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Corrigenda of 'Explicit wave-averaged primitive equations using a Generalized Lagrangian Mean'

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Abstract :

Ardhuin et al. (2008) gave a second-order approximation in the wave slope of the exact Generalized Lagrangian Mean (GLM) equations derived by Andrews and McIntyre (1978), and also performed a coordinate transformation, going from a from GLM to a 'GLMz' set of equations. That latter step removed the wandering of the GLM mean sea level away from the Eulerian-mean sea level, making the GLMz flow non-divergent. That step contained some inaccuarate statements about the coordinate transformation, while the rest of the paper contained an error on the surface dynamic boundary condition for viscous stresses. I am thankful to Mathias Delpey and Hidenori Aiki for pointing out these errors, which are corrected below.

1 Surface boundary condition and virtual wave stress

In their equation (61) Ardhuin et al. (2008) wrongly separated the average stress as a horizontal force and a vertical force acting on the surface. Instead, it is more appropriate to follow the many investigations on the subject and to separate the stresses as a normal and a tangential stress (e.g. Jenkins, 1992). The horizontal component of the normal stress is the usual pressure-slope correlation that we will denote τ_w, and which is often called 'wave-supported stress' which is slightly incorrect given the other (small) part of the wave-supported stress that is the oscillatory shear stress. The intesting part here, and the source of error in the paper is the tangential stress, which is along the surface has a local value given by Longuet-Higgins (1953), and also discussed in Xu and Bowen (1994), among others. Here we use the expression given by Dore (1970, eq. 3.18), valid at the instaneous sea surface $z = \zeta$

$$\tau_t = \mu \left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - 4 \frac{\partial \zeta}{\partial x} \frac{\partial u}{\partial x} \right]. \tag{1}$$

As stated by Dore (1970), the proper boundary condition is thus given by the continuity of both the normal stresses and tangential stresses. In the case of the GLM equations, the boundary condition for the tangential stress is thus the horizontal projection of the GLM of eq. (2).

$$\overline{\tau_t}^L = \mu \left[\overline{\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}}^L - 4 \overline{\frac{\partial \zeta}{\partial x} \frac{\partial u}{\partial x}}^L \right]. \tag{2}$$

This tangential stress is clearly of second order in the wave slope ka, and thus its projection on the horizontal only gives third or higher order correction. For the purpose of averaging, we can thus consider this stress to be horizontal.

For the case of a zero quasi-Eulerian current, the first of these two terms was already evaluated by Ardhuin and Jenkins (2006) using linear wave theory, which is sufficient here given the small deviation caused by viscosity (e.g. Dore, 1970, 1978). We thus only need to add the quasi-Eulerian mean current $\widehat{u}$ in the average, namely

$$\overline{\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}}^L = \frac{\partial (U_s + \widehat{u})}{\partial z} \tag{3}$$

with U_s the Stokes drift velocity, which, for monochromatic waves of amplitude a, radian frequency σ and wavenumber k is given by

$$U_s = \sigma k \frac{a^2}{2} \frac{\cosh(2kz + 2kh)}{\sinh^2(kD)}. \tag{4}$$

In the second term, due to the product of the surface slope and velocity gradient, which are each first-order terms in the wave slope ka, it is enough to use an Eulerian approximation to the Lagrangian average, based on velocities in the water, extrapolated to the mean sea surface at $\zeta = 0$. A rigourous Taylor expansion up to $\zeta = 0$ yields extra term which are at least of order $(ka)^2$ for the velocity gradient, and thus $(ka)^3$ when multiplied by the slope. This has already been evaluated by e.g. (Xu and Bowen, 1994, eq. 48),

$$4\overline{\frac{\partial \zeta}{\partial x}\frac{\partial u}{\partial x}} = \frac{\partial U_s}{\partial z}. \tag{5}$$

Hence the two terms of τ_t that contain $\partial U_s/\partial z$ cancel, leaving $\partial \widehat{u}/\partial z = 0$ at this order, on $z = \overline{\zeta}$.

We thus obtain the mean continuity equation across the air-sea boundary in which the wind stress τ is the sum of the mean tangential and wave-supported stresses right at the sea surface

$$\tau = \overline{\tau}^L = \mu \frac{\partial \widehat{u}}{\partial z} + \tau_w \quad \text{at} \quad z = \overline{\zeta}. \tag{6}$$

We note, however, that the subsurface shear, at a small distance δ from the free surface is increased by the momentum lost by waves associated to wave dissipation. Indeed, in the absence of wind, $\tau = \tau_w = 0$ and $\partial \widehat{u}/\partial z = 0$ right at the surface. But this shear increases away from the surface, as measured in the laboratory by Longuet-Higgins (1970), who found, at the base of the viscous layer, $\partial\widehat{u}/\partial z = \partial U_s/\partial z$. Indeed, the glm2z momentum equations (eq. (42) in Ardhuin et al., 2008) can be integrated from $z = \overline{\zeta}$ down to $z = \overline{\zeta} - \delta$, provided that the force $\widehat{X}_\alpha$ is understood to represent all the momentum lost by the wave field, including viscous dissipation. In the absence of wind, and for constant wave conditions we have,

$$\mu \frac{\partial \widehat{u}}{\partial z}\bigg|_{z=\overline{\zeta}-\delta} = -\int_{\overline{\zeta}-\delta}^{\overline{\zeta}} \widehat{X} \mathrm{d}z. \tag{7}$$

For viscous dissipation, the momentum X lost by the wave field per unit volume is concentrated near the surface, and its integral is the rate of loss of wave energy divided by the phase speed, i.e.

$$\int_{\overline{\zeta}-\delta}^{\overline{\zeta}} \widehat{X} \mathrm{d}z = 2\mu g k^2 a^2/(\sigma/k) = \mu \frac{\partial U_s}{\partial z}, \tag{8}$$

where the first equality is only valid for monochromatic waves in the x direction, while the second equality holds for a random sea state.

2 Vertical coordinate transformation from GLM to GLMz

In the transformation of the vertical coordinate Ardhuin et al. (2008) incorrectly defined the new coordinate by their eq. (48),

$$s = z^\star + \overline{\xi_3}^L \tag{9}$$

That transformation does not satisfy the desired equation (46) because of the vertical stretching of ξ_3. Instead one may introduce a vertical displacement ζ_c

attached to a constant vertical coordinate c,

$$\zeta_c = \xi_3(x_\alpha, z = c, t). \tag{10}$$

As a result the Stokes correction of ζ_c, just like for the position of the free surface, will only contain terms associated with horizontal gradients,

$$\overline{\zeta_c}^S = \overline{\partial \frac{\zeta_c}{x_\alpha} \xi_\alpha|_{z=c}} + O(k^{-1}\epsilon^3) = \overline{\partial \frac{(\xi_3|_{z=c})}{x_\alpha} \xi_\alpha|_{z=c}} + O(k^{-1}\epsilon^3). \tag{11}$$

Using linear wave theory this gives

$$\overline{\zeta_c}^S = \overline{(-ak_\alpha m F_{SS} \sin\psi) \times \left(-am \left[\frac{k_\alpha}{k} F_{CS} + \frac{m}{\sigma}\frac{\partial \overline{u}_\alpha}{\partial z} F_{ss}\right] \sin\psi\right)} + O(k^{-1}\epsilon^3), \tag{12}$$

where ψ is the phase of waves and all notations are defined in Ardhuin et al. (2008).

Neglecting the effect of current shear, $m = 1$, $\partial \overline{u}_\alpha / \partial z = 0$

$$\overline{\zeta_c}^S = \frac{1}{2} a^2 k (F_{CS} F_{SS})_{z=c} + O(k^{-1}\epsilon^3). \tag{13}$$

and we finally have the desired relation

$$\frac{\partial}{\partial z}\left(\overline{\zeta}^S\right) = \frac{1}{2} a^2 k^2 \left(F_{CS}^2 + F_{SS}^2\right) + O(\epsilon^3) = J_2. \tag{14}$$

We may thus define implicitly the vertical coordinate $z^\star$ for each $z = c$,

$$z = s(z^\star) = z^\star + \overline{\zeta_c}^S \tag{15}$$

to obtain a non-divergent set of equations, and, in particular, a mean sea level that is at the Eulerian mean sea level.

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