Supporting Information for "Low-mode internal tides and balanced dynamics disentanglement in altimetric observations: synergy with surface density observations"

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Introduction This supplementary material describes the stratification profiles used for initiation and relaxation in the numerical simulations, as well as the result of a stability calculation based on initial conditions.

Text S1.

We start by computing the following intermediate profiles:

$$\rho_s^0(z) = \rho\text{max,s} - \lambda_s(z+h) - \Delta_s \frac{1+\tanh(\zeta_s - z_{1,s})/\delta_s}{2} \tag{1}$$

$$\zeta_s(z) = z_{1,s} + (z - z_{1,s}) \times \sqrt{1 + \frac{(z-z_{1,s})^2}{(1.3\delta_s)^2}} \tag{2}$$

$$\rho_n^0(z) = \rho\text{max,n} - \lambda_n(z+h) - \Delta_n \frac{1+\tanh(\zeta_n - z_{1,s})/\delta_n}{2} \tag{3}$$

$$\zeta_n(z) = z_{1,n} + (z - z_{1,n}) \times \sqrt{1 + \frac{(z-z_{1,n})^2}{(1.3\delta_n)^2}} \tag{4}$$

where parameters are described in table S1 and:

$$\Delta_n = -2\frac{\rho_s^0(z=0) - \rho\text{max,n} + \lambda_n h}{1 + \tanh\{(\zeta_s(z=0) - z_{1,n})/\delta_n\}} \tag{5}$$

The final profiles are given by:

$$\rho_s(z) = \rho_s^0(z) - \Delta_s^s \alpha_1 e^{(z-z_0)/|z_0|}/e^1 - \Delta_s^s \alpha_2 \frac{1 + \tanh(z - z_0)/|z_0|}{2\tanh(1)} \tag{6}$$

$$\rho_n(z) = 0.5 \times \left\{\rho_n^0(z) - \Delta_n^s \alpha_1 e^{(z-z_0)/|z_0|}/e^1 - \Delta_n^s \alpha_2 \frac{1 + \tanh(z - z_0)/|z_0|}{2\tanh(1)}\right\} + 0.5 \times \rho_s(z) \tag{7}$$

The profiles are joined together in order to construct the full three-dimensional density field, according to:

$$\rho_i(x, y, z) = [1 - \gamma(y)]\rho_{\text{south}}(z) + \gamma(y)\rho_{\text{north}}(z), \quad \text{with :} \tag{8}$$

$$\gamma(y) = 0.25 \times (1 + \tanh\frac{y - y_m + L_y/2}{5 \times 10^4}) \times (1 - \tanh\frac{y - y_m - L_y/2}{5 \times 10^4}) \tag{9}$$

where $L_y = 800$ km and it the coordinate in the y direction at the center of the domain. Pressure is computed by vertical integration of the density field with a constraint of no pressure at the sea floor. Horizontal velocities are estimated assuming geostrophy.

References

Smith, K. S. (2007), The geography of linear baroclinic instability in earth's oceans, *J. Mar. Res.*, *65*, 655–683.

Table S1. Northern and southern profile parameters. Additional parameters: $\alpha_1 = 0.0075$, $\alpha_2 = 1$, $z_0 = -300$ m.

Profile	ρmax	λ	Δ	z_1	δ	Δ^s
North	27.7573	9.8×10^{-6}	see (5)	-400	300	0
South	27.75	9.8×10^{-6}	1.4	-1000	700	1.5

jet1_instab.eps

Figure S1. Stability analysis based on initial conditions at the center of the jet. The calculation follows *Smith* [2007]. Left: growth rate of unstable of modes as a function of the zonal wavenumber. Right: modal structure of the deep mode ($k = 2.1 \times 10^{-5}\mathrm{m}^{-1}$, black) and shallow mode ($k = 2.0 \times 10^{-4}\mathrm{m}^{-1}$, grey); full and dashed lines are real and imaginary parts. Note that the shallow mode is barely resolved by the ROMS numerical simulations.