

## NUMERICAL MODELLING OF EQUILIBRIUM AND EVOLVING LIGHTWEIGHT SEDIMENT LABORATORY BEACH PROFILES

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### Abstract

The recent advances of numerical beach profile models allowed the simulation of on/offshore sandbar migrations on timescales of weeks to months with fair success. These models were systematically applied to natural, persistently evolving, beaches. In this contribution, we apply our model to small-scale laboratory experiments for which coarse and lightweight sediment is used to satisfy the laws of similitude in the flume. Such experiments can result in equilibrium beach profiles and provide detailed information on the respective role of undertow and wave nonlinearities on sediment transport and the resulting cross-shore sandbar migration. Here we first apply the coupled, wave-averaged, cross-shore waves-currents-bathymetric evolution model 1DBeach to an equilibrium beach profile. The model simulates an equilibrium beach profile with reasonable success. Yet, when applying the best fit parameters to a subsequent rapid onshore sandbar migration, the model fails in reproducing the overall beach profile evolution. Further model calibration on the evolving beach profile sequence shows that the model can actually reproduce the rapid onshore sandbar migration with a significant contribution of acceleration skewness. This suggests that a number of misspecifications of the physics remain in coupled, wave-averaged, cross-shore waves-currents-bathymetric evolution model. In addition, given that best-fit model free parameters are of the same order of magnitude of those found on natural beaches, our study suggests that small-scale experiments with coarse and lightweight sediment can be used to further explore the respective contribution of wave nonlinearities and undertow to sediment transport and the overall beach profile evolution

**Key words:** Beach profile, numerical model, physical modelling, equilibrium profile, sandbar migration.

### 1. Introduction

In recent years, numerical sandy beach profile models (*e.g.*, Ruessink *et al.*, 2007; Walstra *et al.*, 2012; Kuriyama, 2012; Dubarbier *et al.*, 2012; Dubarbier *et al.*, 2013; Castelle *et al.*, 2013) have succeeded in simulating on/offshore surfzone sandbar migrations on timescales of weeks to months with fair success. Typically, (1) offshore sandbar migration occurs during storms when large waves break on the bar due to the feedback between waves, undertow, suspended sediment transport, and the sandbar and (2) onshore bar migration is predicted for energetic, weakly to nonbreaking conditions due to the feedback between near-bed wave skewness, bedload transport, and the sandbar (*e.g.*, Ruessink *et al.*, 2007). In nature, the sandbar morphology is never in equilibrium with the hydrodynamic forcing given the persistent changes in natural wave and tide conditions. Accordingly, beach profile models were systematically applied to natural, persistently evolving, beaches. For each field site, model calibration is performed finding the best fit values of the free model parameters minimizing the difference between observed and predicted bed evolution using a given method (*e.g.*, global search algorithm, simulating annealing). The range of beach profile dynamics used for calibration typically encompasses a number of on/offshore migration .

A large number of laboratory experiments addressed beach profile evolutions. To satisfy the laws of similitude, experiments at scale 1 (*e.g.*, Wang and Kraus, 2005; Guannel *et al.*, 2007; Masselink *et al.*, 2013) or small-scale experiments with coarse and lightweight sediment (*e.g.*, Grasso *et al.*, 2009) have

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been performed to address cross-shore sandbar behaviour. For instance, these experiments allowed investigating equilibrium beach profiles (e.g., Wang and Kraus, 2005; Grasso *et al.*, 2009) and the impact of beach nourishments (Grasso *et al.*, 2011a). The respective role of undertow and wave nonlinearities on sediment transport and resulting cross-shore sandbar migration was also addressed through physical modelling (e.g., Grasso *et al.*, 2011b). Interestingly, the respective hydrodynamic contributions were quantified in detail for a given equilibrium beach profile (Michallet *et al.*, 2011).

In this paper, our objectives are: (1) to address the ability of our model to obtain an equilibrium beach profile; (2) to test if our calibrated model successfully simulates the subsequent evolving beach profile involving a rapid onshore sandbar migration ; (3) to compare our sediment transport best-fit parameters with those obtained on natural beaches (e.g., Duck, North Carolina, USA, and Egmond, The Netherlands, Dubarrier *et al.*, 2012) and recent full-scale laboratory experiments (Masselink *et al.*, 2013; Dubarrier *et al.*, 2013) and further discuss the validity of the coarse lightweight sediment strategy in small-scale laboratory experiments.

## 2. Methods

### 2.1. Wave flume experiment

The experimental set-up is extensively described in Grasso *et al.* (2009). The experiments are carried out in the 36 m long, 55 cm wide LEGI flume, equipped with a piston wave generator (Figure 1). The still water depth at the wave-maker is 55.3 cm. The sediment bottom consists of loose material of low density ( $1.19 \text{ g.cm}^{-3}$ ) with a median diameter  $d_{50} = 0.64 \text{ mm}$ . The sediment is chosen such that the Shields number  $\theta$  and the Rouse number  $Rou$  are of the same magnitude as those of natural environments. The different sediment transport regimes (bed load, sheet flow, suspension) are reproduced in the experiments. A Froude similitude links the time and length scales that are roughly 1/3 and 1/10, respectively. Irregular waves are generated (jonswap spectrum; peak enhancement factor of 3.3). The generated wave series are characterized by their significant wave height  $H_s$  and peak wave period  $T_p$ . Bottom profiles are recorded between wave runs using an acoustic profiler mounted on a motorized trolley. Two contrasting experiments are used in this contribution and are briefly described below.

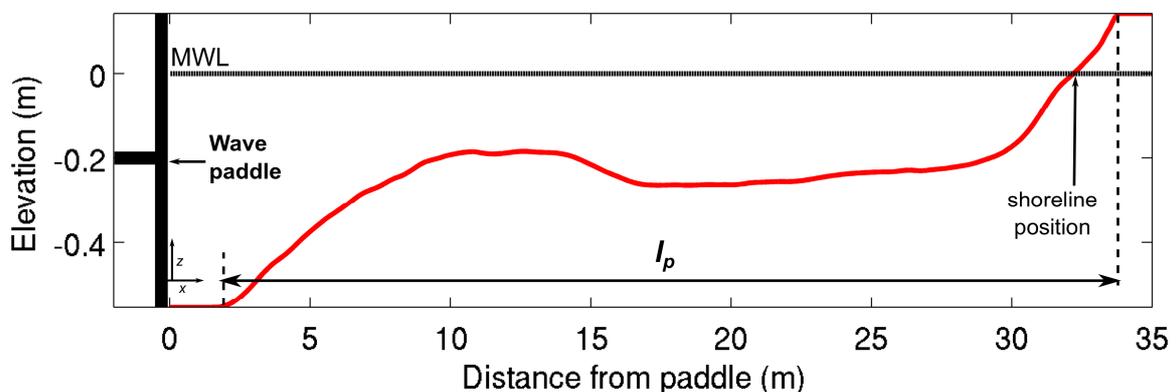


Figure 1. Schematic diagram of the LEGI wave flume:  $l_p$  is the active profile length.

#### 2.1.1. Equilibrium barred-beach profile experiment

The first experiment is the barred-beach quasi-equilibrium profile extensively described in Michallet *et al.* (2011) with  $H_s = 16 \text{ cm}$  and  $T_p = 2.5 \text{ s}$ . The same wave sequence of 30 min was repeated continuously for a total of 68 h. Starting from a terrace profile, a sandbar formed after 11 h of experiment. The bar grew until  $t \sim 24 \text{ h}$  and then started to migrate onshore (see Figure 5a). Here we used wave and flow measurements of the last 30 hours of experiment when the bar did not evolve significantly. This experiment is used to calibrate our coupled, wave-averaged, cross-shore waves-currents-bathymetric evolution model 1DBeach (Section 2.2).

### 2.1.2. Onshore sandbar migration experiment

The second experiment is the case of climate B2 ( $H_s = 16$  cm and  $T_p = 2.0$  s) in Grasso *et al.* (2009). During this sequence, the bar initially located offshore migrated rapidly onshore. During migration, the bar developed an asymmetric shape with a steeper slope shoreward (Figure 7a). This sequence is used to challenge our model previously calibrated on the equilibrium barred-beach profile.

## 2.2. Numerical model

### 2.2.1. Wave and flow module

The cross-shore distribution (along the axis  $x$ ) of the root mean square wave height  $H_{rms}$  is computed through the wave energy flux balance equation assuming that the wave field spectrum is narrow in frequency and direction. The breaking-induced wave dissipation is computed following Baldock *et al.* (1998) with the breaker wave height  $H_b$  computed from Battjes and Janssen (1978) using the depth-limited breaking parameter  $\gamma$  suggested by Ruessink *et al.* (2003). This parametrisation of wave dissipation was found to accurately reproduce wave height cross-shore distribution on a number of contrasting wave-dominated barred-beaches (*e.g.*, Ruessink *et al.*, 2003). The still water level  $\eta$  is computed using the conservation of momentum fluxes accounting for the roller contribution (Michallet *et al.*, 2011).

The mean return flow (undertow) that compensates the wave mass flux in the surface layer is solved through the mass conservation equation (Phillips, 1977). To account for the time short waves need to break due to the local variation of bed profile that subsequently affect the cross-shore distribution of the mean return flow, we used the delayed mean Stokes drift concept (Reniers *et al.*, 2004a) which depends on an integrated distance  $\lambda$ , proportional to the local wave length. We use the quasi-1DV mean current model proposed by Reniers *et al.* (2004b) to estimate a mean current value at the top of the bottom boundary layer, with the undertow injected in the set of equations to close the system. As the phase-averaged wave model is unable to estimate time series of orbital velocities, we used the relation between the *Ursell* number and an analytical formulation of wave orbital velocity time series (Abreu *et al.*, 2010) through the recent parameterization of the free-stream non-linear wave motion (Ruessink *et al.*, 2012) deduced from natural field conditions. These two flow components drive sediment transport and bottom changes described below.

### 2.2.2. Sediment transport and bottom change module

Sediment transport estimation is based on the work of Hsu *et al.* (2006) that accounts for bedload and suspended-load sediment transport, both associated with (1) the wave orbital velocities only and (2) mean current and interactions with oscillatory current. The gravitational downslope sediment transport contribution is also taken into account. Each sediment transport contribution is assigned to distinct friction coefficients here considered as free user parameters. The bottom changes at each time step are obtained by resolving the sediment mass conservation equation with the modified non-oscillatory central scheme described in Marieu *et al.* (2008).

### 2.2.3. Model set-up and calibration

The model was run on a regular grid with a 10-cm spacing and a morphological time step of 1 minute. First, the hydrodynamic module is validated with the dense wave, flow and surface elevation data gathered on the barred-beach equilibrium profile. For the beach profile evolution, in our present model configuration there are 3 free parameters: the sediment transport friction factors associated (1) with velocity skewness that control onshore sediment transport  $C_w$ , (2) with the mean current that governs offshore sediment transport  $C_c$  and (3) with the slope effect that control sandbar amplitude decay  $C_f$ . A simulated annealing (SA) algorithm (Bertsimas and Tsitsiklis, 1993) was used to find the best fit parameters. The advantage of this method is the possibility for the system to overcome local minima to eventually reach a global minimum in the error with measurements.

### 3. Results

#### 3.1 Calibration on the equilibrium barred-beach profile

##### 3.1.1. Hydrodynamics

Figure 2 shows the comparison of the simulated cross-shore distribution of wave height, mean water level and undertow with measurements for the equilibrium-barred profile (Figure 2a). Both simulated significant wave height (Figure 2b) and undertow (Figure 2d) are in very good agreement with measurements. Of note, a value of  $\lambda=1.3$  is necessary to accurately estimate the cross-shore maximum undertow position over the bar otherwise leading systematically to an offshore spatial lag (model/measurement) of some centimeters, finally this parameter have no impact on the spatial distribution of other hydrodynamic parameters. The mean sea level is in good agreement in shape but the model overestimates the set-up elevation over the trough region (Figure 2c). The latter is not an issue for our beach profile evolution simulations as the set-up does not impact sediment transport rate and, conversely, cross-shore distribution of both undertow and wave height is a critical component to the morphodynamics.

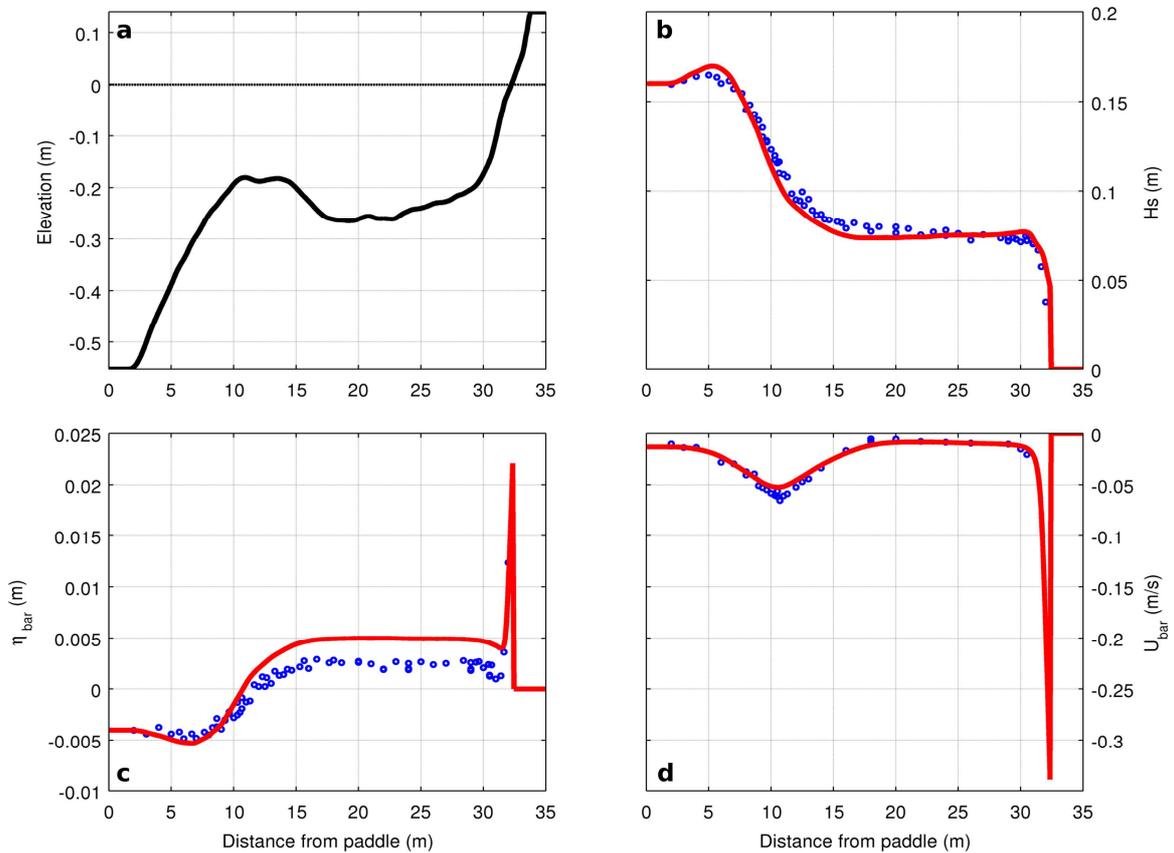


Figure 2. Validation of the hydrodynamic module of 1DBeach for the equilibrium barred-beach profile obtained in the LEGI flume: (a) seabed profile and cross-shore distribution of (b) significant wave height, (c) mean sea surface elevation and (d) undertow. In (b, c, d) model results and measurements are indicated in red and blue, respectively.

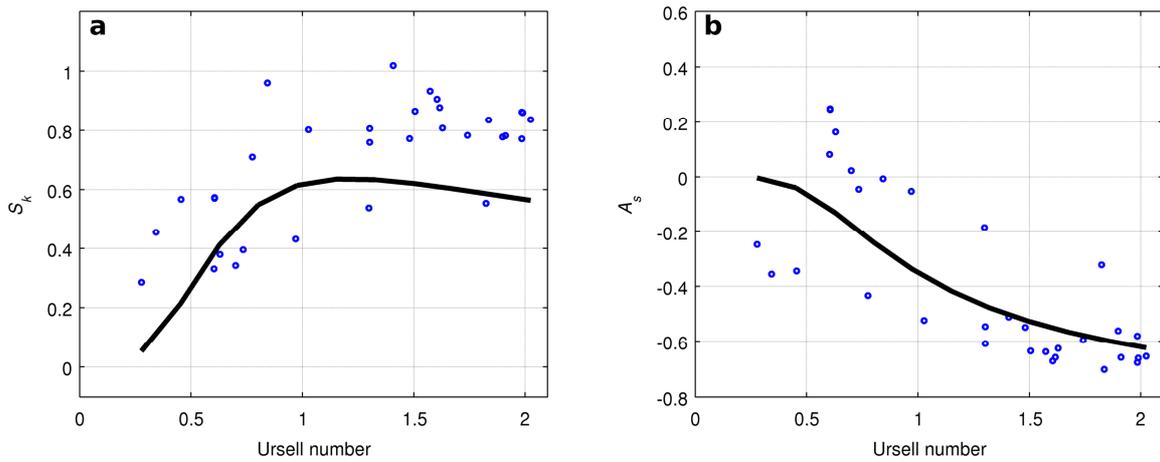


Figure 3. Velocity skewness ( $Sk$ ) and acceleration skewness ( $As$ ) versus Ursell number for the equilibrium barred-beach profile obtained in the LEGI flume. In both panels the model results and measurements are indicated by the solid black line and the blue circles, respectively.

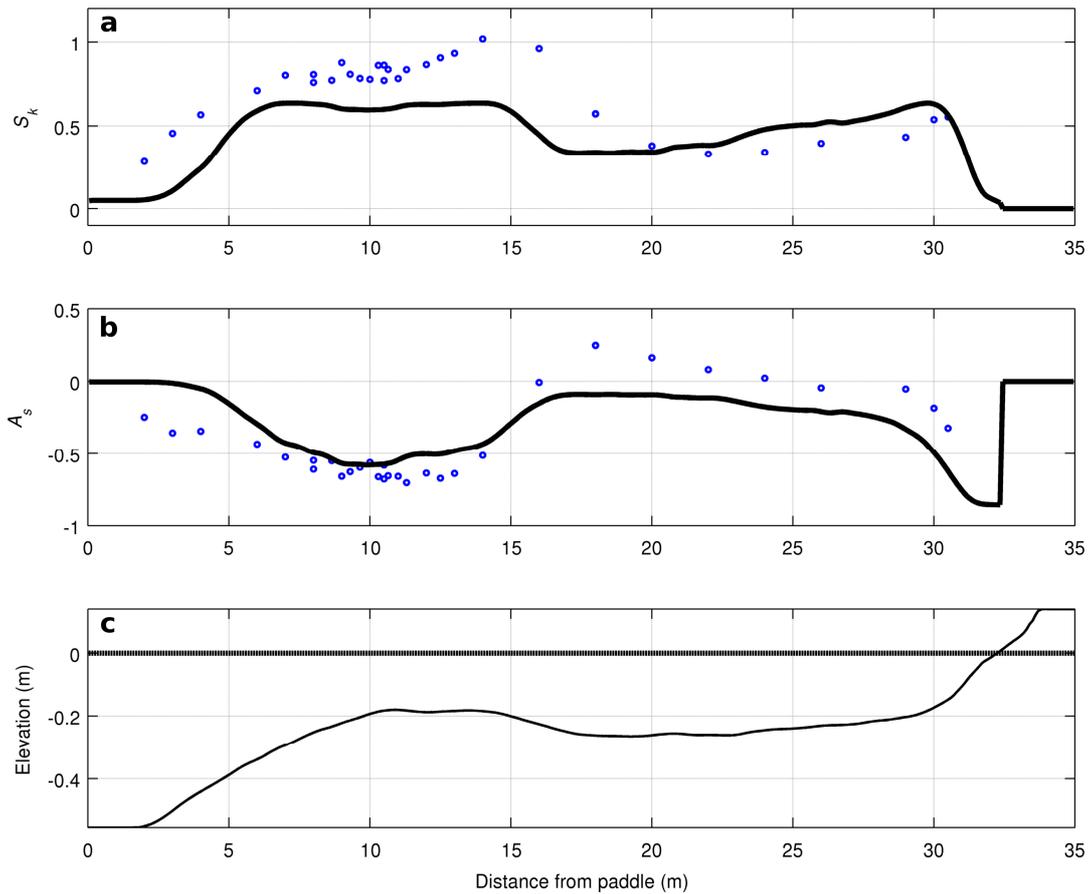


Figure 4. Cross-shore distribution of velocity skewness ( $Sk$ ) and acceleration skewness ( $As$ ) for the equilibrium barred-beach profile obtained in the LEGI flume. In (a,b) the model results and measurements are indicated by the solid black line and the blue circles, respectively.

For the same configuration, Figure 3 shows velocity skewness ( $Sk$ ) and acceleration skewness ( $As$ ) versus Ursell number. Despite the apparent scattering in the data, the model (fitted on  $Ur$  dependence only) predicts the overall shape and magnitude of  $Sk$  and  $As$ , besides we pointed out that the velocity skewness/asymmetry measured in the LEGI flume fall into the standard deviation ( $\pm 0.25$  for  $1 \leq Ur \leq 2$ ) of both respective parametric function derived from various field experiments (Figure 1a, b in Ruessink *et al.*, 2012). This is further confirmed in Figure 4 that shows the corresponding cross-shore distribution of  $Sk$  (Figure 4a) and  $As$  (Figure 4b) for the equilibrium barred-beach profile (Figure 4c). The latter further reveals that the model slightly underestimates  $Sk$  both in the shoaling zone and across the sandbar. Of note, this underestimation of the parametric velocity skewness function, derived from orbital velocity collected on natural beaches, can be explained by the absence of wave directional spreading in wave flume experiments (Ruessink *et al.*, 2012). In contrast, the model predicts good  $As$  values across the sandbar with estimations degrading in both the trough and the shoaling zone. Yet, overall the model predicts the cross-shore distribution of both  $As$  and  $Sk$  with fair accuracy.

### 3.1.2. Morphodynamics

Figure 5b shows the time evolution of the simulated beach profile for the equilibrium barred-beach equilibrium experiment. Results show that seabed changes are barely visible with no significant cross-shore migration of the patterns. This is further illustrated in Figure 6 that shows the comparison of the model results with measurements at  $t = 43$  h and 61 h together with initial profile ( $t = 29$  h). It clearly shows that the bar did not evolve significantly during the course of the experiment as only a tiny onshore sandbar migration can be depicted in the right-hand panel. The model successfully reproduces this steady bar situation. It is important to notice that, to obtain this quasi-steady state, the simulating annealing algorithm did not find 0 values to minimize sediment transport rates and therefore beach changes. Instead, values of friction factors are found to be in order of magnitude as ones found for observed, natural and full-scale experiment, unsteady sand bar dynamics (Dubarbier *et al.*, 2012, 2013), see table.1. Quasi-equilibrium barred-beach profile was therefore simulated with 1DBeach because onshore sediment transport driven by wave nonlinearities and sediment transport driven offshore by the undertow nearly balance in the simulation.

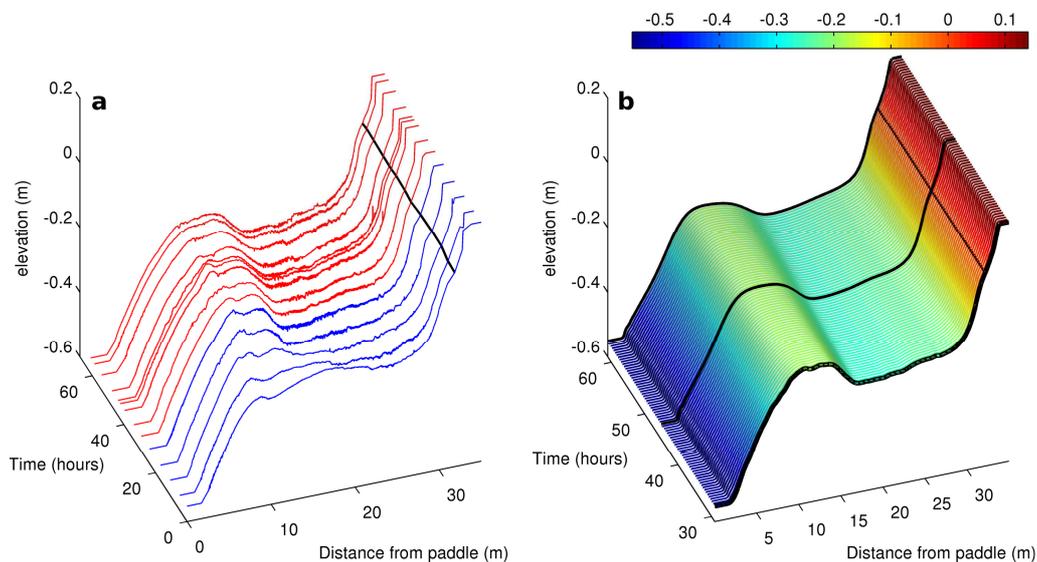


Figure 5. a) Time evolution of measured beach profile evolution from building bar (blue profiles) to quasi-equilibrium bar (red profiles) obtain in the LEGI flume, b) Time evolution of the simulated beach profile for the equilibrium barred-beach profile experiment in the LEGI flume. The 3 solid lines indicate the beach profiles simulated at  $t = 29, 47$  h and 67 h. Colorbar indicates seabed elevation in meters. Horizontal black lines indicate shoreline position.

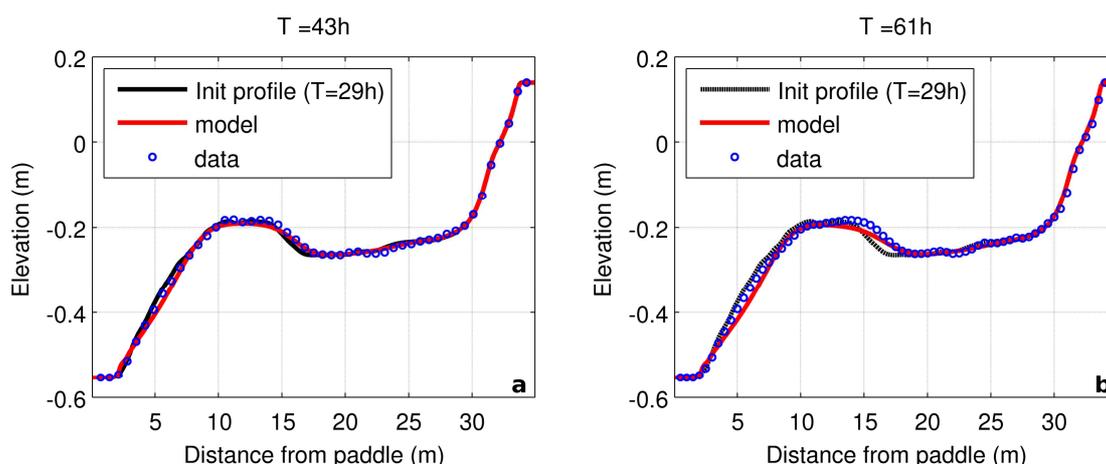


Figure 6. Comparison of simulated and measured beach profiles for the equilibrium barred-beach profile experiment in the LEGI flume at  $t = 43$  h (left-hand panel) and  $t = 61$  h (right-hand panel).

Table 1. Best-fit values of the free model parameters for natural beaches (Dubarbier *et al.*, 2012) and the LEGI flume equilibrium barred-beach profile.

Beach sites	Data set		Free model parameters				
	Resolution	Morphologic features	$C_w$	$C_c$	$C_f$	$\lambda$	$K_a$
LEGI small scale flume	30 hours	Single bar, equilibrium profile	0.0019	0.0027	0.0208	1.3	0
	4 hours	Single bar, continuous onshore migration	0.0019	0.0814	0.0515	2.20	0.0001
BARDEX full scale flume (C2)	3 hours	Terrace bar, on/offshore migration	0.0299	0.0283	0.089	0	0.0003
Duck82	3 months	Single bar, continuous onshore migration	0.0021	0.0024	0.0177	1.37	0
Duck94	10 days	Single bar, on/offshore migration	0.0064	0.0197	0.0332	2.03	0
Egmond	1 month	Double bar, continuous offshore migration	0.0075	0.0157	0.0513	2.80	0

### 3.2 Onshore sandbar experiment using calibration from the equilibrium barred-beach experiment

Figure 7 shows the application of 1DBeach to the rapid onshore sandbar migration event using the best-fit parameters found for the quasi-equilibrium profile. Measurements from the LEGI flume (Figure 7a) show that, starting from single-barred beach configuration, the bar migrates rapidly onshore and welds to the shore at  $t \sim 4$  hours forming a large berm. Results show that the model does not reproduce the onshore migration (Figure 7b). Instead, the bar exhibits decreasing amplitude and remains approximately at the same location, suggesting that model calibration must be performed using data depicting a significantly evolving beach profile sequence.

### 3.3 Calibration on the onshore sandbar experiment

Figure 8 shows the simulated onshore sandbar migration using 1DBeach calibrated on the onshore sandbar migration experiment. Results confirm that 1DBeach can simulate the onshore sandbar migration event with fair accuracy. In particular, the model reproduces the strongly asymmetric shape of the sandbar as well as the water depth of the bar crest. Of note, small numerical instabilities appear seaward of the bar crest. For this calibration, there are only slight changes in the values of  $C_w$  and  $C_f$ . Conversely,  $C_c$  which is related to mean return flow, is found to be nearly 20 times the value found for the equilibrium experiment. The reason for this significant increase in  $C_c$  must be explored further. A critical component was the inclusion of acceleration skewness in our sediment transport formulation, which was not a necessary

requirement for the equilibrium barred-beach profile as well as for natural wave-dominated sandy-beach profile evolution (see Table.1). Similar conclusions were made when applying 1DBeach to BARDEX II experiment (Masselink *et al.*, 2013) during which rapid off/onshore sandbar migration were observed (Dubarbier *et al.*, 2013; Castelle *et al.*, 2013), suggesting that acceleration skewness is important to accurately simulate rapid morphological readjustments involving onshore sandbar migration. This can also support the idea that a particular wave-flume free-stream non-linear parametrization is required when applying a beach profile model to a given wave flume experiment.

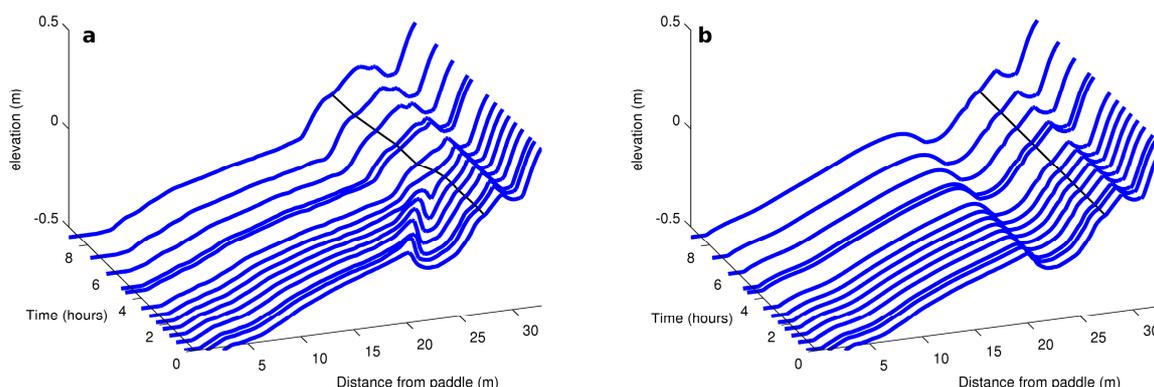


Figure 7. Rapid onshore sandbar experiment with the time evolution of (a) the beach profiles measured in the LEGI flume and (b) beach profile simulated using 1DBeach with best-fit parameters calibrated on the equilibrium barred-beach profile. Horizontal black lines indicate shoreline position.

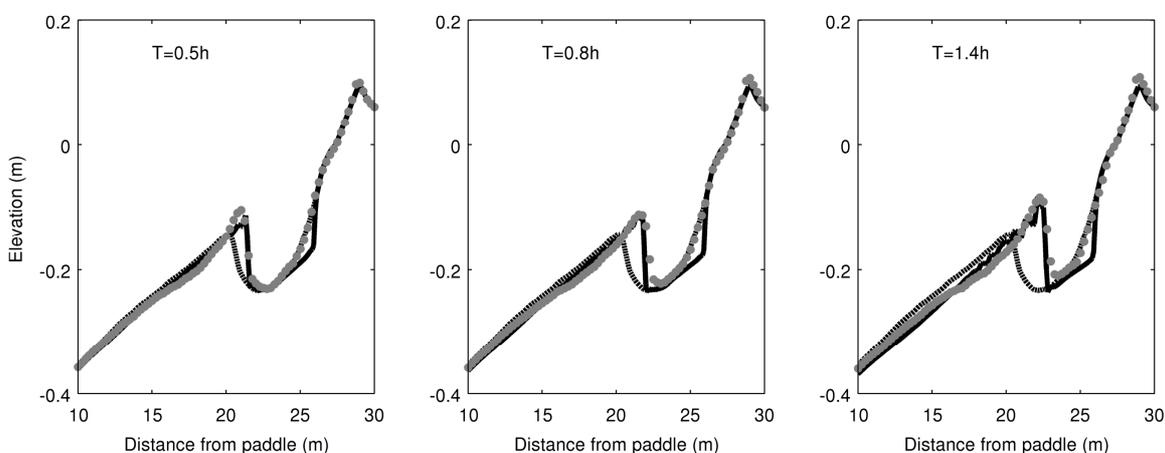


Figure 8. Simulation of the rapid onshore sandbar migration at  $t = 0.5, 0.8$  and  $1.4$  hours with best-fit free parameters determined through simulating annealing on this specific evolution. In all panels, model result (solid black line) and measurements (grey dots) are superimposed

#### 4. Conclusions

We showed that our coupled, cross-shore waves (phase averaged) – currents - bathymetric evolution model 1DBeach can simulate an equilibrium barred-beach profile obtained in the LEGI flume with fair accuracy. Best-fit model free parameters are of the same order of magnitude of those found on natural beaches. Quasi-equilibrium is reached because onshore sediment transport driven by wave nonlinearities and sediment transport driven offshore by the undertow nearly balance. This supports the use of small-scale experiments with coarse and lightweight sediment to explore wave-dominated sandy beach morphodynamics. Yet, using the latter best-fit parameters, the model fails in simulating the subsequent

onshore sandbar migration. Performing calibration on the onshore sandbar migration shows that the model can actually simulate the rapid onshore bar migration with some variations in the free parameter values but the inclusion of sediment transport driven by acceleration skewness appear to be a key factor to simulate accurately onshore sandbar sequence in wave flume condition. Overall, our studies suggests that (1) some misspecification of the physics remain in beach profile evolution models, (2) model calibration must be performed on a range of on/offshore sandbar migration and (3) small-scale experiments with coarse and lightweight sediment can be used to further explore the respective contribution of wave nonlinearities and undertow to sediment transport and the overall beach profile evolution.

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