C2 COMPARISON OF TIME DOMAIN CONTROL LAWS FOR A PISTON WAVE ABSORBER

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ABSTRACT

The final goal of the study is the derivation of efficient time domain control strategies for the absorption of water waves by the motion of rigid bodies. In this paper we focuse our attention on the case of a very simple device, a 2D piston, in order to exploit the well known analytic expressions of the potential as far as possible in the design of novel absorption laws.

We consider a semi-infinite two-dimensional wave tank closed by a mobile vertical plate. An unsteady wave train generated at infinity impinges on this plate. The problem of dynamic wave absorption consists in finding, in real time, the velocity to give to the plate in order that the radiated and the reflected wave train should cancel each other. In the present work, the paddle is supposed to be moved by an external mechanism in response to the dynamical part of the fluid force measured on it.

In 1970, Milgram studied such a wave absorbing rotating plate, but the control system was based on a wave height feedback. For his hinged paddle absorbing device, Salter (1979) used a force feedback which included inertia and hydrostatic components.

In the present study, we first derive a frequency dependent transfer function between the optimal velocity of the paddle and the total force for the case of steady time harmonic incident waves. As a consequence of the linear approach we choose, the time domain paddle velocity leading to the complete absorption of the incident wave train is obtained by convoluting the inverse Fourier transform of this transfer function by the measured hydrodynamic force. Unfortunately, the impulse response function of the ideal absorber derived that way is not causal; thus it cannot be used just as it is as the control loop of a physical absorbing device.

In this paper, we suggest two causal non-ideal approximations of this ideal non-causal controller. These time-domain absorbing relations differ in whether or not one knows a dominant frequency of the incident wave train to be absorbed. Their performances are compared to the absorption efficiency of the low frequency asymptotic Sommerfeld relation. Preliminary results of this study were already presented in Maisondieu and al. (1993).

NOMENCLATURE

f=	total hydrod. force (time domain)
h=	impulse response function of the ideal feedback controller
Q=	Fourier transform of q(t)
K=	transfer function of the feedforward loop

i= complex unit: $i^2=-1$
k= impulse response function of the feedforward loop
p= causal part of k(t)
q= even part of k(t)
u= time domain piston velocity
t= time

F= total hydrod. force (freq. domain)
F_D= hydrod. force due to reflected potential
F_I= hydrod. force due to incident potential
F_R= hydrod. force due to radiation potential
H= transfer function of the ideal feedback controller

M= added mass
N= damping coefficient
P= Fourier transform of p(t)
U= frequency domain piston velocity
X,Y= spacial coordinates

α= coefficient of the approx. of q(t)
Φ_I= incident wave potential
Φ_D= reflected wave potential
Φ_R= radiated wave potential
ω= dominant frequency in FDFF mode
Ω= radian frequency

* denotes complex conjugate.
~ denotes the optimal regime, i.e 100% absorption in the frequency domain.

1.0 THE IDEAL 2D PISTON WAVE ABSORBER

1.1 frequency domain

Let $\Phi_I e^{i\Omega t}$ be the complex Airy velocity potential of the incident left-going wave train (see Fig.1), and $\Phi_D e^{i\Omega t}$ the potential of the corresponding (right-going) reflected wave with Ω the frequency.

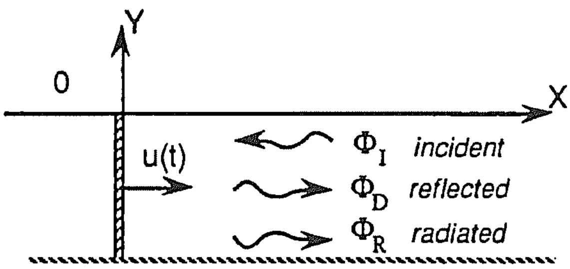

Figure 1: *principle of dynamic absorption*

When a time harmonic motion is imposed to the paddle with a given velocity law: $u(t)=\Re\{U.e^{i\Omega t}\}$, a right-going wave train deriving from the radiated velocity potential $\Phi_R e^{i\Omega t}$ is generated in the basin. The induced linearized hydrodynamic force acting upon the paddle surface may be simply derived by integrating the dynamic pressure $p=-\Phi_t$ from the bottom (Y=-1) to the free surface (Y=0). The radiation force then reads: $F_R e^{i\Omega t}=[N(\Omega)+i\Omega M(\Omega)]U e^{i\Omega t}$, where $N(\Omega)$ and $M(\Omega)$ are the well known damping and added-mass coefficients.

In the linearized theory, the total velocity potentiel is the algebraic sum of the three above mentionned components: $\Phi_T=\Phi_I+\Phi_D+\Phi_R$ and the total force upon the paddle: $F_T e^{i\Omega t}=\{F_I+F_D+[N(\Omega)+i\Omega M(\Omega)]U\}e^{i\Omega t}$

The complete absorption of the incident wave train requires the velocity U to be such that, at least at a certain distance d from the paddle, the reflected waves and the propagating part of the radiated waves cancel each other. Let us denote by $\tilde{U}$ the optimal complex value (amplitude and time phase) of the piston velocity U leading to this ideal result. Thus, for every given frequency Ω, we can determine the complex transfer function $H(i\Omega)$ of the ideal wave-absorber controller which would give access to the optimal velocity from the measured hydrodynamic force:

$$H(i\Omega) = \frac{\tilde{U}(i\Omega)}{F(i\Omega)} = \frac{1}{N(\Omega) - i\Omega M(\Omega)}$$

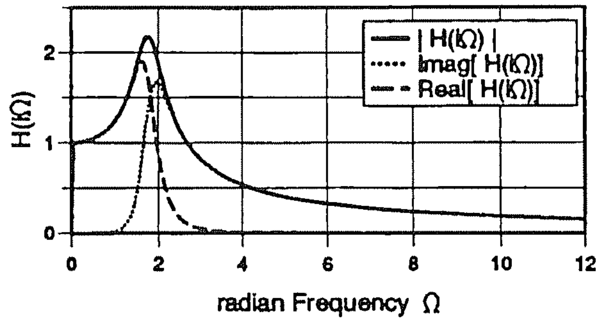

Figure 2: *Force-to-Velocity transfer function of the ideal piston wave-absorber*

1.2 Time domain

The real and imaginary parts of $H(i\Omega)$ are plotted on fig. 2 above. This feedback controller being linear and time invariant, the classical theory of LTI systems results in that its impulse response function $h(t)$ is the inverse Fourier transform of its transfer function in the frequency domain. Then, its output in the time domain $\tilde{u}(t)$ will be given by the following convolution integral:

$$\tilde{u}(t) = \int_{-\infty}^{+\infty} f(\tau)h(t-\tau)d\tau$$

where $f(t)$ is the time varying hydrodynamic force. On figure 3, we have plotted $h(t)$ and $h^*(t)$ which are the inverse Fourier transform of respectively $H(i\Omega)$ and its complex conjugate:$H^*(i\Omega)$. As one can see, $h(t)$ is "anticausal" (*i.e* h(t)=0; t≥0), while $h^*(t)$, which is by construction its symetric with respect to the time variable, is causal.

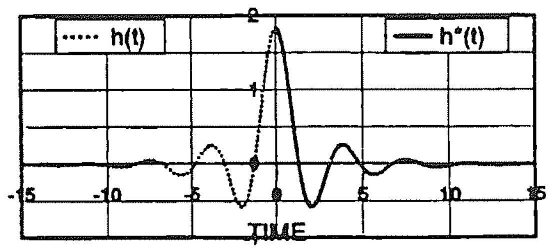

Figure 3: *direct and retrograde impulse response functions of the ideal wave-absorber.*

Refering to the convolution integral above, that means that the calculation of the optimal velocity $\tilde{u}(t)$ to be given to the paddle to achieve total absorption involves all the future values of the input $f(t)$, but none from the past ! In other words, we may always derive the transfer function of the ideally wave-absorbing plate, but we cannot practically realize it.

2.0 THREE CAUSAL APPROXIMATIONS OF THE IDEAL WAVE ABSORBER

2.1 Low Frequency Limit (LFL) mode

On figure 2, one can see that in the low frequency range (i.e $0 \le \Omega \le 1.$), the phase lag between force and optimal velocity is negligible while the gain remains in the range: $1 \le |H(i\Omega)| \le 1.2$ .Thus, a very first rough approximation of a time domain absorption relation between measured force and velocity stands in the limit relation: $\bar{u}(t) = f(t)$. This is nothing but the well known Sommerfeld relation between local pressure and normal velocity [see e.g Orlanski (1976)], integrated from $Y=-1$ to $Y=0$; it is exact in the limit $\Omega \to 0$. As a logical consequence, the performances of this absorption relation are very good in this low frequency range, as we shall see later (fig. 6).

Decomposition of the transfer function: The inverse Fourier transform $k(t)$ of the function $K(i\Omega)$ defined below can be shown to be the sum of a causal function $p(t)$, and an even function of time $q(t)$ (fig. 4). Thus, introducing $K(i\Omega)$ in the transfer relation we obtain: $\bar{U} = P(i\Omega)\bar{U} + Q(i\Omega)\bar{U} - H^*(i\Omega)F$.

$$K(i\Omega) = 1 + \frac{N(\Omega) - i\Omega M(\Omega)}{N(\Omega) + i\Omega M(\Omega)}$$

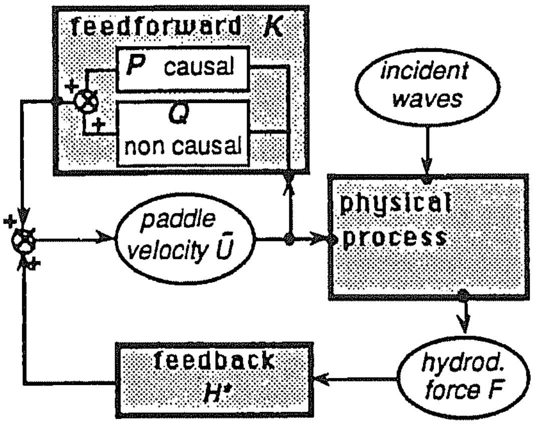

Figure 5: *decomposition of Ideal controller*

In the time domain, assuming the fluid to be at rest for $t \le 0$, we have by inverse Fourier transforming:

$$\bar{u}(t) = \int_0^t p(t-\tau)\bar{u}(\tau)d\tau + \int_0^{\infty} q(t-\tau)\bar{u}(\tau)d\tau - \int_0^t h^*(t-\tau)f(\tau)d\tau$$

The first two terms calculated from $\bar{u}(t)$ may be regarded, for the whole system, as a feedforward control loop with a non-causal part Q (see fig.5) due to the symetry of $q(t)$ with respect to $t=0$ (see fig.4) ; the force feedback term is now causal due to the complex conjugation in the frequency domain ($H \leftrightarrow H^*$). At that stage, the motion controller is not yet realizable, but a substancial step has been made toward this goal with this decomposition. No approximation has been introduced, and the instantaneous velocity defined by the above relation is always the *optimal* one in that sense that, if we were able to compute it at time t from the knowledge of the past, it would absorb 100% of any incident wave train. We shall now propose two different causal approximations of $q(t)$ and evaluate their absorption efficiency.

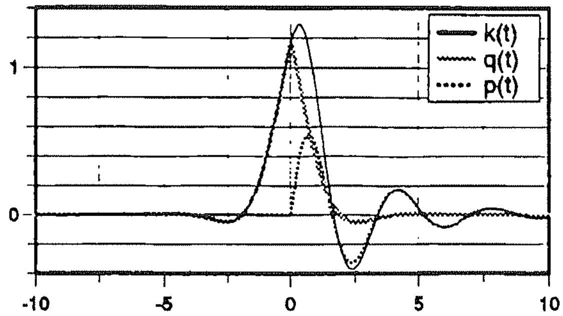

Figure 4: *impulse responses of the feedforward loop.*

2.2 Purely Unsteady Feedback-Feedforward (PUFF) causal approximation.

The sharp shape of the even function $q(t)$ around the origin (fig. 4) suggested us to approximate it by a Dirac δ function, and write: $q(t) \equiv \alpha\delta(t)$. From energetic arguments, the weight α was set equal to the surface under the curve $q(t)$ which is itself equal to $Q(0)$. The absorption relation is now given by :

$$u(t) = \frac{1}{1-\alpha}\left[\int_0^t p(t-\tau)\ddot{u}(\tau)d\tau - \int_0^t h^*(t-\tau)f(\tau)d\tau\right]$$

This absorption mode is said to be *purely unsteady* because, as the *LFL* mode, it does not require any spectral knowledge of the incident wave train.

2.3 Frequency Dependent Feedback-Feedforward (FDFF) causal approximation.

In this second approach, the incident wave train spectrum was assumed to present a known dominant frequency ω. In that case, the coefficient α of the *PUFF* relation was tuned to the exact real value $Q(i\omega)$ of the frequency domain transfer function previously defined. The *FDFF* absorption relation is the same as the *PUFF* relation above after the substitution: $\alpha \mapsto Q(i\omega)$.

3.0 RESULTS

These absorption relations were implemented in a "2D linearized numerical wave tank" (Dommermuth and al 1988). At one end, a piston wavemaker generated a short wave train consisting in a monochromatic harmonic wave modulated by a linear up and down ramp window. The motion of the opposite piston end, initially at rest like the fluid itself, was deduced from the absorption laws studied herein in response to the total hydrodynamic force. The measured wave amplitude absorption coefficients are plotted on fig. 6. They have to be squared in order to obtain the corresponding energy coefficients.
It is clear from this figure that the purely unsteady feedback-feedforward method brings only a little improvement with respect to the low frequency limit.

On the other hand, the FDFF control scheme give excellent results even in the high frequency range as soon as one can identify a priori a dominant frequency in the incident wave train. This conclusion agrees with the conclusions of preceeding studies about non radiating numerical boundary conditions for unsteady water waves simulations. For broad banded incident spectrum, we are convinced that a more efficient relation remains to be developped.

An experimental prototype of the device is being developped at LHN laboratory in a narrow wave tank, in order to check the feasibility of this approach.

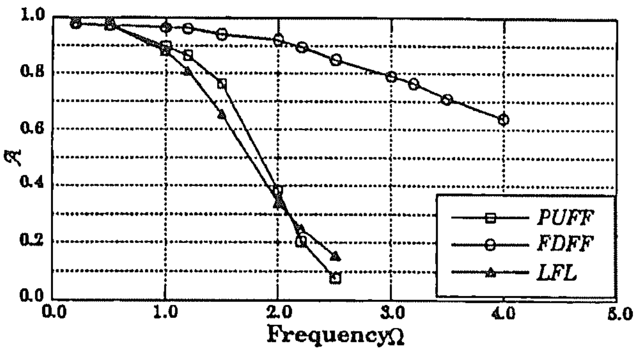

Figure 6: *absorption efficiency (amp. ratio)*

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