ICES Journal of Marine Science, 56: 489–499. 1999
Article No. jmsc.1999.0482, available online at http://www.idealibrary.com on IDEAL

An efficiency model of a scallop (*Pecten maximus*, L.) experimental dredge: Sensitivity study

S. Fifas and P. Berthou

Fifas, S. and Berthou, P. 1999. An efficiency model of a scallop (*Pecten maximus*, L.) experimental dredge: Sensitivity study. – ICES Journal of Marine Science, 56: 489–499.

In order to estimate the abundance of scallops it is necessary to estimate the efficiency of the experimental fishing gear (scallop dredge with rings of 50 mm diameter, 2 m wide and a toothed bar with 30 teeth). The age–size structure of catches does not usually correspond to that of the actual population because of selectivity of the gear. Several previous studies on the Saint-Brieuc scallop stock (English Channel, France) have used a ratio estimator per age group: an assumption of increase in efficiency vs. scallop size is taken into account. The evaluation of the gear efficiency used for this stock since 1986 has been carried out by diving according to a random sampling plan. Factors such as sedimentary heterogeneity, tide, current, and depth are neglected. There are drawbacks as regards to the definition of dredge efficiency by this method: (1) a ratio estimator is biased when the number of samples is small such as the number of diving units in this study; and (2) the definition of the estimator using age rather than scallop size limits the period of assessment validity: intra-annual extrapolations are not valid. These two difficulties are resolved when dredge efficiency is defined against scallop size. A logistical curve is used and requires three parameters: (1) maximum asymptotical efficiency (e_{max}); (2) parameter describing the selection deviation (α) and; (3) size corresponding to 50% of e_{max} (L_{50}). The matrix of variances–covariances of three parameters is also obtained.

Key words: scallop, dredge, efficiency, logistical curve, variance, Taylor's polynomial.

Received 6 July 1997; accepted 2 April 1999.

S. Fifas and P. Berthou: IFREMER – Centre de Brest – DRV/RH – BP 70, 29280 Plouzané, France. Correspondence to S. Fifas: tel: +33 (0) 298 22 43 78; fax: +33 (0) 298 22 46 53; e-mail: sfifas@ifremer.fr

Introduction

In order to assess the abundance and biomass of scallop populations, it is necessary to know the efficiency and selectivity of the fishing gear. The age and size composition of the experimental catches does not correspond to that of the actual population. On the one hand, a multiplicative correction has to be introduced in order to estimate the population abundance from catches because only a part of the individuals are retained by dredging; on the other hand, this term is different according to age group because available information shows that the efficiency of the fishing gear increases according to individual length.

We report on an evaluation of dredge efficiency experiment using divers in the Saint-Brieuc Bay (Western English Channel, France; Figure 1); a similar method has also been used to evaluate densities of scallop beds (Scottish waters: Mason *et al.*, 1979; Western English Channel: Dare and Palmer, 1994). Our experiment estimates the number of individuals per age group escaping from the dredge which is then used in a ratio estimator of efficiency as presented by Laurec and Le Guen (1981) and Buestel *et al.* (1985):

$$\text{efficiency} = \frac{\text{number of scallops in the dredge}}{\text{total number of scallops (dredge + sea bottom)}}. \quad (1)$$

Materials and methods

The experimental fishing gear

The experimental fishing gear is similar to the dredge with a pressure plate used by fishermen during the fishing season (Figure 2). It differs in terms of the diameter of the metal rings (50 instead of 85 mm) in order to reduce the effects of selectivity for small sized

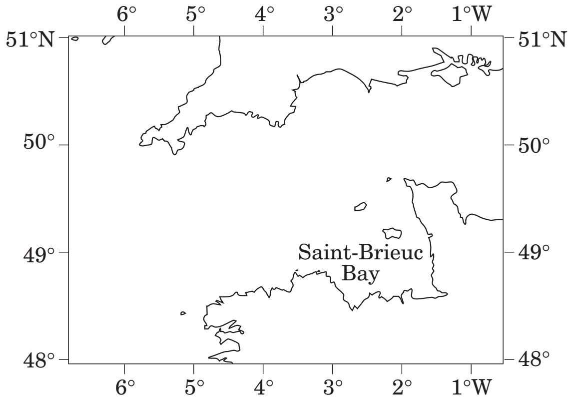

Figure 1. The area studied: the Saint-Brieuc Bay in the western English Channel.

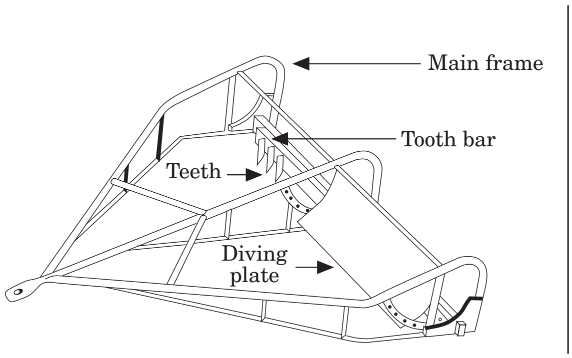

Figure 2. The experimental fishing gear. A dredge with a pressure (diving) plate.

scallops. Likewise, the dredge consists of a 2 m wide toothed bar with 30 teeth instead of 22 used by fishermen. In order to improve dredging conditions, tooth length is variable according to the nature of the sea bottom: for soft sea bottom, tooth length is 13 mm, and for rough sea bottom, the length is 9 mm.

The dredge efficiency against scallop age and size

Using results from diving operations carried out for stock assessment of Saint-Brieuc Bay in 1986 and 1987, according to a sampling technique which has been assumed to be a simple random scheme, it was possible to apply ratio estimators [Equation (1)] and to estimate two values of dredge efficiency vs. scallop age (Fifas, 1991).

(1) A first value was related to scallops of age-group 2. In this case, dredge efficiency was equal to 0.558.
(2) A second value was related to older scallops (age-groups 3+). Efficiency was equal to 0.675.

Ratio estimators were characterized by low values of variance (in both cases, coefficients of variation were less than 10%), but, on the other hand, bias was relatively high because their values were close to those of the standard deviations of the numerator and denominator variables.

The use of a ratio estimator induces three main problems:

(1) It is recommended to have a relatively high number of diving samples available, i.e. about 20, because ratio estimators are influenced by sample size. Cochran (1977) has written that a low sample size (less than 30 observations) combined with high values of coefficients of variation (greater than 10%) for numerator and denominator variables, can induce a non-normal distribution because of the positive value of skewness. In fact, the constraints of diving (limited number of qualified divers, diving during low tide in areas with strong current, the long duration of operations, etc.) mean that only about 10 dives can be carried out during the annual stock assessment.
(2) The fact that estimators are defined according to age and scallop size limits the period of validity of estimators for short time intervals (i.e. in 1986 and 1987, divings were usually carried out in June). Consequently, extrapolations are not valid.
(3) Efficiency has been estimated according to a simple random sampling plan. Its variability with respect to the type of the sea bottom has not been taken into account.

As regards this latter disadvantage, it is necessary to draw up a sampling plan based on the stratification of the Saint-Brieuc Bay according to sedimentary units; some recent references (Thouzeau, 1989) are useful in this case. In the absence of a redefinition of the sedimentary strata, it is assumed that the total area under study is homogeneous and the impact of the nature of the sea bottom is residual. On the other hand, difficulties linked to the first two disadvantages can be overcome if efficiency is defined against scallop size.

Components of efficiency

An efficiency model of dredges using scallop size as an independent variable has to be represented by an increasing monotonic function. Efficiency is made up of the following components:

(1) A certain number of small sized scallops can escape through the 50 mm rings: this phenomenon defines the *selectivity of dredge mesh* which describes a component of *catchability*, called *vulnerability* (Laurec and Le Guen, 1981). This term includes parameters related to the behaviour of the animals and to their size. The sedentary life of the studied species reduces selectivity and the study describes the passive escape of individuals through the rings.

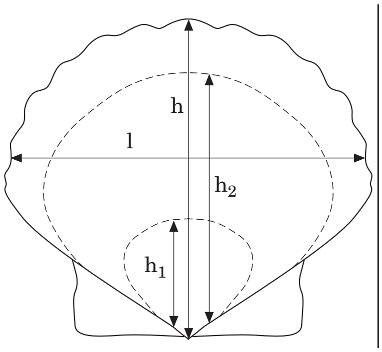

Figure 3. Different dimensions on a scallop shell. l=marketable size; h=symmetrical dimension defining the individual growth; h_1, h_2=sizes corresponding to winter rings.

(2) Nevertheless, scallop dredges present a second type of selectivity, namely through the teeth which can be called the *selectivity of the toothed bar*. On the one hand, this phenomenon is similar to selectivity through the rings and can be described by an increasing monotonic function of the scallop size, but, on the other hand, it depends on the mechanical and physical conditions of dredging (the setting of the dredge due to the length of the teeth, meteorological conditions, etc.). This two-stage selection and retention process (by toothed bar and meshes) has already been described in the case of spring-loaded dredges (Dare *et al.*, 1993).

(3) Note that for the first two components of efficiency, selectivity is defined according to the smaller dimension of the scallop defining the individual linear growth (h) and is different from larger one which defines minimum marketable size (l) (Figure 3).

(4) A final component of efficiency is related to the *mechanical and physical characteristics* of dredging which are independent of scallop size. In fact, even when individuals attain a large size and cannot escape through the teeth or the rings, all scallops are not caught on a dredged area: a certain number of them are still left on the bottom. The third term is defined as efficiency depending on the physical (nature of sea bottom, direction and speed of winds and currents, etc.) and mechanical (speed and direction of ship during the dredging, etc.) characteristics. If we consider that these different parameters have a residual effect during stock assessment, we assume that, unlike the first two components of efficiency, the physical and mechanical components can be represented by a constant term in our model.

Mathematical formulation

An efficiency model using scallop size as an independent variable (e_i vs. L_i) is represented by:

$$e_i = \frac{e_{max}}{1+\exp[-\alpha(L_i - L_{50})]} + \xi_i, \quad (2)$$

where:
e_{max}=maximum asymptotical efficiency; this term is considered to be independent of scallop size; it depends on physical and mechanical characteristics during dredging.
α=parameter linked to the *deviation of selection* of experimental dredge, defined by the difference $L_{75} - L_{25}$ of scallop sizes where efficiency is equal to 75 and 25%, respectively, of maximum asymptotical efficiency [$\alpha = 2\ln(3)/(L_{75} - L_{25})$].
L_{50}=size corresponding to 50% of the maximum asymptotical efficiency.
ξ_i=unexplained residual error.

For a size L_i, the observed efficiency, e_i, is written as:

$$e_i = \frac{N_{i1}}{N_{i1} + N_{i2}}, \quad (3)$$

where:
N_{i1} and N_{i2}=number of individuals of size L_i, caught by the dredge and those left on the sea bottom, respectively.

The proposed model is a logistical function, similar to the one used for selectivity studies, but the number of parameters is three instead of two. Its curve has an inflection point at size L_{50}.

The available data

In the past there were problems with the efficiency curve vs. size, due to the limited number of size classes which were frequently represented in experimental catches of stock assessment (individuals belonging to age-groups 2 and plus). The mean size of 2-year-old individuals is generally around 65 to 75 mm. As a result, previous samples gave a mean efficiency value per age-group, but did not provide either an estimate of the deviation of selection (and of the parameter α) nor the size L_{50} which is probably smaller than the mean size of age-group 2.

This problem has been partially resolved. Stock assessments during the 1990s have provided a considerable amount of data concerning age-group 1 whose mean size is 30–60 mm. In order to complete the data series, it has been assumed that the mechanical conditions of dredging on the sea bottom have not been modified significantly over the last few years and data collected in 1987 and 1990 have been pooled.

The fitting of the efficiency function was carried out for size classes between 27 and 104 mm. The total number of scallops caught in the dredge was 247 and 494 scallops were sampled by divers from the sea bottom after dredging.

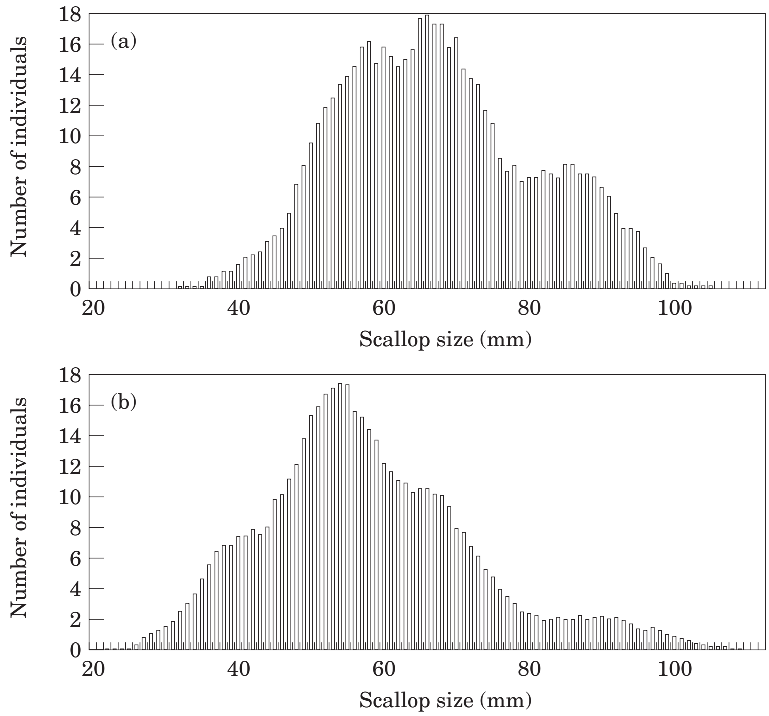

Figure 4. Distribution of length frequencies of scallops. Samples used for fitting model. (a) Scallops caught by dredges. (b) Escaping scallops.

Preliminary data have been smoothed by moving means (Antoine, 1979). This method has advantages because it is possible to distribute uniformly systematical bias introduced by measurement. Furthermore, smoothing reduces large differences in efficiency values between the two contiguous size classes, induced usually by sampling errors. A smoothing step has been fixed as 1 cm.

Figure 4 presents the size frequency histograms of smoothed data for scallops caught by dredge and those which escaped.

Fitting of the logistical curve

Smoothing has not eliminated all sampling errors because the larger were size classes under-represented in the catch. In fact, size measurements in the past were systematically limited to the first three age-groups which have a more biological and marketable applications for forecasting the future fishing season. This limitation may cause problems for fitting because the effect of the large size classes is important in estimating a maximum asymptotical efficiency. An examination of the smoothed data (Figure 4) shows that for the larger size classes, large variations in efficiency due to a low sample size are not eliminated. However it does not seem that efficiency decreases with scallop size (Buestel *et al.*, 1985; Fifas, 1991). Therefore, we have arbitrarily fixed an acceptable minimum number of five individuals and a maximum size of 90 mm for the model.

For the fitting of logistical curves expressing size-selectivity of fishing gears, different algorithms based on conditional maximum likelihood model (Millar, 1991) or multinomial likelihood method (Perez-Comas and Skalski, 1996) have been developed. A comparison of direct and linear fitting methods has been presented by Caddy and Defoe (1996).

The fitting of the logistical curve has been conducted according to the following principles:

(1) The fitting has been done using the Simplex method presented by Nelder and Mead (1965) and it has been chosen because it does not need any statistical constraint (e.g. no formal statistical distribution of estimates needs to be assigned).

(2) The Simplex algorithm proceeds by minimizing the sum of the weighted residual squares: for a given size class i, weight is the corresponding total number of scallops, including scallops caught by dredge (N_{i1}) plus those sampled on the sea bottom by divers (N_{i2}). This procedure attributes greater weight to intermediate size classes and probably helps to reduce uncertainty in parameters α and L_{50}.

The vector of parameters (e_{max}, α, L_{50}) is estimated by minimizing the following quantity:

$$\sum_{i=1}^{nc}(N_{i1}+N_{i2})\cdot\left[e_i-\frac{e_{max}}{1+\exp[-\alpha(L-L_{50})]}\right]^2, \qquad (4)$$

where: nc = number of size classes taken into account for fitting.

Matrix of variances–covariances of parameters. Variance of efficiency

The simplex method does not give an estimate of the matrix of variances–covariances of the three parameters. In this case, it is often recommended to apply non-parametric techniques such as the Bootstrap method (Caddy and Defoe, 1996). In this paper, the calculation of the matrix of variances–covariances was carried out according to a parametric procedure used by Lin (1987) and Fifas (1991).

These authors indicate that matrix of variances–covariances is obtained by the following relationship:

$$(M)=s^2\cdot(I)^{-1}, \qquad (5)$$

where:
(M) = matrix of variances–covariances;
$(I)^{-1}$ = inverse of *matrix of information*;
s^2 = sum of mean residual squares of the fitted function:

$$s^2=\frac{\sum_{i=1}^{nc}\left[e_i-\frac{e_{max}}{1+\exp[-\alpha(L_i-L_{50})]}\right]^2}{nc-3}. \qquad (6)$$

In order to calculate the matrix of information (I), it is necessary to derive formula (2) relative to the three estimated coefficients (e_{max}, α, L_{50}). This operation gives a matrix (Z) of nc lines and three columns:

$$(Z)=\begin{bmatrix}\frac{\partial e_1}{\partial e_{max}} & \frac{\partial e_1}{\partial\alpha} & \frac{\partial e_1}{\partial L_{50}}\\ \frac{\partial e_2}{\partial e_{max}} & \frac{\partial e_2}{\partial\alpha} & \frac{\partial e_3}{\partial L_{50}}\\ \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot\\ \frac{\partial e_{nc}}{\partial e_{max}} & \frac{\partial e_{nc}}{\partial\alpha} & \frac{\partial e_{nc}}{\partial L_{50}}\end{bmatrix}. \qquad (7)$$

The matrix of information is obtained by:

$$(I)=(Z)'\cdot(Z).$$

[(Z)′ = transpose of (Z)].

The matrix of information is inverted using an algorithm presented by Lefebvre (1980).

The matrix of variances–covariances of the three parameters of the model and the use of partial derivatives of order 1 provide an approximate estimate of the variance of the variable Ψ(L) representing efficiency against size L. This procedure is possible using limited developments of order 1 in Taylor's series (Laurec, 1986; Laurec and Mesnil, 1987; Chevaillier, 1990; Chevaillier and Laurec, 1990; Fifas, 1991).

Let Φ be a function and $\theta_1, \theta_2, \ldots, \theta_k$ its parameters. By using Taylor's polynomial it is possible to present the variance of Φ by:

$$V[\Phi]\approx\sum_{i=1}^{k}\left(\frac{\partial\Phi}{\partial\theta_i}\right)^2\cdot V[\theta_i] + 2\cdot\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}\frac{\partial\Phi}{\partial\theta_i}\cdot\frac{\partial\Phi}{\Phi\theta_j}\mathrm{Cov}[\theta_i,\theta_j]. \qquad (9)$$

In the case of the efficiency of the scallop experimental dredge, (9) is equivalent to:

$$\begin{aligned}V[\Psi(L)]\approx&\left(\frac{\partial\Psi(L)}{\partial e_{max}}\right)^2\cdot V(e_{max})+\left(\frac{\partial\Psi(L)}{\partial\alpha}\right)^2\cdot V(\alpha)\\&+\left(\frac{\partial\Psi(L)}{\partial L_{50}}\right)^2\cdot V(L_{50})\\&+2\cdot\frac{\partial\Psi(L)}{\partial e_{max}}\cdot\frac{\partial\Psi(L)}{\partial\alpha}\cdot\mathrm{Cov}(e_{max},\alpha)\\&+2\cdot\frac{\partial\Psi(L)}{\partial e_{max}}\cdot\frac{\partial\Psi(L)}{\partial L_{50}}\cdot\mathrm{Cov}(e_{max},L_{50})\\&+2\cdot\frac{\partial\Psi(L)}{\partial\alpha}\cdot\frac{\partial\Psi(L)}{\partial L_{50}}\cdot\mathrm{Cov}(\alpha,L_{50}).\end{aligned} \qquad (10)$$

Sensitivity study

Let Ψ be a function containing the $\Theta_1, \Theta_2, \ldots, \Theta_n$ parameters; a response, Y_Θ, is associated to this function: $\Psi\{\Theta_1, \Theta_2, \ldots, \Theta_n\}=Y_\Theta$. Let Θ_i be a parameter which is subject to a variation according to $\Delta\Theta_i$ and let $Y'_{\Theta+\Delta\Theta}$ be the new response to the Ψ function.

By using a sensitivity study, it is possible to estimate the modification of the Y_Θ response into $Y'_{\Theta+\Delta\Theta}$ against the variation in Θ_i parameter, according to $\Delta\Theta_i$. Absolute or relative sensitivities exist; in the latter case, the modification of the Y_Θ response into $Y'_{\Theta+\Delta\Theta}$ is expressed in percentage with respect to the variation of Θ_i into $\Theta_i+\Delta\Theta_i$, which is equally expressed in percentage ($100\cdot\Delta\Theta_i/\Theta_i$).

In the case of non-linear models, i.e. the efficiency model for scallop dredges, analytical investigation of errors is impossible and approximative methods must be used, e.g. Delta methods based on the approximate nature of a function in an infinite Taylor series (see references previously quoted: Laurec, 1986; Laurec and Mesnil, 1987; Chevaillier, 1990; Chevaillier and Laurec, 1990; Fifas, 1991); those approximations are valid only if variation increment is too small comparative to the value of the parameter Θ_i.

The terms $\partial e_i/\partial e_{max}$, ∂e_i,$\partial\alpha$, $\partial e_i/\partial L_{50}$ used above for calculation of efficiency variance are called *absolute coefficients of sensitivity of order 1* of the parameters e_{max}, α, L_{50}. If we represent by $\Psi(L)$ the function "efficiency of dredge using scallop size L as an independent variable" and by $a(e_{max})$, $a(\alpha)$, $a(L_{50})$ the coefficients of sensitivity, we can write:

$$\frac{\partial\Psi(L)}{\partial e_{max}}=a(e_{max})=\frac{1}{1+\exp[-\alpha(L-L_{50})]}. \quad (11)$$

$$\frac{\partial\Psi(L)}{\partial\alpha}=a(\alpha)=\frac{e_{max}(L-L_{50})\exp[-\alpha(L-L_{50})]}{(1+\exp[-\alpha(L-L_{50})])^2}. \quad (12)$$

$$\frac{\partial\Psi(L)}{\partial L_{50}}=a(L_{50})=-\frac{\alpha e_{max}\exp[-\alpha(L-L_{50})]}{(1+\exp[-\alpha(L-L_{50})])^2}. \quad (13)$$

The absolute coefficients of sensitivity are involved in calculating the variance of a dependent variable, but, it is more common to use the *relative coefficients of sensitivity of order 1* which provide a more concrete sense. For a parameter θ of a function Φ, the relative coefficient of sensitivity of order 1, written $b(\theta)$, is given by:

$$b(\theta)=a(\theta)\cdot\theta/\Phi. \quad (14)$$

For the parameters e_{max}, α, L_{50}, these coefficients are equal to:

$$b(e_{max})=1. \quad (15)$$

$$b(\alpha)=\frac{\alpha(L-L_{50})\exp[-\alpha(L-L_{50})]}{1+\exp[-\alpha(L-L_{50})]}. \quad (16)$$

$$b(L_{50})=-\frac{\alpha L_{50}\exp[-\alpha(L-L_{50})]}{1+\exp[-\alpha(L-L_{50})]}. \quad (17)$$

Results and discussion

Fitting of the efficiency model and the matrix of variances–covariances

The results of fitting are presented in Table 1 which also gives the main parameters of the logistical curve (deviation of selection, factor of selection) and the matrix of variances–covariances and of correlations between the parameters. Fitting is also presented in Figure 5.

Table 1. Fitting of the efficiency function. Number of size classes nc=64. Sum of weighted residual squares SRQ=0·2564.

Parameter	Value	Standard deviation	C.V.
e_{max}	0.646	0.0157	0.0244
α	0.088	0.0043	0.0489
L_{50}	58.620	0.8254	0.0141
Factor of selection: $L_{50}/L_{mesh}=1.172$			
Deviation of selection: $2.\ln(3)/\alpha=24.97$ mm			
Matrix of variances–covariances			
	e_{max}	α	L_{50}
e_{max}	0.249E-03	−0.551E-04	0.119E-01
α		0.185E-04	−0.263E-02
L_{50}			0.681E+00
Matrix of correlations			
	e_{max}	α	L_{50}
e_{max}	1.0000	−0.8126	0.9115
α		1.0000	−0.7409
L_{50}			1.0000

The estimated relationship between efficiency e_i and size L_i is written as follows:

$$e_i=\frac{0.646}{1+\exp[-0.088\cdot(L_i-58.620)]} \quad (18)$$

A general examination of the results shows that a fitting has been satisfactorily obtained. In fact, the maximum asymptotical efficiency 0.646 is close to that of previous studies on the same area for larger sized scallops which were not selected by rings or a toothed bar: Buestel *et al.* (1985) estimated this value around 0.7. Furthermore, an efficiency study using a ratio estimator has given a value of 0.675 for scallops of age-group 3 and plus (Fifas, 1991). For scallops of age-group 2, if we use growth parameters as presented by Antoine (1979), the efficiency calculated by the fitted equation is approximatively equal to 0.571 (Buestel *et al.*, 1985: 0.50–0.55; Fifas, 1991: 0.558).

Our results and those of other researchers (Dare *et al.*, 1993; Shafee, 1979) show that efficiency estimates for scallop dredges cover a wide range. Dredge efficiency is highly dependent upon complex interactions between the gear, the seabed, hydrodynamic forces (i.e. tide level and coefficient), and the behaviour of the scallops. In the case of spring-loaded dredges used in the Western English Channel, Dare *et al.* (1993) indicated efficiencies ranging from 6% for rough ground to 41% on smooth muddy gravel bottom for scallops greater than 90 mm (legal size). For other Pectinid species, a relationship between efficiency and duration of hauls has been shown. For example, Shafee (1979) observed a decrease in efficiency with duration for black scallop beds. Studies of dredge efficiencies for other marine populations show that density can also be

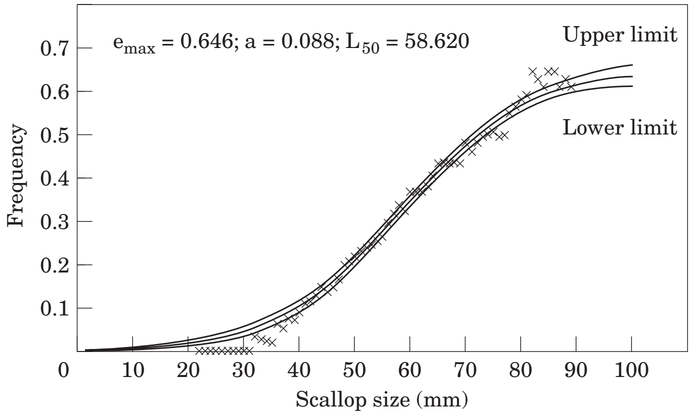

Figure 5. The fitted efficiency model with mean confidence intervals (confidence level of 0.95).

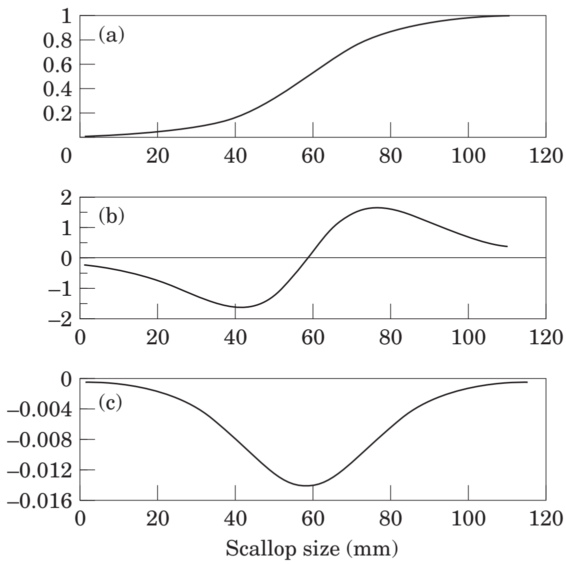

Figure 6. Absolute sensitivity coefficients of the three model parameters. (a) $a(e_{max})$; (b) a(a); (c) $a(L_{50})$.

an explanatory factor. For example, for some crustacean populations, Zhang *et al.* (1993) report that dredge efficiency seems to have declined exponentially as crab density increased.

Absolute sensitivities of order 1

Figure 6 presents the curves of the absolute coefficients of sensitivities of order 1 relative to the three parameters. These coefficients approach 0 asymptotically except for the parameter e_{max} which is equal to 1 when L approaches $+\infty$.

The *coefficient* $a(e_{max})$ is a logistical function. The fact that the maximum asymptotical value of this function is equal to 1 indicates that for a given absolute error of e_{max}, the error of the dependent variable (efficiency) increases against scallop size. Consequently, a divergence of efficiency values is induced.

The *coefficient* $a(\alpha)$ is represented by a periodic function; its curve is non-symmetrical at around L_{50}. When the scallop size approaches 0 or $+\infty$, this coefficient is close to 0. In fact, if the scallop size is not within intermediary values, the absolute error of α contributes less significantly to the efficiency error. The coefficient is equal to zero at L_{50} and for this size, the contribution of the absolute error of α to the efficiency error is zero (in this case, efficiency is always equal to $e_{max}/2$).

The coefficient $a(\alpha)$ presents two extreme values which are non-symmetrical at around L_{50}. After derivation of $a(\alpha)$ relative to scallop size L, the equation is:

$$1-\alpha(L-L_{50})+\exp[-\alpha(L-L_{50})] + \alpha(L-L_{50})\cdot\exp[-\alpha(L-L_{50})]=0. \quad (19)$$

If $\alpha(L-L_{50})$ is replaced by x, Equation (19) is equivalent to:

$$x=\frac{\exp(x)+1}{\exp(x)-1}, \quad (20)$$

which can also be represented by:

$$x-\coth(x/2)=0, \quad (21)$$

where coth is called "hyperbolical cotangent".

The development of the formula (21) using Taylor's series gives:

$$\sum_{k=0}^{\infty}\left[\frac{x}{2}\right]^{2k}\cdot\frac{4k-1}{(2k)!}\approx 0, \quad (22)$$

where: (2k)! = factorial product of 2k.

For every presentation of this function [formulae (20) to (22)], the value of x maximizing (or minimizing) $a(\alpha)$

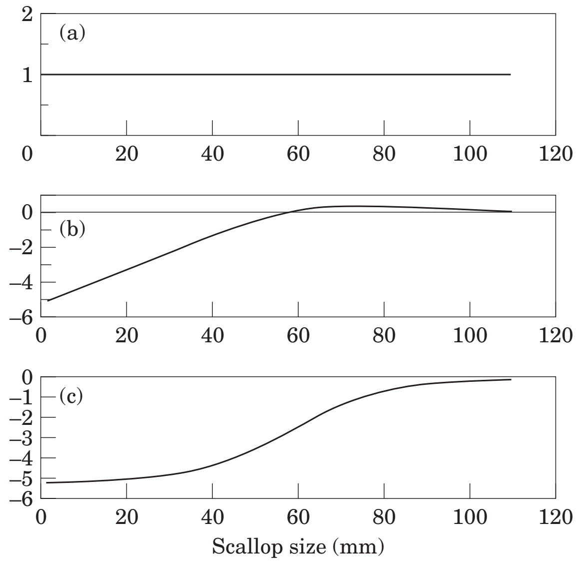

Figure 7. Relative sensitivity coefficients of the three model parameters. (a) $b(e_{max})$; (b) b(a); (c) $b(L_{50})$.

is independent on α and L_{50}, that is, *its value is constant for every scallop dredge*.

A presentation of the analytical expression of the two solutions of x is not straightforward but x can be solved for it actively as:

a(α) is minimized by: $x = -1.543 \rightarrow L \approx 41.1$ mm (efficiency ≈ 0.114)

a(α) is maximized by: $x = -1.543 \rightarrow L \approx 76.2$ mm (efficiency ≈ 0.532)

The *coefficient* $a(L_{50})$ is always negative and it is represented by a curve which is symmetrical at around L_{50}. It approaches 0 when L is close to 0 or to $+\infty$; and its function is minimized if $L = L_{50}$. For $L = L_{50}$ the contribution of the absolute error of L_{50} is at its maximum.

Relative sensitivities of order 1

Figure 7 presents the patterns of the relative coefficients of sensitivity of order 1 for the three parameters.

The *coefficient* $b(e_{max})$ is constant. The contribution of a relative error of e_{max} to a relative error of efficiency is independent of scallop size. Furthermore, its value is always equal to 1 such that an overestimate of e_{max} always induces an equivalent overestimate of efficiency.

The *coefficient* b(α) is negative for sizes less than L_{50}, and positive above this size, similar to a(α), but it is not symmetrical. For scallop size less than L_{50}, an overestimate of α produces an underestimate of efficiency; and for a given relative error of α, the relative error of efficiency decreases when size increases. b(α) quickly approaches 0 and is equal to zero for L_{50}. Above L_{50}, the relative error of α is relatively low and does not exceed 0.28: for these scallop sizes, an overestimate of α gives an overestimate of efficiency, the relative error of the latter never exceeds 30% of the relative error of α. For this interval, a maximum value exists and, finally, b(α) approaches 0 asymptotically.

If $\alpha(L - L_{50})$ is replaced by x, the maximum of the function b(α) is calculated by resolving the following equation:

$$\exp(-x) - x + 1 = 0, \quad (23)$$

which has a solution independent of the scallop dredge similar to a(α).

In order to make the presentation of (23) easier using a Taylor's polynomial, we can write:

$$(x-1)\exp(x-1) = \exp(-1), \quad (24)$$

which can be more or less approximated to $x_o = 1$; the polynomial of x can be inverted and presented by a polynomial of $\exp(-1)$ of the same degree. The analytical solution of x is given by:

$$x \approx 1 + \sum_{k=1}^{\infty} \frac{(-1)^{k-1} k^{k-1} \exp(-k)}{k!}, \quad (25)$$

which has certain disadvantages due to a slow convergence.

The value maximizing b(α) is $x \approx 1.279$; it corresponds to a scallop size: $L \approx 73.1$ mm (efficiency ≈ 0.505).

The *coefficient* $b(L_{50})$ is a monotonic increasing function and its value is always negative such that an overestimate of L_{50} induces systematically an underestimate of efficiency. For a given relative error of L_{50}, the relative error of efficiency decreases vs. scallop size. This coefficient is characterized by a sigmoidal curve which approaches 0 asymptotically and includes an inflection point at L_{50}.

Calculation of variance

Description of the function of variance

The variance of efficiency is given in Figure 8. It is an increasing function which has many local extremes for the intermediate size range. However, the analytical formulation is not mathematically simple. Furthermore, in order to explain the characteristics of the curve of variance it is important to take into account the combined effects of the three parameters of the model (effect of covariances, sensitivity coefficients of order 2).

The figure shows that the trends for variance are characterized by three stages. First of all, for small size classes, variance increases monotonically with size. In

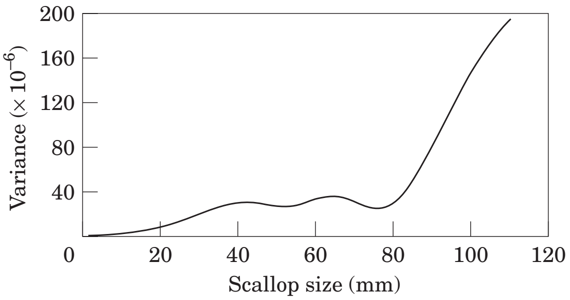

Figure 8. Variance of the efficiency.

the range between 40 and 75 mm, variance is relatively stable and presents local extreme values of low amplitude and there does not seem to be any specific periodicity. During the third stage, concerning large size classes, variance increases quickly and asymptotically approaches that of e_{max}.

The third stage differentiates the variance of the efficiency model from other logistical functions (selectivity) which have only two parameters. The variance of efficiency is described as an increasing function unlike selectivity models which are characterized by a constant maximum asymptotical value and their variance is close to zero when size approaches $+\infty$. The divergence of efficiency constitutes a disadvantage. Nevertheless, this characteristic seems to match reality because the main problems related to the variability of efficiency usually concern large sized scallops.

Simulations of variances and covariances of parameters

In order to better understand the effects of variances and covariances between the three parameters of the model, two types of simulations have been carried out.

(1) In the three cases, one parameter of the model has been cyclically assumed as constant: its variance and covariances with the other two parameters become zero. It is important to note that variances and covariances are referred to as *estimates* of parameters and not as true values which are unknown. For this reason, the simulation is handicapped. In fact, it is impossible to comment on true correlations between e_{max}, α, L_{50} because it is not valid to attribute a biological or physical sense to the three parameters of the model except for e_{max}. The result of the simulation is presented in Figure 9.

(1.1) *Invariability of* e_{max}. It is the variability of e_{max} which contributes mostly to the variance of efficiency and its function even if its variation coefficient is not the highest (Table 1). The invariability of e_{max} completely transforms the curve which presents only a maximum value due to the strong negative correlation between α and L_{50} and, above the size corresponding to this value, it falls asymptotically to zero.

(1.2) *Invariability of* α. Non-significant modifications are induced by this simulation.

(1.3) *Invariability of* L_{50}. As regards around to L_{50} size, the curve becomes similar to that fitted. For size classes below L_{50}, the simulated curve is different. The elimination of terms of limited effect on L_{50} probably contributes in increasing the effect of covariance between e_{max} and α and of the sensitivity coefficient $a(\alpha)$. It is important to note that this simulation produces a variance which is systematically greater than that of the fitted model. The part of $Cov(e_{max},L_{50})$ seems to be significant because it is the only negative term which has been eliminated given there is a invariability of L_{50}.

(2) The second type of simulation is based on the modification of correlation coefficients between the three parameters. In order to simplify calculations, we have kept unchanged the values of variation coefficients of parameters equal to those obtained by fitting and we have also considered the same signs of covariances between the three parameters [$Cov(e_{max},\alpha)<$, $Cov(e_{max},L_{50})>0$, $Cov(\alpha,L_{50})<0$]. Many cases have been studied and extreme situations correspond, on the one hand, to a total absence of correlations and, on the other hand, to a perfect correlation between the estimates of the three parameters. The results are presented in Figure 10. Only the two extreme cases have been commented upon.

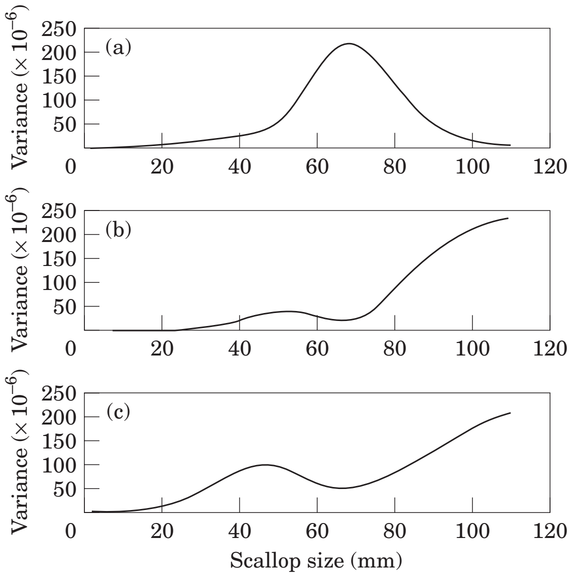

Figure 9. Simulation of variance of the efficiency with alternative invariability of one model parameter. (a) $V(e_{max})=0$; (b) $V(a)=0$; (c) $V(L_{50})=0$.

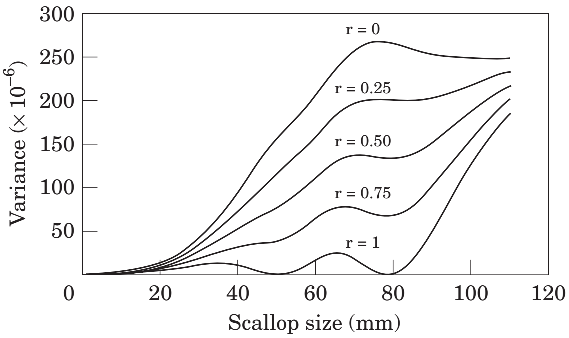

Figure 10. Simulation of variance of the efficiency for different values of correlation between the parameters. The signs of correlations are the same to fitting results.

(2.1) *Absence of correlations*. This simulation produces a sharp increase in variance beginning from the small size classes. After a maximum value (around 75 mm), the curve becomes stable and approaches asymptotically $V(e_{max})$.

(2.2) *Perfect correlations*. This simulation will be commented upon in more details because it is close to the results obtained in the fitted model. The curve of variance of efficiency seems to be stable up to 75 mm and it increases very sharply, it approaches asymptotically $V(e_{max})$ for the larger size classes. The variance obtained by this simulation was always lower than that calculated by zero correlations.

Opposite signs of correlations gave two sizes with a variance equal to 0. Perfect correlation allows the sum of the limited development of order 1 to be transformed as:

$$V(\Psi(L)) \approx [\sigma e_{max} \alpha(e_{max}) - \sigma\alpha a(\alpha) — \sigma L_{50} a(L_{50})]^2, \quad (26)$$

where σe_{max}, $\sigma\alpha$, σL_{50} are the standard deviations (estimated by values presented in Table 1).

After developing Equation (26) and replacing $L - L_{50}$ by y, the solutions of the equation are obtained by:

$$\exp(\alpha y) \approx \frac{y\sigma\alpha + \alpha\sigma L_{50}}{CV(e_{max})} - 1, \quad (27)$$

with: $CV(e_{max})$ = variation coefficient of e_{max}.

Values resolving this equation were:

$y \approx -8.523 \rightarrow L \approx 50.1$ mm (efficiency ≈ 0.207)

$y \approx -18.980 \rightarrow L \approx 77.6$ mm (efficiency ≈ 0.543)

The study of correlations between the three model parameters shows that:

- The variance of efficiency decreases according to correlations between the three parameters. In fact, Figure 10 indicates the total absence of an intersection between the simulated curves.
- For sizes greater than L_{50}, the increase in variance is greater as the correlations are high; the inverse situation is produced for sizes less than L_{50}.
- Simulated curves converge asymptotically to $V(e_{max})$, but, for the intermediate values of scallop sizes, a divergence is observed.

The main local extreme curve values, for sizes greater than L_{50}, are relatively stable in the modification of correlations between the model parameters.

Conclusion

The fitting of the efficiency model is satisfactory because it gives parameter coefficients of variation from 1 to 5%. The fact that data have been collected over several years probably reduces the effects of interannual variability of efficiency. On the other hand, it has not demonstrated that seasonal variability is reduced because data were sampled during the same season. It is important to verify, using samples taken during other seasons, whether this variability can be neglected or not. In order to achieve an unbiased estimate of parameters it is necessary to apply re-sampling techniques (Jackknife, Bootstrap). These methods have not been chosen in this paper because of the absence of information concerning the matching of data to a given sample and sensitivity studies were limited to the order 1 (variances).

The examination of sensitivities, both absolute and relative, and the study of the function of variance show the dominant part of maximum asymptotical efficiency (e_{max}) in the error induced by the fitted function, mainly as for the larger sized classes. This point constitutes an important difference between the fitted function and selectivity curves which include only two parameters. The variance of efficiency increases sharply above 75–80 mm and approaches asymptotically that of e_{max}. This point proves it is difficult to reconstitute abundances of scallops for old age-groups. Nevertheless, strong correlations between the three parameters of the model produce a significant decrease in variance, mainly for the intermediate size classes which are the most frequently represented in scallop fishery in the Saint-Brieuc Bay.

References

Antoine, L. 1979. La croissance de la coquille Saint-Jacques *Pecten maximus* (L.) et ses variations en mer Celtique et en Manche. Thèse 3e cycle, COB, UBO. 148 pp.

Buestel, D., Dao, J. C., and Gohin, F. 1985. Estimation d'un stock naturel de coquilles Saint-Jacques per une méthode combinant les dragages et la plongée. Traitement des résultats par une approche géostatistique. CIEM, Thème X. 33 pp.

Caddy, J., and Defoe, O. 1996. Fitting the exponential and logistic surplas yield models with mortality data: Some explorations and new perspectives. Fisheries Research, 25: 39–62.

Chevaillier, P. 1990. Méthodes d'étude de la dynamique des espèces récifales exploitées par une pêcherie artisanale tropicale: le cas de la Martinique. Thèse Docteur-Ingénieur, EN.S.A. Rennes. 367 pp.

Chevaillier, P., and Laurec, A. 1990. Logiciels pour l'évaluation des stocks de poisson. ANALEN: Logiciel d'analyse des données de capture par classes de taille et de simulation des pêcheries multi-engins avec analyse de sensibilité. F.A.O., Document Technique sur les Pêches, 101 (suppl): 124 pp.

Cochran, W. G. 1977. Sampling techniques, 3rd edition. Wiley and Sons, New York. 413 pp.

Dare, P. J., and Palmer, D. W. 1994. The use of dredge efficiency factors for estimating indirectly population composition and abundance of scallops, *Pecten maximus* (L.). International Pectinid Workshop, Nanaimo, BC (Canada) 22–27 April 1993, Volume 2: pp. 137–142.

Dare, P. J., Key, D., and Connor, P. M. 1993. The efficiency of spring-loaded dredges used in the western English Channel fishery for scallops, *Pecten maximus* (L.). ICES CM 1993/B:15, 17 pp.

Fifas, S. 1991. Analyse et modélisation des paramètres d'exploitation du stock de coquilles Saint-Jacques (*Pecten maximus*, L.) en baie de Saint-Brieuc (Manche Ouest, France). Thèse de Doctorat d'Université, U.B.O., Brest. 422 pp.

Laurec, A. 1986. Les méthodes delta en halieutique. Evaluation des sensibilités, approximation des biais et variances à l'aide des développements limités. Rapp. int. IFREMER, DRV – 86.002, RH/Nantes. 64 pp.

Laurec, A., and Le Guen, J. C. 1981. Dynamique des populations marines exploitées. Tome I. Concepts et modèles. Rapports scientifiques et techniques. No 45, CNEXO. 118 pp.

Laurec, A., and Mesnil, B. 1987. Analytical investigations of errors in mortality rates estimated from length distribution of catches. *In* Length based methods in Fisheries Research, pp. 239–282. Ed. by D. Pauly, and G. R. Morgan. ICLARM Conf. Proc. 13, Manila, Philippines and Kuwait Institute for Scientific Research.

Lefebvre, J. 1980. Introduction aux analyses statistiques multidimensionnelles. Ed. Masson, Paris. 259 pp.

Lin, X.-Q. 1987. Etude de la biologie de la plie (*Pleuronectes platessa*, Linné) de la baie de Douarnenez: Croissance, Régime alimentaire, Reproduction. Thèse Univ. Bordeaux I: 181 pp.

Mason, J., Chapman, C. J., and Kinnear, J. A. M. 1979. Population abundance and dredge efficiency studies on the scallop, *Pecten maximus* (L.). *In* Population of shellfish stocks, pp. 91–96. Rapp. P.-V. Reun. CIEM, 175.

Millar, R. B. 1991. Estimating the size-selectivity of fishing gear by conditioning on the total catch: The SELECT (Share Each Lengthclass's Catch Total) model. ICES CM 1991/B:57, 21 pp.

Nelder, J. A., and Mead, R. 1965. A simplex method for function minimization, pp. 308–313. National Vegetable Research Station. Wellesbourne, Warwick.

Perez-Comas, J. A., and Skalski, J. R. 1996. A parametric multinomial model for size selection in alternate-haul experiments. Fisheries Research, 27: 113–129.

Shafee, M. S. 1979. Underwater observation to estimate the density and spatial distribution of black scallop, *Chlamys varia* (L.), in Lanveoc (Bay of Brest). Bulletin Off. Natl. Pêches (Tunisie), 3: 143–156.

Thouzeau, G. 1989. Déterminisme du pré-recrutement de *Pecten maximus* L. en baie de Saint-Brieuc. Thèse Doc. Univ., U.B.O., Brest. 525 pp.

Zhang, C. I., Ault, J. S., and Endo, S. 1993. Estimation of dredge sampling efficiency for blue crabs in Chesapeake Bay. Bulletin of the Korean Fish Society, 26: 369–379.