir) and \( F_0 \) is the total flux of mass leaving per unit time. It is often used as an equivalent to the “residence time” or “average transit time” being the mean time the water particles have been in the sea area at the moment they are leaving it (as the name residence time already suggests), but this equivalence is, as was shown by Bolin and Rodhe (1973) only valid under steady state conditions, when the total mass, the fluxes and the internal processes within the reservoir do not vary with time. This is usually not the case in the sea, but often it is tacitly assumed that the variations are small and that the mean values of \( M_0 \) and \( F_0 \) can be used.

It should be mentioned here that the term residence time is used in a different sense by Eriksson (1971),...
where it is equivalent to the "average age" of the particles in a reservoir.

It can be of value for studies where this residence time concept is used to apply some refinement, by not just taking the average transit time, but to consider the frequency distribution of transit times for the various water elements (or the "age" distribution of the elements leaving the reservoir). This study aims at a determination of such a frequency distribution for the sea area to the west of the Netherlands coast. We suppose that variations (on a month-to-month basis) of the mass within this area and of the sea water flux through this area are small enough to allow a steady-state approximation.

THE FREQUENCY DISTRIBUTION FOR THE TRANSIT TIMES

Bolin and Rodhe define the function \( F(t) \) as the mass leaving the reservoir per unit time after it has spent a time \( \leq t \) in the reservoir. Per definition

\[
\lim_{t \to \infty} F(t) = F_0. \tag{2}
\]

The frequency distribution \( \varphi(t) \) for the transit times \( t \) is

\[
\varphi(t) = \frac{1}{F_0} \frac{dF(t)}{dt}. \tag{3}
\]

Where

\[
\int_0^\infty \varphi(\xi) d\xi = 1. \tag{4}
\]

The average transit time \( \tau \), is given by

\[
\tau = \int_0^\infty \tau \varphi(\tau) \, d\tau. \tag{5}
\]

If a conservative tracer is continuously added at the inflow of the reservoir, at a (variable) concentration \( Z(t) \), the concentration observed at the outflow, \( X(t) \), is, according to Eriksson (1971):

\[
X(t) = \int_0^\infty \varphi(\tau) Z(t-\tau) \, d\tau. \tag{6}
\]

So the regular observation of the addition of some tracer at the inflow and of its concentration at the outflow gives the possibility to estimate \( \varphi(\tau) \).

If the observations are made at discrete intervals \( \delta \) we may estimate the value \( \varphi_\tau \), the value of \( \varphi \) integrated over the intervals \( \tau - (1/2)\delta \) to \( \tau + (1/2)\delta \) from:

\[
X_\tau = \sum_{j=0}^{\infty} \varphi_j Z_{\tau-j}. \tag{7}
\]

Here \( X_\tau \) and \( Z_\tau \) are the mean concentrations over the interval centered around time \( \tau \).

HYDROGRAPHIC CONDITIONS IN THE SOUTHERN BIGHT OF THE NORTH SEA ALONG THE NETHERLANDS COAST

The hydrographic conditions in the Southern Bight are relatively well-studied. The average residual current from the Channel towards the Central North Sea and German Bight is manifest in the mean salinity distribution (e.g. ICES, 1962). However, considerable variations of this residual current do occur, largely under influence of the changing wind conditions, as for instance is illustrated by the results of current estimates from the Dover-Calais telephone cable and of the measurements at the Netherlands current meter station (53°24'N-03°58'E) presented in the Report of the ICES working group on permanent, moored current-meter stations in the North Sea (Ramster and Koltermann, 1976). The waters along the Netherlands coast have low salinities because of the admixture of fresh water derived from Scheldt, Meuse and Rhine, having mean discharges according to Grindley (1972) of respectively about 10, 10 and 65 km³/year (or about 300, 300 and 2000 m³/s). The mixture of river water and sea water, moving in northerly direction, is concentrated along the coast: in a section near IJmuiden 88% of the river discharge lies within 35 km off the coast (van Bennekom, Gieskes, Tys...
On the basis of just salinity data we cannot discriminate between the different fresh water sources. However, using optical properties, especially "natural fluorescence", we find that the fresh water derived from the Scheldt is mainly found further offshore (Otto, 1967) and we may assume that the 88% of the river discharge mentioned above cover nearly the whole discharge of Rhine and Meuse. The waters of Rhine and Meuse mix already to a large extent in the estuaries and these two rivers can be considered as one single source of fresh water.

In 1970, after the observations discussed in the next chapter were finished, the Haringvliet, one of the principal estuarine branches, was closed, which has influenced the regime of fresh-water discharge to some extent.

**OBSERVATIONS AT “TEXEL” SECTION**

From 1964 till 1969 a programme of frequent salinity and temperature observations was carried out at the “Texel” section, a line stretching from Den Helder in a WNW-ly direction along the Texel lightvessel up to about 35 km from the coast (Fig. 1). At 7 stations along this line samples were taken at the surface, at 10 and at 20 m depth (bottom depth between 25 and 30 m). A preliminary report on the results of these observations has been given by Visser (1971) and a final report (in Dutch) has recently been completed (Visser, 1977). The observations were usually made twice during a cruise, once westbound and once eastbound (although occasionally sampling took place only once), so tidal effects are cancelled out to some extent.

Although it was the intention to sample at monthly intervals this could not be accomplished in practice, because of available ship’s time, adverse weather conditions, etc. The observations are distributed as follows:

- December, January, February: 7 cruises
- March, April, May: 11 cruises
- June, July, August: 14 cruises
- September, October, November: 12 cruises.

So there is a bias against the winter, mainly because of the higher frequency of gales.

Figure 2 (from Visser, 1977) gives some examples of the density (sigma-t) distribution along the section, under various conditions (unstratified and stratified).

The observations permit an estimate of the mean of the concentration of fresh water in the sea water over the section if we know the salinity of "undiluted" sea water. These variable concentrations of the "tracer" being here fresh water, together with available data of the river run-off can be used in the way as explained p. 32 to estimate the transit time frequency distribution for otherwise steady-state conditions (Fig. 3).
SCHEMATIZATION

The coastal waters are considered as a reservoir, at the eastern side bounded by the coastline, in the north by the "Texel" section, in the south by a section running from the island of Goeree in a west-north-westerly direction, whereas at present we cannot indicate a definite western boundary, but only can say that it runs at a distance preliminarily taken to be 35 km (the length of the Texel section) from the coast. The influx passes through the southern boundary and the outflux through the northern, so the western boundary must be along a streamline of the mean residual current pattern. The general character of the residual circulation is illustrated by Figure 4 (from Otto, 1970). Although, according to recent information, this figure is subject to some modifications, the general pattern can be considered to be established. The (quasi) turbulent exchange through the boundaries is neglected here.

The tracer added at the southern boundary is the river water from Rhine and Meuse. The Scheldt is left out of consideration: its contribution to the fresh water content in the southern bight is small, and remains largely outside the reservoir defined above (see p. 33). The run-off of Rhine and Meuse is measured at Lobith and Lith respectively, at distances of roughly 150 and 100 km from the sea. We cannot use these run-off figures directly, but have to consider what happens between the gauging stations and the sea.

Also we have to consider the exchange with the Wadden Sea, which causes some "leakage" around the Texel section.

These points are further explained below.

Information on the fate of the Rhine waters between Lobith and the sea is given in a report by Rijkswaterstaat (Anonymous, 1970). About 10% of the Rhine discharge goes to the Yssel Lake, the other 90% discharge via the estuaries in the south-western part of the Netherlands. It is estimated that in one or two days the run-off reaches the estuarine region.

From there the fresh water moves through the, rather complicated, network of estuaries to the sea.

At the time the data were taken, that is before the closing of the Haringvliet estuary, roughly 50% followed a northern route, and entered the sea by the Rotterdam Waterway at the Hook of Holland. The other 50% joined the Meuse run-off and entered the sea via the Haringvliet. The mean turn-over time of the water in the northern estuary system is estimated, from the quotient of fresh water volume and river discharge, to be of the order of some days, the mean turn-over time of the southern system is estimated to be of the order of 10 days. These rough figures show that we have to consider a time delay between the observations of run-off and the addition of the "tracer" fresh water in the sea area along the coast of the order of one or two weeks. Furthermore...
short-period fluctuations in the run-off will be largely damped-out in the estuarine region. The variations of the monthly means are considered to be long-periodic in comparison with the time constant of the estuaries. However, in a study like this a better understanding of the estuarine processes might result in some corrections in the computations undertaken here.

Another point to be considered in this schematization is the complication due to possible water exchange with the Wadden Sea either Marsdiep.

Under certain assumptions van Bennekom et al. (1974) estimated the percentage of Rhine water in the Marsdiep at about 15%. Zimmerman and Rommets (1974) estimated on the basis of salinity-fluorescence relationships this percentage in January 1973 at 5.5%. This water, by advective or diffusive processes, penetrates into the Wadden Sea. On the basis of recent experiences van Bennekom (personal communication) estimates that about 14% of the Rhine water leaves the Wadden Sea. This water and the fresh water discharged from the Yssel Lake via sluices partly leaves the Wadden Sea through the more eastern inlets, and partly leaves through the Marsdiep again after a certain residence time (Zimmerman, 1976).

A correction for the Rhine water entering the Wadden Sea can be applied by assuming that we do not use the tracer "fresh water", but "fresh water, not having the Wadden Sea as its destination", or in other words, by reducing the concentration at the entrance by 14%.

This does not affect the transit time frequency distribution, but only the resulting estimate of the total flux of sea water, \( F_o \). Of course, we assume here that the chance to reach the Wadden Sea is the same (0.14) for all water elements leaving the estuaries.

The fresh water leaving the Wadden Sea via the Marsdiep, especially variations in this exchange, may give complications if it contributes significantly to the fresh water content of the Texel section. Here it is assumed that most of this water remains close to the shore and shallows, that is between the easternmost station of the Texel section and the coast. There is some justification for this assumption in the water-mass analysis by Zimmerman and Rommets (1974) where we find a rapid decrease of the Yssel Lake water in the Marsdiep area from 10 to 2.5%.

In a similar way we may have to reduce the quantity of the tracer added because of the movement of riverwater on the west of the Texel-section. In this study we assume no leakage at the western end (see p. 34).

COMPUTATIONS

The following computations are made on a monthly basis. As will follow from our results a shorter time interval would have been better, but our observations are not frequent enough.

Using equation (7) estimates are made of \( \varphi_1 \). We introduce for \( X_1 \), the concentration of fresh water in the Texel section estimated as explained on p. 34 and using as a preliminary reference salinity 35%o (later to be adjusted). The observations were arranged in monthly groups, each group containing the observations made between the 14th of one month and the 14th of the succeeding month (thereby taking into account the mean delay between the observation of the run-off and the introduction of the fresh water in the coastal waters). In most cases only one vice versa series of observations in the section was made for a monthly group, only ten monthly groups contain two of such series. This means that by using the average values of the fresh water concentration as a monthly mean we certainly introduce an important error. Only by taking this mean over a much longer series of observations during one monthly period we may assume to have a good estimate of the mean monthly fresh water concentration. However, it is assumed that the error is accidental and will not significantly influence the results of the computations.

For \( Z_1 \) we use the sums of the monthly mean discharges of Rhine and Meuse, reduced with a factor 3/4, because of the flow to the Yssel Lake and the "leakage" east of the Texel section. These values have to be divided by the mean flow \( F_\infty \) of sea water entering at the southern boundary of our reservoir. This is allowed, as long as \( F_\infty \) is much larger than the run-off (fresh water concentrations are given by the quotient of run-off and \( F_\infty \) plus run-off). Because of the time-delay in the seaward sections of the rivers and the estuarine region, the time \( \tau \) is taken as the end of each monthly period for which the discharge is given, whereas half the interval width \( \delta \) is half a month. The discharge data were provided by "Rijkswaterstaat". They are given in cubic meter per second, with sufficient accuracy this is equivalent to \( 10^3 \) kg/s.

In this way 34 data-sets could be formed, from which by multiple regression technique the value of \( \varphi \) can be estimated, using equation (7). However, there is a problem in the length of the series of monthly periods \( j=0, 1, \ldots, k \) that has to be taken into account. In principle this number should go to infinity, but because of the limited number of data-sets and the
inherent periodicity in the run-off, we may obtain unrealistic results. Therefore we have made the analysis for 4, 5 and 6 monthly periods. The results were most satisfactory for 4 periods \((k=3)\); for \(k=4\) the correlation became worse, whereas for \(k=5\) negative correlation between fresh water concentration and run-off 6 months earlier clearly was the result of the normal seasonal periodicity and did not give useful data for the analysis. A modified equation (7) that was used (taking volume fluxes in stead of mass fluxes for \(F\) and \(R\)) is

\[
X_i = X_0 + 1/F_x \\
\times \{ \varphi_0 R_i + \varphi_1 R_{i-1} + \varphi_2 R_{i-2} + \varphi_3 R_{i-3} \} \quad (8)
\]

Here \(Z_j = R_j/F_x\), where \(R_j\) is the (reduced) run-off, while furthermore an extra term \(X_0\) is introduced, which takes into account a correction which should be applied if the salinity of the sea water coming from the south and mixing with the fresh water is different from \(35\%\). The results of the analysis are given in Table 1.

In analogy with (4) we may write

\[
\varphi_0 + \varphi_1 + \varphi_2 + \varphi_3 = 1,
\]

so

\[
10.8 \times 10^{-6} F_0 = 1,
\]

or

\[
F_0 = (\text{as volume flux}) \ 92 \times 10^3 \text{ m}^3/\text{s},
\]

or

\[
2900 \text{ km}^3/\text{year}.
\]

This figure depends, however, on the magnitude of the corrections for "leakage" east (via the Wadden Sea) and possibly west of the Texel section.

If no tracer is added,

\[
X_i = X_0 = \frac{35 - S_R}{35} = 0.0156,
\]

where \(S_R\) is the salinity (in per mile) of sea water available for mixing with the run-off, being lower than the preliminary reference salinity of \(35\%\).

We find

\[
S_R = 34.45\%.
\]

The above figures for \(F_0\) and \(S_R\) appear to be of the right order of magnitude, although \(F_0\) is somewhat on the high side, which could mean that "leakage" is underestimated:

from the chart of the residual transport pattern (Fig. 4) drawn-up by the author (Otto, 1970) on the basis of current observations a residual transport of between 60 and \(70 \times 10^3 \text{ m}^3/\text{s}\) is estimated for the Texel section and in the Belgian programme for the modelling of the Southern Bight a value of about \(50 \times 10^3 \text{ m}^3/\text{s}\) is usually applied for the flux passing compartment "1 N", largely coinciding with the reservoir used in our schematization (JO PODAMO, 1974). At the westernmost point of the Texel section the mean salinity is about \(34.4\%\), which is in good agreement with our \(S_R\) value.

The mean transit time can be estimated from the values of \(\varphi\) on the basis of equation (5):

\[
\tau(\text{in months}) = \frac{1}{2} \times 0.63 + \frac{1}{2} \times 0.20
\]

\[
+ 2 \frac{1}{2} \times 0.10 + 3 \frac{1}{2} \times 0.07 = 1.11.
\]

This value is in agreement with estimates on the basis of residual transport (van Bennekom et al., 1974), giving 1 month, and on the basis of equation (1), applied to compartment "1 N" of the Belgian model, giving 0.8 month.

On the basis of equation (1) we may now estimate the dimensions of the reservoir, provisionally defined p. 34. For fluxes of \(92 \times 10^3\) and \(60 \times 10^3 \text{ m}^3/\text{s}\) we estimate the volume of the reservoir to be respectively 280 and 180 \(\text{km}^3\). At an average depth of 25 m and a distance between northern and southern boundary of 135 km this gives a western boundary at 83 and 53 km offshore. So we may conclude that the Texel section may have been somewhat too short for the purpose of this estimate.

Nevertheless, the values of \(\varphi\) will be not much influenced by this fact.

The shortest possible transit time may be estimated as the time it takes for a water element to traverse the reservoir if it only moves northward with the flood current, but remains at its position if the ebb (southerly) current flows.
This is found to be 5 or 6 days. The best guess for the transit time frequency distribution is therefore as given in Figure 5 (where the first segment of the histogram is somewhat distorted correspondingly if compared with the value of \( \phi \) in Table 1).

### Table 1

Results of multiple regression analysis of the relation between river discharge and fresh water concentration in Texel section [equation (8)].

\[
X_0 = 0.0156 \\
\phi_0/F_s = 6.7 \times 10^{-6} \text{ s/m}^3 \\
\phi_1/F_s = 2.2 \times 10^{-6} \text{ s/m}^3 \\
\phi_2/F_s = 1.1 \times 10^{-6} \text{ s/m}^3 \\
\phi_3/F_s = 0.8 \times 10^{-6} \text{ s/m}^3 \\
F_0 \text{ eliminated} \\
\phi_0 = 0.63 \\
\phi_1 = 0.20 \\
\phi_2 = 0.10 \\
\phi_3 = 0.07 \\
\text{Correlation coefficient} = 0.552. \\
n = 34. \\
95\% \text{ confidence level} = 0.521.
\]

### DISCUSSIONS

Some assumptions have been made in the above chapters in order to obtain these estimates. Most of them do not essentially influence the results. However, there may be a differential movement of the fresh water admixture, compared with the body of the water. This may invalidate the results for the reservoir as a whole, but they still remain applicable to the run-off water itself and the substances carried into the sea by the rivers.

There are several points where this estimate, on the basis of new data, could be improved.

The new regime of the river discharge in the North Sea, created by the closing of the Haringvliet by a system of sluices that regulate the discharge gives the possibility to obtain better estimates of the fresh water input. This, combined with a programme of regular and more frequent observations along a somewhat longer and more southerly section (in order to avoid problems of "leakage" and exchange with the Wadden Sea) might result in better figures than could be given here.

Of course the variability of the water circulation in the Southern Bight of the North Sea makes important deviations possible from the results given here. If one wants to take into account such variations, circulation models based upon actual wind information have to be applied.

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REFERENCES


