The generation of residual vorticity by the combined action of wind and bottom topography in a shallow sea

Southern North Sea
Residual currents
Vorticity
Windstress
Bottom topography

Mer du Nord, partie méridionale
Courants résiduels
Tourbillon
Tension du vent
Topographie du fond

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ABSTRACT

A study is presented on the generation of wind induced vorticity by topographic elements in the residual current field of a shallow, infinite sea. It appears that the vertically averaged equations of motion with linear bottom friction can not describe the generation of wind induced residual vorticity, unless the bottom friction parameter r is depth dependent. To obtain the depth dependence of r we study the three dimensional equations of motion. The resulting expression for the residual vorticity is linear in the wind stress and in the gradient of the bottom topography. The values found for the Southern Bight of the North Sea agree reasonably with results obtained during a current measuring program.

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RÉSUMÉ

Production de tourbillon résiduel dans une mer peu profonde sous l'action combinée du vent et de la topographie du fond

Une étude est présentée sur la production de tourbillons par le vent et les éléments topographiques dans le champ de courant résiduel d'une mer infinie et peu profonde. Il apparaît que les équations du mouvement moyennées sur la verticale, avec un frottement linéaire sur le fond, ne peuvent décrire la production de tourbillons résiduels par le vent si le paramètre de frottement r est indépendant de la profondeur. Pour tenir compte de cette dépendance, nous étudions les équations tridimensionnelles du mouvement. L'expression du tourbillon résiduel qui en résulte est une fonction linéaire de la tension du vent, et de la pente du fond. Les valeurs calculées dans la partie méridionale de la mer du Nord s'accordent raisonnablement avec celles obtenues à partir des mesures de courant.

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INTRODUCTION

In this paper theory and observations are presented on the vorticity in the residual current field in the Southern Bight of the North Sea. The Southern Bight is the shallow southern part of the North Sea with depths between 20 and 40 m. The area is dominated by semi-diurnal tidal currents (M_2) of the order of 50 to 70 cm/sec., depending on position. Because of the shallow depths and the relatively large tidal currents bottom turbulence prevents stratification except for some limited areas, where river outflow occurs. Over the Southern Bight, the wind exerts forces on the water that lead to complex residual water movements. These residual water movements are also influenced by nonlinear effects resulting from the tidal currents (Nihoul, Ronday, 1975) and by oceanic inflow. Depending on the wind force, meteorologically induced residual currents are of the order of 0-20 cm/sec., with average values between 5-10 cm/sec. Several types of bottom topography are present, ranging from flat bottom to sandwaves with a typical wavelength of 500-1 000 m and amplitudes of 3-10 m. On a large scale deepening towards the north is a dominant feature.

In recent years, several current measuring programs were undertaken, which were the result of national as well as international efforts. Two examples of the latter were the Joint North Sea Data Acquisition Programs Jonsdap 73 and Jonsdap 76. During Jonsdap 76 about 180 current meters were deployed over the whole North Sea basin. From these programs it became clear that at certain times and places in the Southern Bight appreciable spatial variability of residual currents could occur over horizontal distances of the order of 10 km. Residual currents in this paper are defined as averages over two M_2 tidal periods. In studying this spatial variability we found it useful to consider the vorticity of the two-dimensional residual current field. The importance of mesoscale eddies and their interaction with the large scale residuals has been emphasized by Nihoul (1980).

Zimmerman (1978) showed that in an area where tidal currents dominate residual vorticity is generated by the combined action of topography and tidal flow. An application of Zimmerman's theory is given in Komen and Riepma (1981) where they found that the tidal residual signal is rather weak and that wind induced residuals may often dominate.

In this paper we focus on the problem of wind induced residual vorticity. With a specially designed current meter array we obtained measurements which we used to determine the residuals vorticity. First these measurements will be discussed and it will be shown that there exists a good correlation between wind stress and residual vorticity. Then we study vertically averaged equations to see if this correlation can be explained. It turns out that the parameterization of the bottom friction is crucial. To obtain better insight into the problem we will study the three-dimensional equations next. Solutions of these equations are then presented first for a very shallow sea. To avoid the necessity of using numerical techniques we simplify to a shallow sea of infinite horizontal extent. We do not take account of tides explicitly, and study the *local generation* of vorticity only. This approach differs from earlier work on topographic effects by Weenink (1958), Birchfield (1972) and Bennet (1974), (see also Simons, 1980) in that we do not consider effects of coastal geometry. Our approach should be valid on intermediate timescales when the local response to wind forcing has become stationary, but when important surface slopes have not built up yet. We study several types of bottom friction parameterization in the three dimensional case. Later we extend our calculations to the general Ekman problem for a shallow sea with bottom topography. The resulting expression for the residual vorticity is linear in the wind stress and the bottom slope. The proportionality constants depend on the Coriolis force, on the depth and on the effective bottom friction. They are given in Figure 3. Then we compare theory and measurements. A problem arises because a typical value of the residual vorticity is 10^{-6} sec.$^{-1}$, on all scales. On small scales the related velocity shears cannot be determined because of instrumental errors, which are of the order of 1 cm/sec. On scales of 10 km, however, residual current shears becomes appreciable. We show that the observed vorticity can be understood as determined by a weighted average of the bottom slopes. We end with a summary of our conclusions.

MEASUREMENTS

During the period from 12 October until 30 November 1978 four current meter stations were deployed by KNMI in the Southern Bight of the North Sea on positions shown in the chart of Figure 1. On each rig one current meter was suspended 5 m above the bottom and another at a depth of about 10 m below the sea surface. The mean water depth was about 38 m. The self recording meters were of the type Plessey and NBA. They were programmed to sample current velocity and current direction every 10 minutes. By applying a running average over two M_2 tidal periods we calculated residual currents centered at 12.00 GMT every day for the entire period. Tidal residuals and density induced residuals are of course not removed by this averaging process. However, as stated before the tidal and density induced residuals are quite small, in comparison with the wind induced residual.

An estimate of the maximum error for individual values of the residual current is 20 % or 1 cm/sec. whichever is the greatest. The source of these errors is partly stochastic, partly systematic for a certain instrument. Between different instruments the error is always stochastic.

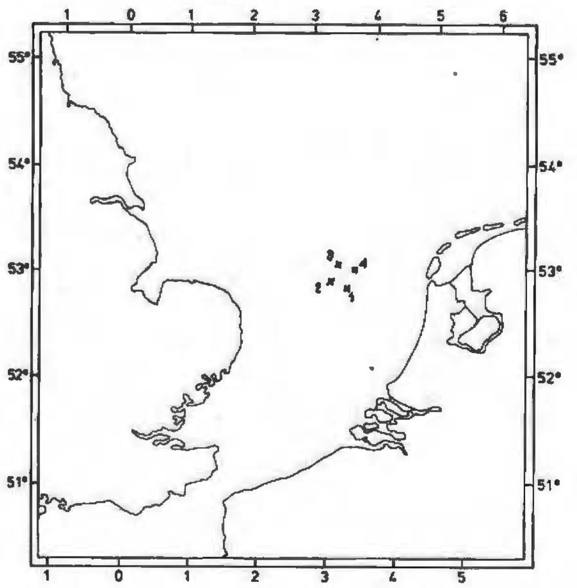

Figure 1
Position of the current meter station in the Southern Bight of the North Sea.

STREAM FUNCTIONS

In general, and certainly during this experiment, the mean sea level shows non-tidal meteorologically induced variations which are small (less than 1 % of the water depth). As a consequence the mass flux field is practically free of divergence so that a stream function can be used to analyze the residual current data. Accordingly we put:

$$F_x = -\frac{\partial \psi}{\partial y}, \qquad F_y = \frac{\partial \psi}{\partial x}, \tag{1}$$

in which F_x=east component of the measured flux parallel to the x-axis and F_y=north component of the measured flux parallel to the y-axis.

It was assumed that the stream function ψ in a small neighborhood around a fixed point could be approximated by a second order Taylor expansion:

$$\psi=\psi_0+b_1x+b_2y+c_{11}x^2+c_{12}xy+c_{22}y^2. \quad (2)$$

Fitting such a quadratic stream function is equivalent with fitting a linear function to the observed residual fluxes, but with the flux divergence forced to vanish.

In (2) the unknowns b_1, b_2, c_{11}, c_{12} and c_{22} must be chosen so as to give an optimal fit of the measurements. Use has been made of a least square fitting method to fit the fluxes as defined by (2) with the observed residual fluxes. With four measured residual fluxes it is possible to derive eight equations for the five unknown coefficients.

On the average the difference between the flux components which are calculated after adjustment of ψ with (2) and the measured residual fluxes, equals about 0.2 m^2/sec., which is equivalent to about 0.5 cm/sec. in terms of the vertically averaged current components. This shows that (2) can give a useful fit to current data in not too large an area, within the instrumental errors.

RESIDUAL VORTICITY

From (1) and (2) it is possible to equate the curl of the residual flux field to:

$$\xi=\Delta\psi=2c_{11}+2c_{22}. \quad (3)$$

For later reference we also introduce ζ as ξ divided by the water depth H:

$$\zeta=\xi/H, \quad (4)$$

which has the dimension of a vorticity (sec.$^{-1}$). Figure 2 shows ζ as a function of time together with the east and north components of the daily averaged wind stress. Wind stresses were calculated from wind data of a nearly oil rig according to:

$$\tau=\rho_{air}C_D U_{10} U_{10}. \quad (5)$$

Here U_{10} is the wind speed at 10 m height, ρ_{air} is the air density and C_D is the drag coefficient ($\rho_{air}.C_D=2.2\times10^{-3}$ kg/m^3). Using a 20% error in the residual currents it can be shown that the maximum error in ζ is about 1.4×10^{-6} sec.$^{-1}$, for the length scale involved. The real error is probably smaller because of statistically averaging of individual errors over a greater number through use of the stream function fit. A reduction by a factor of 3 looks reasonable. The degree in which horizontal derivatives can be estimated can be checked with the amounts of water flowing into and out of the current meter arrays. The actual surplus of net water inflow is 3 times less than the surplus which is estimated on the basis of instrumental errors.

From Figure 2 we note the close resemblance between ζ and the easterly component of the wind stress. In fact we obtain a correlation coefficient of 0.69. Also the northerly component correlates with ζ with a correlation coefficient of 0.50. Taking into account the number of data the correlation coefficient should be larger than 0.28 for 95% significance. This suggest a mechanism in which the wind stress in combination with the bottom topography is responsible for the generation of residual vorticity. The possible mechanism will be studied below.

THE TWO-DIMENSIONAL, VERTICALLY AVERAGED, EQUATIONS

Zimmerman (1978) discussed the generation of residual vorticity by the combined action of the tidal flow over bottom topography. He started from the vertically integrated equations of motion for flow in a rotating system of a constant density liquid, in the absence of

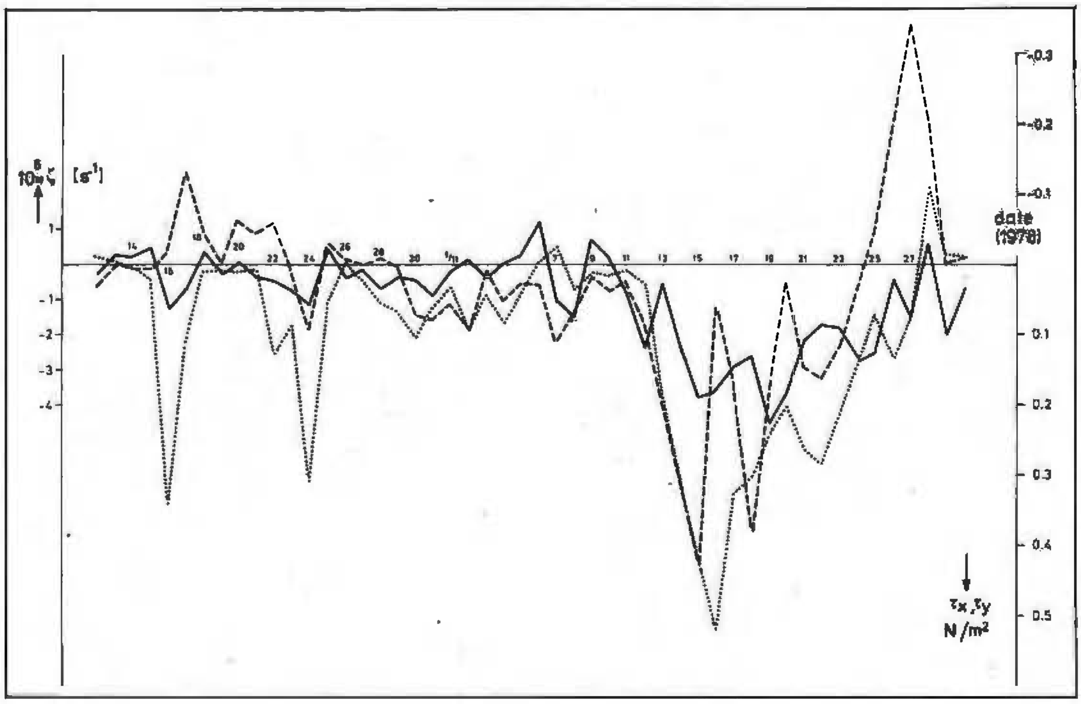

Figure 2
Residual vorticity ζ(—) as a function of time. The values are based on a quadratic stream function fit of residual current measurements at the stations indicated in Figure 1. The x(...) and y(---) component of the wind stress were determined from measurements on a nearby oil rig.

atmospheric effects. The depth was taken as $D(x,y)=H+h(x,y)$ with H the average depth and h a small perturbation. Solutions were obtained with the help of an expansion in h/H.

Here we study the same equations, but with a prescribed constant wind stress τ:

$$\left.\begin{aligned} \frac{\partial \bar{\mathbf{u}}}{\partial t} &= -(\mathbf{f}\times\bar{\mathbf{u}})-g\,\nabla\eta+\frac{\tau-\tau_{\text{bottom}}}{\rho\,(D+\eta)}, \\ \nabla.\{(D+\eta)\,\bar{\mathbf{u}}\} &= -\frac{\partial\eta}{\partial t}, \end{aligned}\right\} \tag{6}$$

where:

$$\bar{\mathbf{u}}(x,y,t)=\frac{1}{D+\eta}\int_{-D}^{\eta}\mathbf{u}(x,y,z,t)\,dz. \tag{7}$$

Here $\bar{\mathbf{u}}=(\bar{u},\bar{v})$ is the horizontal velocity averaged over the instantaneous water column, extending from the bottom at $z=-D(x,y)$ to the position $\eta(x,y,t)$ of the sea surface. In (6) $\mathbf{f}=(0,0,f)$ with f the Coriolis parameter, g is the acceleration of gravity, τ_{bottom} is the bottom friction stress, and ρ is the water density. Non-linear terms in the velocity field have been omitted. Their inclusion in our calculation leads to the possibility of vorticity advection. We do not consider this here since our interest is in vorticity production. Also the Rossby number is of the order of 10^{-2}. The neglect of horizontal friction is justified, when compared with the Coriolis force. The ratio of horizontal friction and Coriolis force is given by $A_H/L^2 f$, which is of the order of 10^{-3} to 10^{-2}. We will look for stationary solutions of (6). The periodic tide will not be taken into account explicitly, however, we allow for its presence in the parameterization of the bottom friction (Bowden, 1953; Weenink, 1958; Groen, Groves, 1962; and Hunter, 1975).

$$\tau_{\text{bottom}}=r\,\rho\,\bar{\mathbf{u}}+s\,\tau. \tag{8}$$

The effect of the term with s is to introduce an effective wind stress $\tau_{\text{eff}}=\tau(1-s)$. In practice $s\ll 1$ (Weenink, 1958). Therefore, we can take account of the second term of (8) by defining τ to mean the effective wind stress in the following.

As we discussed in the introduction we restrict ourselves to the case of a sea with infinite horizontal extent. First we solve the stationary case (6) for a flat bottom $D(x,y)=H$. In that case we expect a homogeneous solution ($\nabla_i\eta=0$ and $\nabla_i u_j=0$) so that (6) reduces to:

$$\left.\begin{aligned} if\,\bar{V}_0 &= \frac{T}{H\rho}-\frac{r}{H}\bar{V}_0, \\ \bar{V}_0 &= \bar{u}_0+i\bar{v}_0, \qquad T=\tau_x+i\,\tau_y, \end{aligned}\right\} \tag{9}$$

where $i^2=-1$. This has the well-known solution:

$$V_0=\frac{1}{r_0+ifH}\cdot\frac{T}{\rho}. \tag{10}$$

Depending on the value of $\alpha=r_0/f\,H$, which measures the relative importance of friction and Coriolis force, the response varies in magnitude and in direction.

In the next step we investigate the perturbation of the flow field which results from an uneven bottom topography:

$$D(x,y)=H+h(x,y), \qquad h/H\ll 1. \tag{11}$$

In general r will depend on h and therefore on x and y. So in addition to (11) we write:

$$r(x,y)=r_0+r'(x,y). \tag{12}$$

As always in a perturbative approach we look for a solution of the form $(\bar{\mathbf{u}},\eta)=(\bar{u}_0+\bar{\mathbf{u}}',\eta')$ where $\bar{\mathbf{u}}_0$ is given by (10). Substitution in (6) and omitting second order terms gives:

$$0=-(\mathbf{f}\times\bar{\mathbf{u}}')-g\,\nabla\eta -(\mathbf{f}\times\bar{\mathbf{u}}_0)\left\{\frac{h+\eta}{H}\right\}-\frac{r_0\bar{\mathbf{u}}'+r'\bar{\mathbf{u}}_0}{H}, \tag{13a}$$

$$\bar{\mathbf{u}}_0.\nabla(h+\eta)+H\,\nabla.\bar{\mathbf{u}}'=0. \tag{13b}$$

For the calculations that follow it is convenient to introduce the residual vorticity in the same way as was done by Zimmerman (1978):

$$\bar{\omega}=\nabla\times\bar{\mathbf{u}}. \tag{14}$$

This definition differs from (4). In fact we have:

$$\zeta=\frac{1}{D+\eta}\nabla\times\{(D+\eta)\,\bar{\mathbf{u}}\}. \tag{15}$$

Here ζ is the curl of the flux field divided by depth. For later reference it is convenient to introduce a velocity field $\tilde{\mathbf{u}}$, such that to lowest order in h/H:

$$\zeta=\nabla\times\tilde{\mathbf{u}}. \tag{16}$$

It is easy to see that this field is just:

$$\tilde{\mathbf{u}}=\bar{\mathbf{u}}'+\frac{1}{H}\bar{\mathbf{u}}_0.h. \tag{17}$$

To lowest order in h/H we have:

$$\zeta=\bar{\omega}-\frac{1}{H}\bar{\mathbf{u}}_0\times\nabla h, \tag{18}$$

and this will be taken into account when we compare the results of our calculations with available measurements.

To zeroth order we find a residual vorticity identically equal to zero. This is a consequence of our choosing $\nabla\times\tau=0$. We have checked that this is a reasonable approximation for the scales we are interested in. For the perturbed velocity we expect to find a non-vanishing vorticity. In fact, with the help of (13) the following equation is obtained:

$$\bar{\omega}=\frac{1}{r_0}(\bar{\mathbf{u}}_0\times\nabla r'). \tag{19}$$

For constant r the vorticity generation due to the bottom friction torque, the Coriolis torque and the wind stress torque per unit mass balance each other (cf. Weenink, 1958). In this case, $\bar{\omega}=0$ can be used to introduce a potential φ' by:

$$\bar{\mathbf{u}}'=\nabla\varphi'. \tag{20}$$

The flow problem then reduces with the help of (13 *b*) to:

$$\Delta\varphi'=-\frac{1}{H}(\bar{\mathbf{u}}_0.\nabla)(h+\eta). \tag{21}$$

In practice we always have $\nabla\eta \ll \nabla h$, so that it is straightforward to solve the complete flow problem. This is shown in Appendix A.

In reality $r' \neq 0$ and thus $\bar{\omega}\neq 0$. From (19) we can see that the depth dependence of r is crucial to a better understanding of vorticity generation. In order to study this depth dependence we propose to consider the three-dimensional equations of motion.

THE THREE-DIMENSIONAL EQUATIONS

The three-dimensional equations of motion are:

$$\frac{\partial \mathbf{u}}{\partial t}=-(\mathbf{f}\times\mathbf{u})-g\,\nabla\eta+\frac{1}{\rho}\frac{\partial}{\partial z}A\frac{\partial \mathbf{u}}{\partial z}, \tag{22}$$

$$\nabla.\mathbf{u}+\frac{\partial w}{\partial z}=0, \qquad \mathbf{u}.\nabla\eta=w(\eta),$$

$$w(-D)+\mathbf{u}.\nabla D=0,$$

in which $\mathbf{u}$ is the horizontal velocity (2−D) and w is the vertical velocity. As before non-linear terms have been omitted. "A" is the turbulent exchange coefficient (for a discussion see e.g. Krauss, 1973). The quantity $\tau(z)=A(\partial\mathbf{u}/\partial z)$ parameterizes the (turbulent) vertical momentum transport. The magnitude of A is of the order of 10 $kg.m^{-1}.sec.^{-1}$, but its value can vary by an order of magnitude depending on the specific oceanographic situation. Solutions of (22) are fixed after specification of additional boundary conditions. As in the previous section, dealing with the vertically averaged equations, we will consider a shallow sea of infinite horizontal extent, such that $\nabla\eta\neq 0$ only if bottom topography is present. At the sea surface $z=\eta$ we take the momentum flux in the sea equal to the wind stress τ:

$$\tau(\eta)\equiv A\left.\frac{\partial\mathbf{u}}{\partial z}\right|_{\eta}=\tau. \tag{23}$$

We are interested in consequences of the specific bottom friction parameterization. We therefore consider several choices:

$$\mathbf{u}(-D)=0, \tag{24 a}$$

$$\tau(-D)=\rho\, r_1\,\bar{\mathbf{u}}, \qquad r_1=\text{Const.}, \tag{24 b}$$

$$\tau(-D)=\rho\, r_2\,\mathbf{u}(-D), \qquad r_2=\text{Const.} \tag{24 c}$$

Which of these conditions is better will depend on the particular choice of problem. For an aerodynamically smooth bottom (24 *a*) seems most appropriate. Then A must also include the molecular viscosity. In case of a rough bottom (24 *b*) is frequently used. It corresponds to the choice of the previous section. If one is interested in effects of bottom topography it may be better to have the bottom friction fixed in terms of the velocity near the bottom, so that (24 *c*) would be an appropriate choice. Naturally $r_2>r_1$.

Several authors have studied (22) under varying assumptions (Ekman, 1905; Welander, 1957; Murray, 1975; Thomas, 1975; Madsen, 1977; Lindijer, 1979; Svensson, 1979; Simons, 1980). Welander (1957) extended the classical Ekman (1905) theory to shallow water situations. Later authors studied the effect of variable A(z). Some authors consider parameterizations in which A acquires its maximum value near the sea surface and decreases with increasing depth (Thomas, 1975; Lindijer, 1979). This can be justified perhaps for deep seas, in situations where one is not interested in the precise behaviour near the surface. For shallow, well-mixed seas, a different approach is preferable. Svensson (1979) considered the structure of the turbulent Ekman layer with the help of the turbulent energy balance. One of his conclusions is that for certain purposes a depth independent A is quite adequate. Recently, considerable progress has been made in the understanding of the 3-dimensional equations for general A (see e.g. Nihoul, 1977 and Davies, 1980). In this paper we will consider, for simplicity, constant A only. This is most realistic outside boundary layers. As a consequence, combination of A=Const. with the bottom boundary conditions (24 *b*) and (24 *c*) (where the bottom boundary layer need not be resolved) is probably superior to combination with (24 *a*). Extension to non trivial depth dependence of A is straightforward, in principle, but it renders the mathematics somewhat complicated which is not needed to obtain the sort of insight that we want to obtain at this stage.

Before we tackle the general Ekman problem we first solve (22) in the limit of a very shallow sea, in which limit the effect of the Coriolis force is negligible, as compared to the term with the second order z-derivative. This can be seen with the help of simple scale analysis, which shows that the magnitude of the Coriolis term relative to the vertical momentum transport term is given by $f\rho D^2/A$. Hence the Coriolis term is negligible for $D\ll(A/f\rho)^{1/2}$, which is the shallow sea limit.

SOLUTION FOR A VERY SHALLOW SEA ($f\rightarrow 0$)

We consider a homogeneous very shallow sea, and neglect the effect of the earth's rotation on the water mass. Looking for a stationary solution, we obtain from (22):

$$\left.\begin{aligned} &g\,\nabla\eta=\frac{1}{\rho}A\frac{\partial^2\mathbf{u}}{\partial z^2};\\ &\nabla.\mathbf{u}+\frac{\partial w}{\partial z}=0,\\ &\mathbf{u}.\nabla\eta=w(\eta), \qquad \mathbf{u}.\nabla D+w(-D)=0. \end{aligned}\right\} \tag{25}$$

The exchange coefficient A has been taken constant. Equation (25) is completed by the boundary conditions (23) and (24).

For a flat bottom we find $u_0=a+bz$, $w_0=\eta_0=0$ and the boundary conditions reduce to:

$$\mathbf{u}_0(-H)=0, \tag{26a}$$

$$\tau(-H)=\rho\, r_1\, \bar{\mathbf{u}}_0, \tag{26b}$$

$$\tau(-H)=\rho\, r_2\, \mathbf{u}_0(-H). \tag{26c}$$

This leads to:

$$\mathbf{u}_0=\frac{\tau}{A}(H+z), \tag{27a}$$

$$\mathbf{u}_0=\frac{\tau}{r_1\rho}+\frac{\tau}{A}\left(\frac{1}{2}H+z\right), \tag{27b}$$

$$\mathbf{u}_0=\frac{\tau}{r_2\rho}+\frac{\tau}{A}(H+z). \tag{27c}$$

The following expressions for the vertically averaged velocities and bottom friction parameters result:

$$\bar{\mathbf{u}}_0=\frac{\tau H}{2A}, \qquad r_0=\frac{2A}{\rho H}, \tag{28a}$$

$$\bar{\mathbf{u}}_0=\frac{\tau}{r_1\rho}, \qquad r_0=r_1, \tag{28b}$$

$$\bar{\mathbf{u}}_0=\frac{\tau}{r_2\rho}+\frac{\tau H}{2A}, \qquad r_0=\left(\frac{1}{r_2}+\frac{1}{2}H\rho/A\right)^{-1}. \tag{28c}$$

In the next step we calculate corrections to (27) which result from the bottom topography. This is done along the lines of the section in which the two dimensional problem was studied i.e. we set $(u, w, \eta)=(u_0+u', w', \eta')$ and $\nabla\eta' \ll \nabla h$ and obtain:

$$\left.\begin{aligned}
&\rho g \nabla\eta'=A\frac{\partial^2\mathbf{u}'}{\partial z^2},\\
&\nabla.\mathbf{u}'+\frac{\partial w'}{\partial z}=0,\\
&w'(0)=\mathbf{u}_0(0).\nabla\eta'\simeq 0,\\
&w'(-H)=-\mathbf{u}_0(-H).\nabla h,
\end{aligned}\right\} \tag{29}$$

with the boundary conditions:

$$A\left.\frac{\partial\mathbf{u}'}{\partial z}\right|_0=0, \tag{30}$$

and:

$$\left.\begin{aligned}
&\mathbf{u}'(-H)=\frac{\tau h}{A},\\
&A\left.\frac{\partial\mathbf{u}'}{\partial z}\right|_{-H}=\rho\, r_1\,\bar{\mathbf{u}}',\\
&A\left.\frac{\partial\mathbf{u}'}{\partial z}\right|_{-H}=-\rho\, r_2\frac{\tau h}{A}+\rho\, r_2\, u'(-H).
\end{aligned}\right\} \tag{31}$$

From these conditions it follows that the perturbations of the velocity field are of the following form:

$$\mathbf{u}'=\frac{\tau h}{A}+\frac{\rho g(z^2-H^2)}{2A}\nabla\eta', \tag{32a}$$

$$\mathbf{u}'=-\frac{gH}{r_1}\nabla\eta'+\frac{\rho g(z^2-1/3.h^2)}{2A}\nabla\eta', \tag{32b}$$

$$\mathbf{u}'=\frac{\tau h}{A}+\frac{\rho g(z^2-H^2)}{2A}\nabla\eta'-\frac{gH}{r_2}\nabla\eta'. \tag{32c}$$

This is not an explicit solution for the perturbed velocity field, since $\nabla\eta'$ is not yet determined in terms of h. It is sufficient, however, for a determination of the perturbed vorticity field. The bottom friction law (24 *b*) gives $\omega'=0$: the two other friction laws lead to identical expression for ω':

$$\omega'=\bar{\omega}=\frac{\nabla h\times\tau}{A}. \tag{33}$$

Thus, as expected, the vorticity depends very much on the choice of the bottom friction law. Which choice is to be preferred is an open question. We will come back to it in the section, in which theory and measurements are compared. For $|\nabla h|=10^{-4}$, $|\tau|=0.1$ Nm^{-2} and $A\simeq 10$ kgm^{-1}.sec.$^{-1}$ equation (33) gives $\omega'\simeq 10^{-6}$ sec.$^{-1}$ which is of the order of magnitude of the observed vorticity. A more detailed comparison between theory and measurements will be postponed for the moment.

In the section on the 2 dimensional problem we have seen that the presence of vorticity in the perturbed velocity field is related to the depth dependence of the friction parameter r. This is consistent with our findings in this Section. The parameterization (24 *b*) implies $r=r_1$ constant so that no vorticity was expected. With the other parameterizations vorticity is found. How in this case r depends on $h(x,y)$ could be checked from the definition:

$$r\rho\bar{\mathbf{u}}=A\left.\frac{\partial\mathbf{u}}{\partial z}\right|_{-D}. \tag{34}$$

Writing $r=r_0+r'(x,y)$ one has:

$$r'\mathbf{u}_0+r_0\mathbf{u}'=A\left.\frac{\partial\mathbf{u}'}{\partial z}\right|_{-H}, \tag{35}$$

which can be used to determine r'. However, the arguments are somewhat involved and are given in Appendix B.

In principle one can continue to solve $\mathbf{u}'$ explicitly. To this end it is best to integrate the continuity equation:

$$w'(z)=-\int_z^0\nabla.\mathbf{u}'\,dz. \tag{36}$$

With the boundary condition at the bottom this leads to:

$$\begin{aligned}
w'(-H)&=-\mathbf{u}_0(-H).\nabla h\\
&=-\int_{-H}^0\nabla.\mathbf{u}'\,dz\\
&=H\nabla.\bar{\mathbf{u}}'+\bar{\mathbf{u}}_0.\nabla h-\mathbf{u}_0(-H).\nabla h,
\end{aligned} \tag{37}$$

or:

$$H\nabla.\bar{\mathbf{u}}'+\bar{\mathbf{u}}_0.\nabla h=0. \tag{38}$$

This of course is just (13 *b*) ($\nabla\eta\ll\nabla h$), which expressed the conservation of mass. To solve $\mathbf{u}'$ one can follow several ways. Use of (32) leads to a Poisson equation for η. Alternatively the divergenceless field $\tilde{\mathbf{u}}=\mathbf{u}'+\mathbf{u}_0\, h/\mathrm{H}$ of (17) can be determined in terms of its curl ζ (18) for which we find [cf. equation (4)]:

$$\zeta=\frac{3}{2}\,\frac{\nabla h\times\tau}{\mathrm{A}}, \tag{39 a}$$

$$\zeta=\frac{\nabla h\times\tau}{2\mathrm{A}}, \tag{39 b}$$

$$\zeta=\left(\frac{3}{2\mathrm{A}}+\frac{1}{r_2\,\rho\,\mathrm{H}}\right)\nabla h\times\tau. \tag{39 c}$$

SOLUTION OF THE GENERAL EKMAN PROBLEM WITH VARYING BOTTOM TOPOGRAPHY

The calculations of the previous section can be repeated for the more general case, where the effect of the Coriolis force is included. The modifications are rather straightforward, in principle. For simplicity we shall consider the problem for the no-slip condition at the bottom only (24 *a*). We will compare the resulting vorticity with case *b* bottom friction law (24 *b*) where we know from the general discussion of the vertically averaged equations that no vorticity in the residual current field is produced. The relevant equation is a special case of (22) which can be conveniently written in complex notation as:

$$\frac{\partial^2 \mathrm{V}}{\partial z^2}-\alpha^2\,\mathrm{V}=\mathrm{C}, \tag{40}$$

$$\mathrm{C}=\frac{g\,\rho(\eta_x+i\,\eta_y)}{\mathrm{A}},\qquad \alpha=(1+i)\left(\frac{f\rho}{2\mathrm{A}}\right)^{1/2}$$

The boundary conditions are as in the section with $f=0$. For a flat bottom one obtains:

$$\mathrm{V}_0=\{\tanh\alpha\mathrm{H}\cosh\alpha z+\sinh\alpha z\}\,\mathrm{T}^*,$$
$$\mathrm{T}^*=\mathrm{T}/(\mathrm{A}\,\alpha)=(\tau_x+i\,\tau_y)/(\mathrm{A}\,\alpha). \tag{41}$$

The correction to this solution as a result of the bottom topography takes the form:

$$\mathrm{V}'=\left(\frac{\mathrm{C}}{\alpha^2\cosh\alpha\mathrm{H}}+\frac{\alpha\, h\,\mathrm{T}^*}{\cosh^2\alpha\mathrm{H}}\right)\cosh\alpha z-\frac{\mathrm{C}}{\alpha^2}. \tag{42}$$

Taking the curl of this expression does not eliminate the surface slope, contrarily to what happened so conveniently in the previous section. This is a consequence of the Ekman rotation. Therefore, in order to obtain an explicit expression for the perturbation vorticity the slope of the sea surface must be eliminated.

This can best be done with the help of relation (38) which fixes $\Delta\eta$, sufficient for elimination of the surface slope from the expression for $\bar{\omega}$. We find:

$$\bar{\omega}=\mathrm{F}_{\mathrm{in}}\frac{\tau.\nabla h}{\mathrm{A}}+\mathrm{F}_{\mathrm{out}}\frac{\tau\times\nabla h}{\mathrm{A}}, \tag{43}$$

with:

$$\mathrm{F}_{\mathrm{in}}=\frac{b}{\mathrm{N}_1\mathrm{N}_2}+\left(\frac{a}{\mathrm{N}_1}+\frac{z_4}{2\beta^2}\right)\frac{2\beta\mathrm{N}_1-\mathrm{N}_4}{\mathrm{N}_2\mathrm{N}_3},$$

$$\mathrm{F}_{\mathrm{out}}=\frac{-a}{\mathrm{N}_1\mathrm{N}_2}+\left(\frac{b}{\mathrm{N}_1}+\frac{z_3-\mathrm{N}_2}{2\beta^2}\right)\frac{2\beta\mathrm{N}_1-\mathrm{N}_4}{\mathrm{N}_2\mathrm{N}_3}, \tag{44}$$

$a=(z_5+z_6-z_7+z_8)/(2\,\beta)$,
$b=-(z_5+z_6+z_7-z_8)/(2\,\beta)$.
$z_1=1-e^{-4\beta}$, $\quad z_5=z_1\,z_3$,
$z_2=e^{-2\beta}\sin 2\beta$, $\quad z_6=2\,z_2\,z_4$,
$z_3=\cosh\beta\cos\beta$, $\quad z_7=z_1\,z_4$,
$z_4=\sinh\beta\sin\beta$, $\quad z_8=2\,z_2\,z_3$,
$\mathrm{N}_1=1+2\,e^{-2\beta}\cos 2\beta+e^{-4\beta}$,
$\mathrm{N}_2=z_3^2+z_4^2$, $\quad \mathrm{N}_4=z_1+2\,z_2$,
$\mathrm{N}_3=z_1-2\,z_2$, $\quad \beta=\mathrm{H}\left(\frac{1}{2}f\rho/\mathrm{A}\right)^{1/2}$.

Solution (33) of the previous section can be recovered in the limit of a very shallow sea: for $\beta\rightarrow 0$ one finds $\mathrm{F}_{\mathrm{in}}=0$, $\mathrm{F}_{\mathrm{out}}=-1$. In Figure 3 F_{in} and $\mathrm{F}_{\mathrm{out}}$ are given as a function of β. For typical southern North Sea parameters one has $\beta\simeq 1$ and $\mathrm{F}_{\mathrm{in}}\simeq -0.3$ and $\mathrm{F}_{\mathrm{out}}\simeq -0.8$. The effect of the Coriolis force is to reduce the $\tau\times\nabla h$ contribution and to add a $\tau.\nabla h$ term.

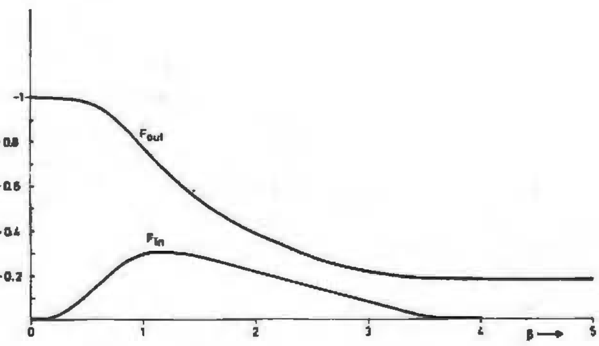

Figure 3
The functions F_{in} *and* $\mathrm{F}_{\mathrm{out}}$ *of* (44) *which determine the depth averaged residual vorticity as a function of* β.

The vorticity of the mass flux field divided by the average depth depends on the choice of bottom friction law. Writing:

$$\zeta=\tilde{\mathrm{F}}_{\mathrm{in}}\frac{\tau.\nabla h}{\mathrm{A}}+\tilde{\mathrm{F}}_{\mathrm{out}}\frac{\tau\times\nabla h}{\mathrm{A}}, \tag{45}$$

we find:

$$\left.\begin{aligned}\tilde{\mathrm{F}}_{\mathrm{in},1}&=(z_3/\mathrm{N}_2-1)/(2\,\beta^2),\\ \tilde{\mathrm{F}}_{\mathrm{out},1}&=-z_4/(2\,\beta^2\,\mathrm{N}_2),\end{aligned}\right\} \tag{46}$$

and:

$$\left.\begin{aligned}\tilde{\mathrm{F}}_{\mathrm{in},2}&=\mathrm{F}_{\mathrm{in}}+\tilde{\mathrm{F}}_{\mathrm{in},1},\\ \tilde{\mathrm{F}}_{\mathrm{out},2}&=\mathrm{F}_{\mathrm{out}}+\tilde{\mathrm{F}}_{\mathrm{out},1}.\end{aligned}\right\} \tag{47}$$

Here $\tilde{\mathrm{F}}_{\mathrm{in},1}$ and $\tilde{\mathrm{F}}_{\mathrm{out},1}$ refer to the standard friction law, (24 *b*) while $\tilde{\mathrm{F}}_{\mathrm{in},2}$ and $\tilde{\mathrm{F}}_{\mathrm{out},2}$ refer to the case with the no-slip condition at the bottom. The functions $\tilde{\mathrm{F}}$ are plotted in Figure 4.

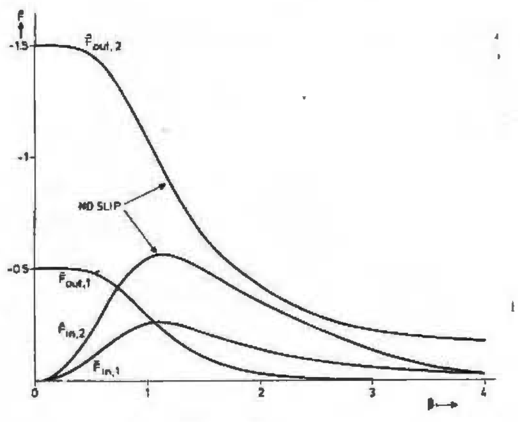

Figure 4

The functions $\tilde{F}$ *of* (46) *which determine the vorticity of the flux field divided by the average depth. The index* 1 *refers to the standard bottom friction law* (24*b*) 2 *results from the no-slip condition at the bottom.*

COMPARISON OF THEORY AND MEASUREMENTS

The comparison between theory and measurements is complicated by the non-existence of a residual vorticity meter. At small scales a vorticity of the order of 10^{-6} sec.$^{-1}$ leads to unobservable current differences. It is for this reason that the measurements were planned on a spatial scale of the order of 10 km.

Experimentally, a good correlation was obtained between wind stress and residual vorticity. This motivated our theoretical work, which led to our expression (45) in which the vorticity is proportional with the wind stress and the local bottom gradient.

The measured vorticity is the spatially averaged vorticity resulting from the action of wind and bottom topography. It was determined with the help of the stream function adjustment, in order to make the flux field divergence free. The stream function was taken quadratic in x and y, so as to take best account of features of the relevant scale. Here, we can calculate the stream function which we could formally expand until terms quadratic in x and y. However, we prefer a slightly different approach in which we start from the assumption that the observed vorticity ζ_{obs} is given in terms of $\tilde{u}$ (17) by:

$$\zeta_{obs}=\frac{1}{2L}[\tilde{v}(\mathbf{X})-\tilde{v}(-\mathbf{X})-\tilde{u}(\mathbf{Y})+\tilde{u}(-\mathbf{Y})]. \tag{48}$$

Here, for convenience, the coordinates are along the diagonal of the current meter array, the origin is in the middle; $\mathbf{X}=(L,0)$ and $\mathbf{Y}=(0,L)$ are current meter positions.

It is straightforward to calculate ζ_{obs}. To this end we first determine the stream function ψ that described the velocity field $\tilde{u}$ of the calculated residual vorticity field ζ. Substitution of (45) in (A 14) gives:

$$\Psi=\frac{1}{2\pi A}[\tilde{F}_{in}\,\tau.\nabla+\tilde{F}_{out}(\tau\times\nabla)] \times\int d\mathbf{x}'\ln|\mathbf{x}-\mathbf{x}'|\,h(\mathbf{x}'), \tag{49}$$

and this can be used with (48) to give:

$$\left.\begin{aligned}&\zeta_{obs}=\frac{1}{A}[\tilde{F}_{in}(\tau.\nabla)+\tilde{F}_{out}(\tau\times\nabla)]\times\mathscr{H}(x),\\&\mathscr{H}(x)=\frac{1}{4\pi L}\int d\mathbf{x}'\left[\frac{\partial}{\partial x}\ln\left|\frac{\mathbf{X}-\mathbf{x}'}{\mathbf{X}+\mathbf{x}'}\right|\right.\\&\left.\qquad-\frac{\partial}{\partial y}\ln\left|\frac{\mathbf{Y}-\mathbf{x}'}{\mathbf{Y}+\mathbf{x}'}\right|\right]h(\mathbf{x}').\end{aligned}\right\} \tag{50}$$

This equation explains the observed correlation between the measured vorticity and the wind stress. In general the exact relationship between ζ_{obs} and τ is a complicated one. There are two different terms $\tau.\nabla$ and $\tau\times\nabla$, which both can contribute. Instead of the depth h, the quantity $\mathscr{H}$ occurs and this is a weighted average of the actual sea depth h. It would be interesting to calculate $\mathscr{H}$ from realistic data on the actual bottom topography. These data are not available, however, so that our conclusion from (50) must remain a qualitative one.

It is interesting to investigate a simple case. This is the situation in which there are dominant westerly winds, while there is a constant positive slope in the sea bottom, towards the north. This is a rough approximation to the situation encountered at our measuring position (Fig. 1). In that case (45) can be used which reduces to:

$$\zeta=\tilde{F}_{out}\,\tau\,\nabla h/A. \tag{51}$$

In our correlation of the vorticity data of Figure 2 with the wind stress we found as a best fit $\zeta=a+b\,\tau$ with $a=0.2\times10^{-6}$ and $b=6.9\times10^{-6}$. Ascribing the constant component to measuring errors, or to the tidally induced residual vorticity (Komen, Riepma, 1981), this yields:

$$\tilde{F}_{out}\,\nabla h/A\simeq-6.9\times10^{-6}\text{ sec.}^{-1}. \tag{52}$$

An estimate for h from topographic charts gives $\nabla h\simeq10^{-4}$, while for $A\sim10$ kg.m^{-1} sec.$^{-1}$ we have $\beta\sim3$, $\tilde{F}_{out,1}=0.001$ and $\tilde{F}_{out,2}=-0.22$.

The smallness of $\tilde{F}_{out,1}$ suggest that the difference between $\bar{\omega}$ and ζ is small. It also suggests that the standard friction law, which gives negligible vorticity, is less realistic for the description of the observed phenomenon. The depth dependent friction law, resulting from the no slip condition at the bottom gives:

$$F_{out,2}\,\nabla h/A\simeq-2\times10^{-6}\text{ sec.}^{-1}. \tag{53}$$

This is an order of magnitude estimate only, since the value of A and ∇h have been guessed. The sign and the order of magnitude agree well with (52).

CONCLUSION

In this paper we studied the spatial variability of residual currents in the Southern Bight of the North Sea on a scale of 10 km. Results of current measurements were used to determine the vorticity of the mass-flux field divided by the average depth. A typical value is several times 10^{-6} sec.$^{-1}$. A good correlation was found between vorticity and wind stress.

Previously Zimmerman (1978) has suggested a possible generation mechanism for vorticity in the residual current field. It is based on the interaction of tidal currents with the bottom. However, Komen and Riepma (1981) found theoretically and experimentally that the resulting vorticity is small ($<10^{-6}$ sec.$^{-1}$) for the greater part of the southern North Sea. In this paper the wind induced residual vorticity is studied therefore. As a first step in understanding the production process we studied situations in which currents that result from the sea surface slope are negligible, and where the turbulent exchange coefficient which parameterizes the turbulent vertical momentum transport is independent of depth. A problem with surface slopes is, that they are determined by the particular geometry of the problem. It was our desire to get some general insight in the problem of vorticity generation, and therefore we concentrated on local effects first, and constant windfields. We found that a linearized bottom friction law in terms of vertically averaged currents with a constant friction parameter leads to a steady state vorticity that equals zero. The stationary case without lateral boundary conditions is characterised by nonzero vorticity only if $r=r(x,y)$. However a study of the equations of motions showed that it is likely that the bottom friction parameter varies with depth. Taking such a depth dependence into account we found an expression for the residual vorticity (45) which is linear both in wind stress and bottom slope.

A detailed comparison of (45) with our measurements is not directly possible. One reason is our theoretical neglect of gradient currents. An other reason is the finite separation of the current meters which were used to determine ζ. Because of this the bottom gradients must be replaced by a weighted average over bottom slopes. An order of magnitude comparison is quite satisfactory, however.

It should perhaps be pointed out that our discussion is still open to extension. In the two-dimensional case we assumed a simple linear bottom friction law and then showed that in order to obtain wind-induced vorticity, the friction coefficient had to depend on depth. Subsequently we studied the friction laws (24) and showed how they can lead to the desired vorticity. Of course one would like to have some justification for the assumed friction laws. One could take the point of view that linear laws like 8 or 24 (for constant A) are empirically valid. Within their range of applicability our conclusions about vorticity generation by wind then hold. A linear law is often derived from the dominant tidal motion by linearizing the quadratic friction laws. The friction parameter is then proportional to the tidal flow. Such an argument is not very appropriate here, because the bottom topography would certainly modulate the tidal motion and this would lead to an additional depth dependence of the friction parameter, leading to additional vorticity generation. Numerical hydrodynamic models (see e.g. Davies, Furnes, 1980), which should use nonlinear friction, have an advantage in this respect, as they can locally adjust the value of A to the prevailing situation. It would be interesting to extend the present calculation with explicit tidal motion. This, however, is beyond the scope of the present paper.

Because a non-zero residual vorticity implies the existence of non-vanishing velocity gradients, residual vorticity is also a measure for the horizontal spatial variability of residual currents. Taking into account the specific values of the residual vorticity one must expect significant current difference over distance of the order of 10 km. Amongst other things this fact has influence on the advection in the Southern Bight, in the sense that one must be careful when deriving Lagrangian transport out of Eulerian point measurements. When water particles move away from the point of the current measurement they can enter flow regimes that differ. The length scales over which this can happen are perhaps smaller than one is used to think of.

Acknowledgements

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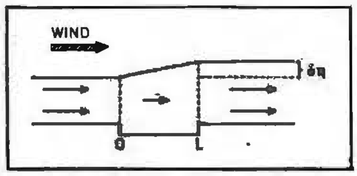

Figure 5
The flow over a square well type of disturbance in the sea bottom (A7).

APPENDIX A

The flow field u'

In the main body of the text we concentrated on a calculation of the vorticity field for most of the time. Here we show how the velocity field itself could be determined.

First we consider a simple example. We specialize (13) to one dimension and constant r. If we take $\nabla\eta \ll \nabla h$ and neglect (non-linear) advection the following simple system results:

$$\left.\begin{aligned} &g\mathrm{H}\,\eta_x = -r\bar{u}', \\ &\bar{u}_0 h_x + \mathrm{H}\bar{u}'_x = 0. \end{aligned}\right\} \qquad (A1)$$

Elimination of $\bar{u}'$ gives:

$$\eta_{xx} = \frac{r}{g\mathrm{H}^2}\bar{u}_0 h_x, \qquad (A2)$$

which is, apart from a normalizing constant, equivalent to (21). (A2) gives $\bar{u}'$ as a divergence. In two dimensions this is equivalent with the vanishing of the vorticity. Using the appropriate boundary conditions at infinity (η = Const., $h = 0$) we obtain as a solution:

$$u' = -\frac{\tau h}{\mathrm{H} r \rho}, \qquad (A3)$$

$$\eta = \frac{\tau}{g\rho\mathrm{H}^2}\int_{-\infty}^{x} h(x')\,dx'. \qquad (A4)$$

The slope in the sea surface is small, $\tau/(\rho g \mathrm{H}^2) \sim 10^{-8}$, but dynamically important. For a square well:

$$\left.\begin{aligned} h = h_0, &\quad 0 < x < \mathrm{L}, \\ 0, &\quad x < 0, x > \mathrm{L}, \end{aligned}\right\} \qquad (A5)$$

we obtain:

$$\left.\begin{aligned} &\bar{u} = \frac{\tau}{r\rho} + \bar{u}' \\ &\bar{u}' = -\frac{\tau h}{\mathrm{H} r \rho}, \end{aligned}\right\} \qquad (A6)$$

$$\left.\begin{aligned} \eta = 0, &\quad x < 0, \\ = \frac{\tau}{g\rho\mathrm{H}^2} h_0 x, &\quad 0 < x < \mathrm{L}, \\ = \delta\eta = \frac{\tau h_0 \mathrm{L}}{g\rho\mathrm{H}^2}, &\quad x > \mathrm{L}. \end{aligned}\right\} \qquad (A7)$$

The flow has been depicted in Figure 5. The small bottom well leads to a small surge in sea level in the down wind direction. This is easily understood. Because of mass conservation the flow is slow above the well and as a consequence of the constant r friction law the friction force is smaller. In order that all forces balance a sea surface slope is required.

To solve the same simplified flow problem with the no-slip condition at the bottom we substitute (32 a) in (38). This gives:

$$\eta_{xx} = \frac{9}{2}\frac{\tau}{\rho g \mathrm{H}^2} h_x, \qquad (A8)$$

which has the solution:

$$\bar{u}' = -\frac{\tau h}{2\mathrm{A}}, \qquad (A9)$$

$$\eta = \frac{9}{2}\frac{\tau}{\rho g \mathrm{H}^2}\int_{-\infty}^{x} h(x')\,dx'. \qquad (A10)$$

For the square well of (A5) this leads to:

$$\bar{u}' = -\frac{\tau h}{2\mathrm{A}}, \qquad (A11)$$

$$\left.\begin{aligned} \eta = \frac{9}{2}\frac{\tau}{\rho g \mathrm{H}^2} h_0\, x, &\quad 0 < x < \mathrm{L}, \\ = 0, &\quad x < 0, \\ = \delta\eta = \frac{9}{2}\frac{\tau}{\rho g \mathrm{H}^2} h_0 . \mathrm{L}, &\quad x > \mathrm{L}. \end{aligned}\right\} \qquad (A12)$$

Mass conservation reduces the flow speed, as before. However, because of the depth dependence of r the resulting sea level slope is slightly different.

In general i.e. in two dimensions and with the Coriolis force included it is best to start from (45) giving the vorticity of the field $\bar{u} = \bar{u}' + \bar{u}_0 . h/\mathrm{H}$ (17), which is divergence free (because it is the mass flux divided by average depth). Defining a stream function as in (1) we have ($\Psi = \psi/\mathrm{H}$):

$$\Delta\Psi = \zeta, \qquad (A13)$$

which is the two dimensional Poisson equation. Ψ is then determined by the known vorticity ζ. The solution of this equation is obtained with the help of the Green's functions:

$$\Psi(\mathbf{x}) = \int d\mathbf{x}'\,\mathrm{G}(\mathbf{x}-\mathbf{x}')\,\zeta(\mathbf{x}'), \qquad (A14)$$

where:

$$\mathrm{G}(\mathbf{x}-\mathbf{x}') = \frac{1}{2\pi}\log|\mathbf{x}-\mathbf{x}'|. \qquad (A15)$$

APPENDIX B

In this Appendix we discuss the relationship between (19) and (33). Both equations give the wind-induced vorticity. The first equation was based on an assumed linear bottom friction law with depth dependent friction coefficient [equations (8) and (12)]; the second started from the no-slip condition at the bottom (24 *a*), and a constant exchange coefficient. An obvious question is: what is the depth-dependence of the friction constant r following from no-slip and constant exchange coefficient. It turns out that to answer this question it is necessary to start from a somewhat more general relation than (8), namely:

$$\tau_{\text{bottom}} = (r_0 + r'_{\text{par}})\,\bar{\mathbf{u}} + r'_{\text{ort}}\,(\mathbf{j} \times \bar{\mathbf{u}}), \tag{B1}$$

which defines a parallel and orthogonal friction coefficient $r_0 + r'_{\text{par}}$ and r'_{ort}. Here $\mathbf{j}$ is a unit vector in the z-z-direction so that the second term represents a friction force at the bottom orthogonal to the mean flow. Physically this is required because the friction $A\,\partial\mathbf{u}/\partial z$ need not be in the direction of $\mathbf{u}$, especially not when there is bottom topography.

With the friction law (B1) as a starting point, repeating the argument of the "Stream functions" paragraph, a different result is obtained from (19). In fact, one finds:

$$\bar{\omega} = \frac{1}{r_0}\,[\bar{\mathbf{u}}_0 \times \nabla r'_{\text{par}} - \bar{\mathbf{u}}_0 \,.\, \nabla r'_{\text{ort}}]. \tag{B2}$$

The question now is to calculate r'_{par} and r'_{ort} in the case of no-slip at the bottom. From the explicit solution (27 *a*) and (32 *a*), and the definition of τ, one finds:

$$-\frac{1}{3}\rho g H \nabla \eta = 2\tau h/H + \tau\, r'_{\text{par}}/r_0 + (\mathbf{j} \times \tau)\, r'_{\text{ort}}/r_0. \tag{B3}$$

This is a vector equation determining both r'_{par} and r'_{ort}. Note that in general $r'_{\text{ort}} \neq 0$. To solve for these, one has to know $\nabla \eta$. This could be determined from (A8):

$$\rho g H^2 \Delta\eta = \frac{9}{2}\tau \,.\, \nabla h. \tag{B4}$$

However, again, we do not have to solve this Poisson equation for a comparison of (B2) and (33). Taking the curl of (B3) we obtain:

$$\frac{\tau \,.\, \nabla r'_{\text{ort}}}{r_0} - \frac{\tau \times \nabla r'_{\text{par}}}{r_0} = \frac{2\tau \times \nabla h}{r_0}, \tag{B5}$$

and this together with (B2) reproduces (33) correctly:

$$\bar{\omega} = \frac{\nabla \mathbf{h} \times \tau}{A}, \tag{B6}$$

so that the consistency of the approach starting from the vertically averaged equations and the approach starting from the 3-dimensional equations has been demonstrated.

Encyclopedia of ocean and atmosphere sciences

Éditée par S. B. Parker

Publiée par McGraw Hill Book Company, Hambourg, New York, 1980.
580 *pages,* 236 *articles,* 500 *illustrations.*
34,50 *dollars US.*

C'est une encyclopédie alphabétique de 580 pages, comportant 236 articles sur les « phases fluides » de la terre. Bien que la jaquette mentionne parmi eux des sujets très actuels, excitant la sensibilité d'un large public (ressources marines, modification du temps, pollution atmosphérique – mais non pollution océanique –, les cycles hydrologiques et de l'énergie, les programmes de satellites...), l'essentiel de la matière de l'ouvrage comporte une documentation très ample, récente et couvrant bien la météorologie et l'océanographie, au sens large, qui en constituent les deux composantes majeures. Une partie de l'information a été empruntée à la 4ᵉ édition (1977) de l'Encyclopédie McGraw-Hill de la science et de la technologie.

Les auteurs, tous de renom international, américains en très grande majorité, sont plus de 200 : 40 % de météorologistes, 40 % d'océanographes (y compris les océanographes appliqués et biologistes). Le reste des auteurs se répartit en géologues et géophysiciens internes, en géophysiciens externes et en hydrologistes.

Un index analytique très détaillé, contenant environ 3 000 noms, et de nombreux renvois au cours des articles, permettent en principe une recherche aisée de l'information désirée par le lecteur. Le grand format permet la présentation d'illustrations, très claires (plus de 500), dont beaucoup sont excellentes et de grande valeur pédagogique. Un grand effort de présentation de nouvelles techniques est accompli.

Le public visé par le volume est apparemment celui de lecteurs cultivés du niveau de l'enseignement supérieur, dans une large gamme, car les commentaires sont de niveau très variable selon les rubriques.

Les articles présentés dans une encyclopédie alphabétique sont fondés sur les dénominations admises pour tel et tel sujet ou phénomène relevant de la matière traitée.

Or les milieux aérien et océanique doivent être d'abord observés, à des échelles d'espace et de temps correspondant à la réalité physique des phénomènes, à la perception que nous en avons et aux besoins qui s'en font sentir; puis les observations doivent être analysées pour que soient déchiffrés leur physique, leur chimie, les mécanismes qui commandent l'évolution de leurs caractères, leur comportement à l'égard des énergies qu'ils reçoivent, leur dynamique en vue de la prévision de leur état futur aux échelles utiles... Or, surtout pour l'atmosphère, des dénominations variées ont été « inventées » pour signifier des phénomènes de nature comparable, ne se distinguant souvent que par leurs différences d'échelles de temps ou d'espace : temps (weather), météorologie, climatologie, atmosphère et sa physique, sa circulation, les tempêtes (storm) et cyclones, la turbulence... autant de rubriques alphabétiques effectivement présentées, qui sont en relation étroite, nécessitant des renvois réciproques fréquents. Il en résulte, dans cette présentation alphabétique, des redites relativement nombreuses qui nuisent à l'homogénéité de l'ouvrage comme à la conception de l'unité planétaire de l'atmosphère. Il est tentant de penser qu'une présentation par « milieu » et par mécanisme, à diverses échelles, serait en l'espèce préférable.

Cette nécessité est moins justifiable pour l'océan (malgré son unité à lui aussi), car les océans et les mers possèdent des noms géographiques, dont il est logique de présenter alphabétiquement la description. Cependant, là aussi, des redites existent : la rubrique « Oceans » présente surtout le relief, la constitution du fond, qu'on retrouve en partie à « Marine geology », à « Marine sediments », voire à « Continental shelf and slope ».

Il reste d'ailleurs que le lecteur désireux de s'informer sur telle rubrique de son choix y trouvera une information solide, récente et raisonnablement complète; si, en se reportant aux renvois, il voit apparaître des redites, il y gagnera par la richesse de la documentation offerte : cette encyclopédie est plus informative que didactique.

Si on trouve avec plaisir dans l'ouvrage d'excellentes rubriques synthétiques sur « Estuarine oceanography, Marine mining, Nearshore processes, Underwater sound, Wind stress over the sea »..., on regrette que les auteurs indiscutés des rubriques géographiques des océans et des mers n'aient pas adopté un plan unique pour la présentation de leur description.

On peut s'étonner de la place faite, dans un ouvrage consacré aux sciences de l'océan et de l'atmosphère, au géomagnétisme, quelle que soit la qualité de cette rubrique : on ne navigue plus guère au compas magnétique... Pourquoi aussi y a-t-il une rubrique « mousson », alors que les alizés se trouvent, curieusement, traités, et l'objet d'un renvoi de l'index, à « Naval meteorology » ?

Cette « Encyclopédie des Sciences de l'Océan et de l'Atmosphère » telle qu'elle est, est un outil de documentation considérable, assorti de références à de nombreux ouvrages à la fin des principales rubriques. Elle fournit aussi des informations souvent difficiles à trouver sur certaines nouvelles techniques d'étude ou d'exploration de l'environnement fluide de notre terre. Elle sera de grande utilité dans le monde entier et constituera un outil efficace pour diffuser, conformément aux désirs de diverses organisations internationales, la connaissance du globe dans tous les pays et notamment dans les pays en développement.

Aussi, malgré nos réserves concernant la présentation plus que le fond, saluerons-nous avec faveur la parution de cette encyclopédie McGraw-Hill, qui servira puissamment à diffuser une documentation à jour concernant l'environnement fluide de la terre, sur lequel l'activité humaine fait peser une menace potentielle qui est l'objet d'une inquiète curiosité et qu'il faudrait parvenir à évaluer.

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General oceanography, an introduction (2nd edition)

by G. Dietrich, K. Kalle, W. Krauss and G. Siedler,

Translated from the German by Susanne and Ulrich Roll
590 *pages* + 33 *pages bibliography,* 8 *maps,* 614 *figures,*
Edited by John Wiley and Sons, New York, 1980.
27 *US dollars.*

The field of oceanography has so much expanded in the past twenty five years that it no longer appears possible to present all aspects of it comprehensively in a volume of reasonable size. The choice made by the authors of this book is to limit the treatment of biological and geological topics to those directly concerned by, or concerning physical and chemical oceanography.

This limitation being recognized, the subjects chosen are well ordered and cover all the classical aspects of physical and chemical oceanography. After setting the scene by a chapter describing the "Geomorphology of the ocean bottom", the following chapters deal with:

- physical properties and chemical composition of sea water;
- oceanographic instruments and observation methods;
- energy and water budgets of the world ocean;
- temperature, salinity, density, characteristic water masses and ice in the world ocean;
- chemical budget of the ocean;
- theory of ocean currents;
- surface waves and internal waves;
- tidal phenomena;
- regional oceanography.

The text is clear, the figures illustrating each chapter are instructive and the mathematical presentation remains at the undergraduate level. The bibliography is extensive and the only serious criticism that may be voiced is that this book, translation of the German text written in 1972, does not cover recent developments concerning surface waves, internal waves and large scale eddies in the oceans.

Nevertheless, this book should be useful both to research scientists and engineers to whom it will provide a sound basis of general information, as well as to undergraduate students who can benefit from its clear presentation and overall approach to the subject.

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