Supporting Information for "Low-mode internal tides and balanced dynamics disentanglement in altimetric observations: synergy with surface density observations"

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Introduction This supplementary material describes the stratification profiles used for initiation and relaxation in the numerical simulations, as well as the result of a stability calculation based on initial conditions.

Text S1.

We start by computing the following intermediate profiles:

\[ \rho_s^0(z) = \rho_{\text{max},s} - \lambda_s(z + h) - \Delta_s \frac{1 + \tanh(\zeta_s - z_{1,s})}{\delta_s} \]

\[ \zeta_s(z) = z_{1,s} + (z - z_{1,s}) \sqrt{1 + \frac{(z - z_{1,s})^2}{(1.3\delta_s)^2}} \]

\[ \rho_n^0(z) = \rho_{\text{max},n} - \lambda_n(z + h) - \Delta_n \frac{1 + \tanh(\zeta_n - z_{1,n})}{\delta_n} \]

\[ \zeta_n(z) = z_{1,n} + (z - z_{1,n}) \sqrt{1 + \frac{(z - z_{1,n})^2}{(1.3\delta_n)^2}} \]
where parameters are described in table S1 and:

\[
\Delta_n = -2 \frac{\rho_s^0(z = 0) - \rho_{\text{max},n} + \lambda_n h}{1 + \tanh \left\{ \left( \zeta_s(z = 0) - z_{1,n} \right) / \delta_n \right\}}
\] (5)

The final profiles are given by:

\[
\rho_s(z) = \rho_s^0(z) - \Delta_s^0 \alpha_1 e^{(z-z_0)/|z_0|/e^1} - \Delta_s^0 \alpha_2 \frac{1 + \tanh(z - z_0)/|z_0|}{2 \tanh(1)}
\] (6)

\[
\rho_n(z) = 0.5 \times \left\{ \rho_n^0(z) - \Delta_n^0 \alpha_1 e^{(z-z_0)/|z_0|/e^1} - \Delta_n^0 \alpha_2 \frac{1 + \tanh(z - z_0)/|z_0|}{2 \tanh(1)} \right\} + 0.5 \times \rho_s(z)
\] (7)

The profiles are joined together in order to construct the full three-dimensional density field, according to:

\[
\rho_i(x, y, z) = \left[ 1 - \gamma(y) \right] \rho_{\text{south}}(z) + \gamma(y) \rho_{\text{north}}(z), \text{ with :}
\]

\[
\gamma(y) = 0.25 \times \left( 1 + \tanh \left( \frac{y - y_m + L_y/2}{5 \times 10^4} \right) \times \left( 1 - \tanh \left( \frac{y - y_m - L_y/2}{5 \times 10^4} \right) \right) \right)
\] (9)

where \( L_y = 800 \) km and \( y \) the coordinate in the y direction at the center of the domain.

Pressure is computed by vertical integration of the density field with a constraint of no pressure at the sea floor. Horizontal velocities are estimated assuming geostrophy.
References

Table S1. Northern and southern profile parameters. Additional parameters: $\alpha_1 = 0.0075$, $\alpha_2 = 1$, $z_0 = -300$ m.

<table>
<thead>
<tr>
<th>Profile</th>
<th>$\rho_{\text{max}}$</th>
<th>$\lambda$</th>
<th>$\Delta$</th>
<th>$z_1$</th>
<th>$\delta$</th>
<th>$\Delta^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>27.7573</td>
<td>$9.8 \times 10^{-6}$</td>
<td>see (5)</td>
<td>-400</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td>South</td>
<td>27.75</td>
<td>$9.8 \times 10^{-6}$</td>
<td>1.4</td>
<td>-1000</td>
<td>700</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Figure S1. Stability analysis based on initial conditions at the center of the jet. The calculation follows Smith [2007]. Left: growth rate of unstable modes as a function of the zonal wavenumber. Right: modal structure of the deep mode ($k = 2.1 \times 10^{-5} m^{-1}$, black) and shallow mode ($k = 2.0 \times 10^{-4} m^{-1}$, grey); full and dashed lines are real and imaginary parts. Note that the shallow mode is barely resolved by the ROMS numerical simulations.