

# Supporting Information S4: Detailed results

Supporting Information for *How do MAR(1) models cope with hidden nonlinearities in ecological dynamics?*  
Grégoire Certain, Frédéric Barraquand and Anna Gårdmark, *Methods in Ecology and Evolution*.

## 1 Correlations between MAR(1) fitted parameters and mechanistic parameters

For each simulation experiment, correlations between the mechanistic parameters of the data-generating model on the one hand, and both fitted  $\mathbf{B}$  matrix elements as well as Granger causality measures (e.g., Detto et al. 2012) on the other hand have been systematically computed and plotted. Scatterplots of these relationships can be seen by running the code in Supporting Information S3, and the Figure S4.1 below provides one example.

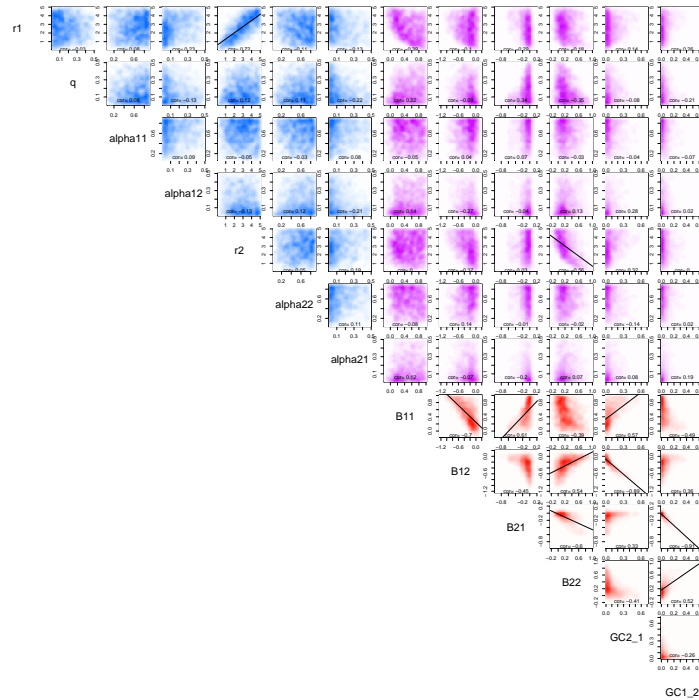


Figure S4.1: Correlations between MAR(1) fitted parameters and 1000 simulations obtained with a Beverton-Holt data generating model. Each simulation has a unique parameter set, and process error on species 1,  $\sigma_1^2$ , equals 0.1. Shade intensity corresponds to point density, and color indicates the type of correlation (blue: among mechanistic parameters; red: among fitted parameters, and purple, mechanistic parameters vs fitted parameters). Regression lines have been drawn when Pearson correlation coefficient was  $> 0.5$ .

## 2 Approximation of the Jacobian matrix $\mathbf{J}$ by $\mathbf{B}$

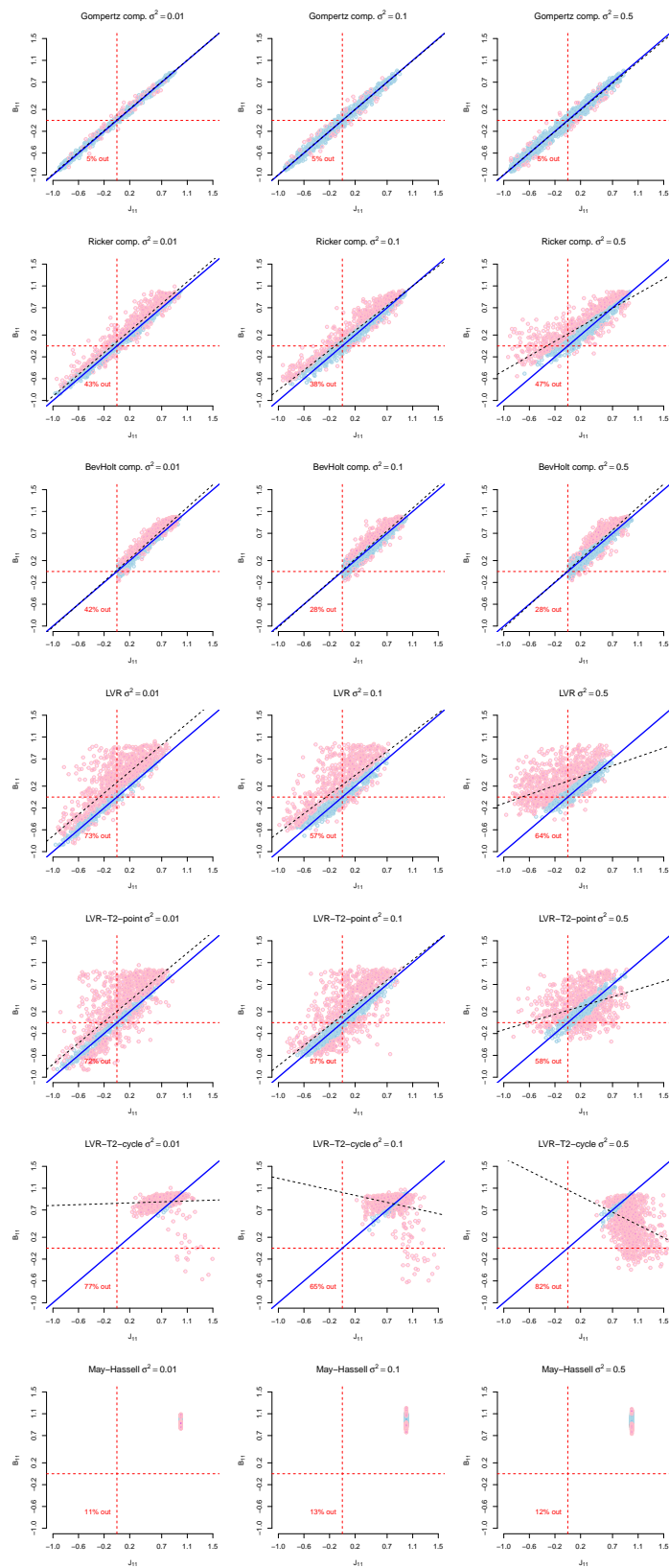


Figure S4.2: Relationship between  $\mathbf{J}$  (x-axis) versus  $\mathbf{B}$  (y-axis) upper-left element across 1000 simulations, for all data generating models (rows) and process error intensities (columns). A dot corresponds to a simulation with a unique parameter set. Dot color indicates whether the true value of the  $\mathbf{J}$  element lies within (blue) or outside (red) the confidence interval provided by the  $\mathbf{B}$  estimate. The blue thick line shows perfect match, while the dashed line shows actual regression between  $\mathbf{B}$  and  $\mathbf{J}$ . Red dashed lines intersect at the origin.

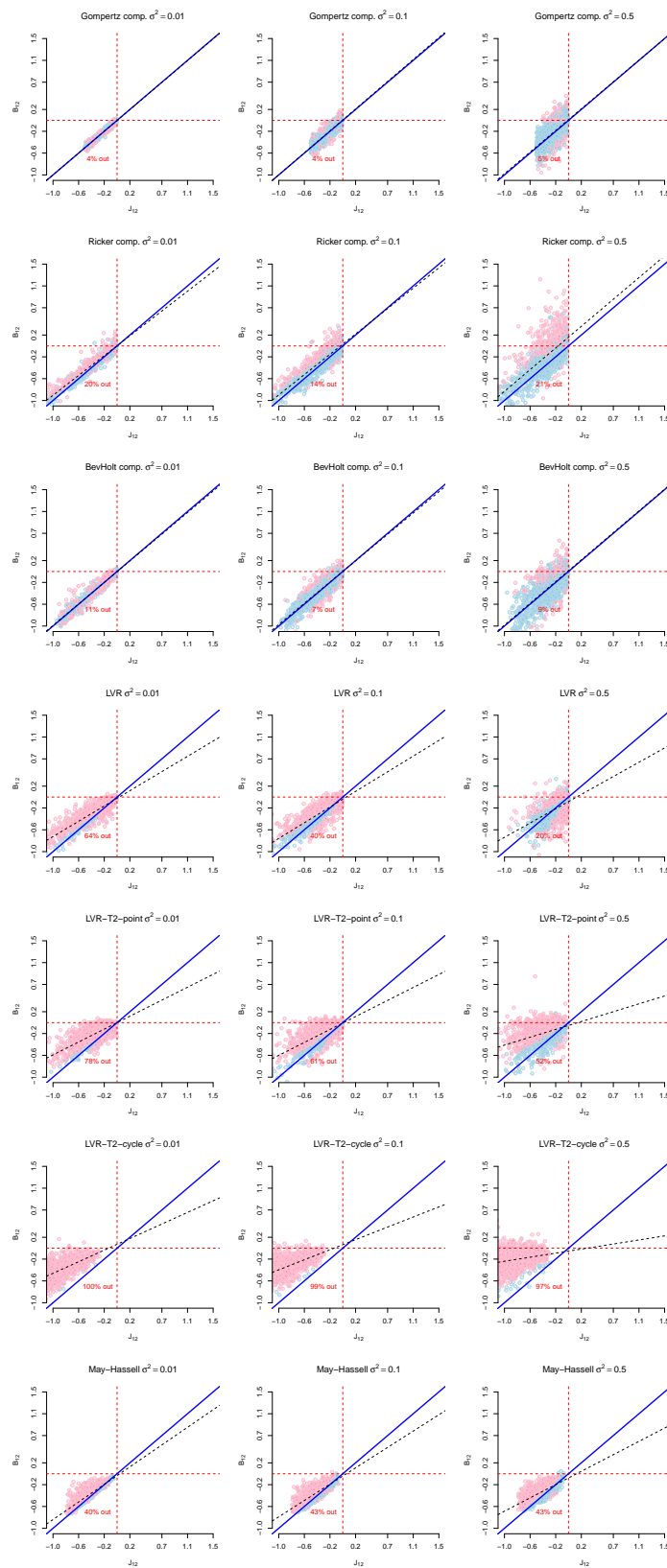


Figure S4.3: Relationship between  $\mathbf{J}$  (x-axis) versus  $\mathbf{B}$  (y-axis) upper-right element across 1000 simulations, for all data generating models (rows) and process error intensities (columns). A dot corresponds to a simulation with a unique parameter set. Dot color indicates whether the true value of the  $\mathbf{J}$  element lies within (blue) or outside (red) the confidence interval provided by the  $\mathbf{B}$  estimate. The blue thick line shows perfect match, while the dashed line shows actual regression between  $\mathbf{B}$  and  $\mathbf{J}$ . Red dashed lines intersect at the origin.

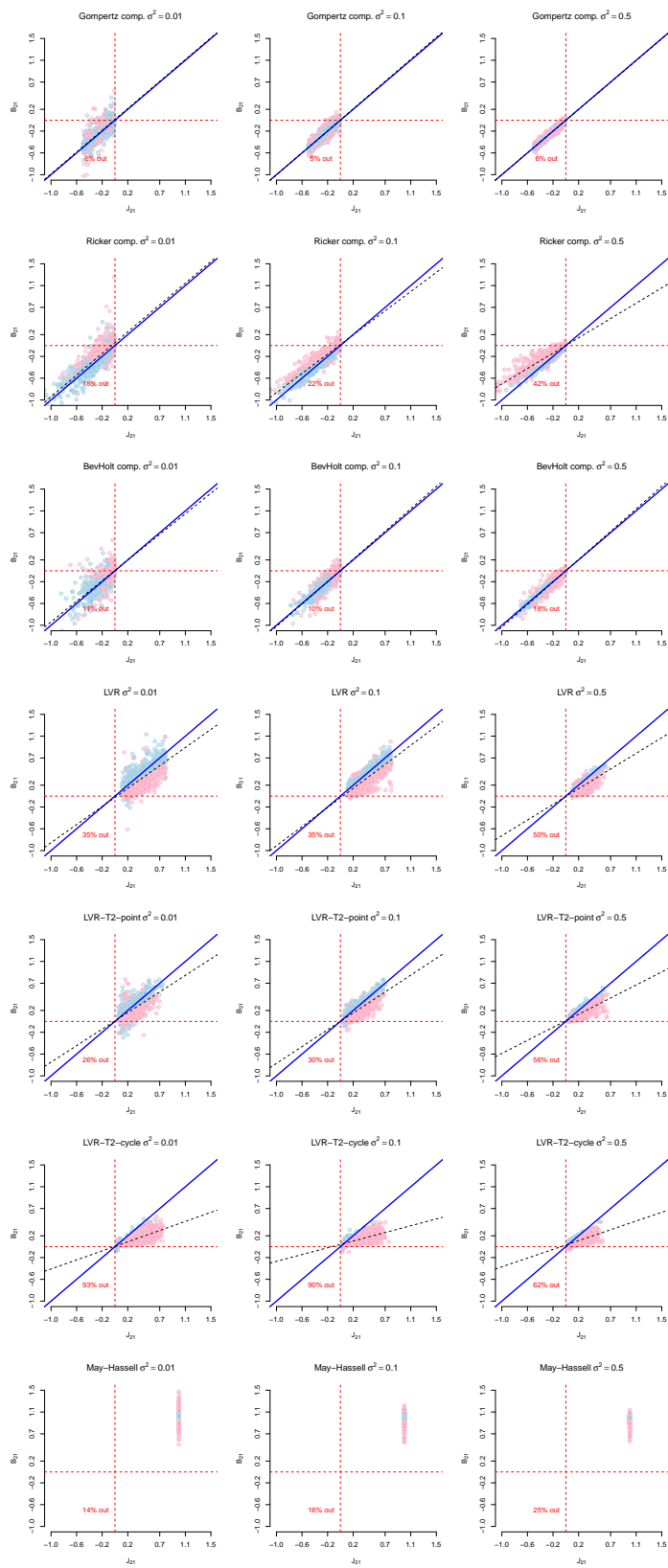


Figure S4.4: Relationship between  $\mathbf{J}$  (x-axis) versus  $\mathbf{B}$  (y-axis) lower-left element across 1000 simulations, for all data generating models (rows) and process error intensities (columns). A dot corresponds to a simulation with a unique parameter set. Dot color indicates whether the true value of the  $\mathbf{J}$  element lies within (blue) or outside (red) the confidence interval provided by the  $\mathbf{B}$  estimate. The blue thick line shows perfect match, while the dashed line shows actual regression between  $\mathbf{B}$  and  $\mathbf{J}$ . Red dashed lines intersect at the origin.

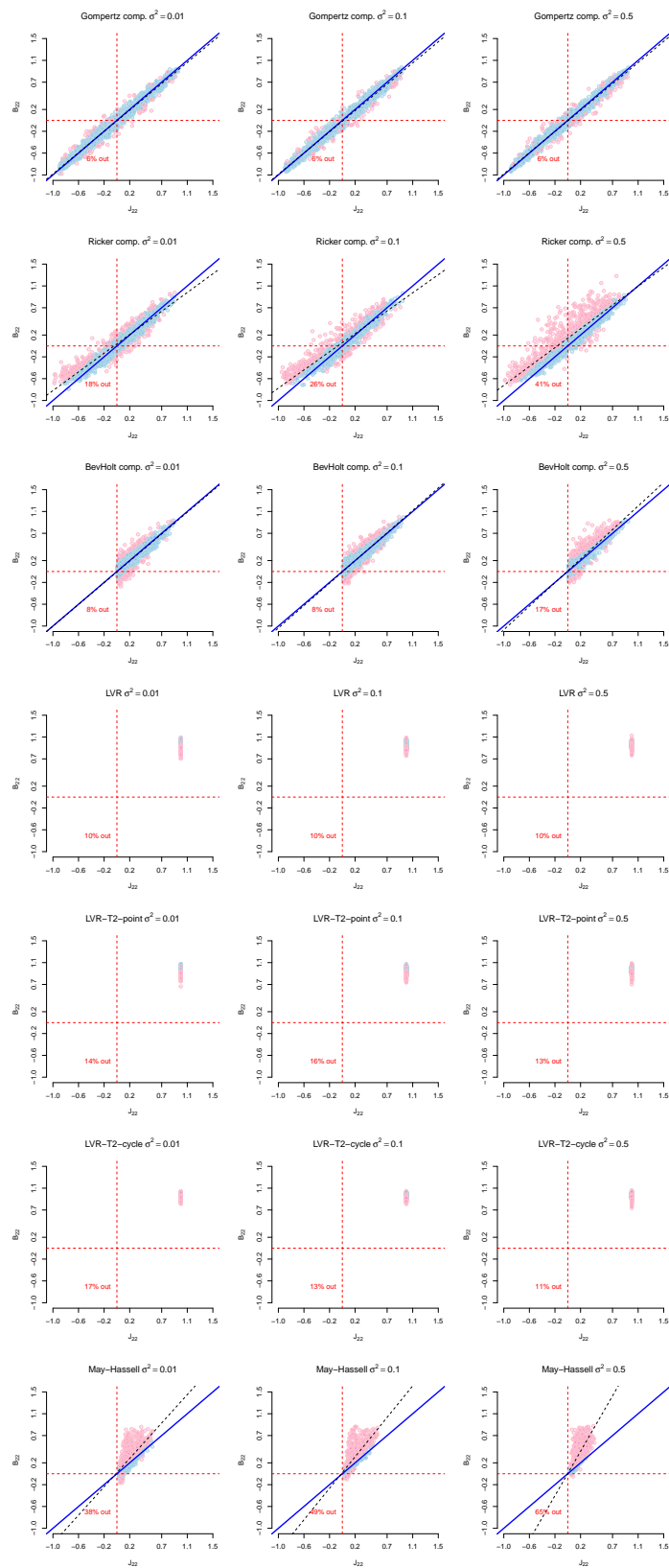


Figure S4.5: Relationship between  $\mathbf{J}$  (x-axis) versus  $\mathbf{B}$  (y-axis) lower-right element across 1000 simulations, for all data generating models (rows) and process error intensities (columns). A dot corresponds to a simulation with a unique parameter set. Dot color indicates whether the true value of the  $\mathbf{J}$  element lies within (blue) or outside (red) the confidence interval provided by the  $\mathbf{B}$  estimate. The blue thick line shows perfect match, while the dashed line shows actual regression between  $\mathbf{B}$  and  $\mathbf{J}$ . Red dashed lines intersect at the origin.

### 3 Long-term PRESS predictions



Figure S4.6: The figure shows, for all species (Sp), process error intensities (columns) and all data generating models (rows), the relationship between the true change triggered by a PRESS perturbation (x-axis) and the predicted change from a fitted MAR(1) model according to eq. 3 in the main paper (y-axis). The colorscale for dots is the same as Fig. 4 of the article. It illustrates the distance between the true change and the median prediction in terms of inter-quantile range: yellow corresponds to 0.5, which means that the true change lies beyond the 95% prediction interval derived from the bootstrapped distribution of the predictions and red corresponds to 0.5, which means that the median of the bootstrapped predictions and the true change are the same.

## 4 Process error estimate across the data-generating models

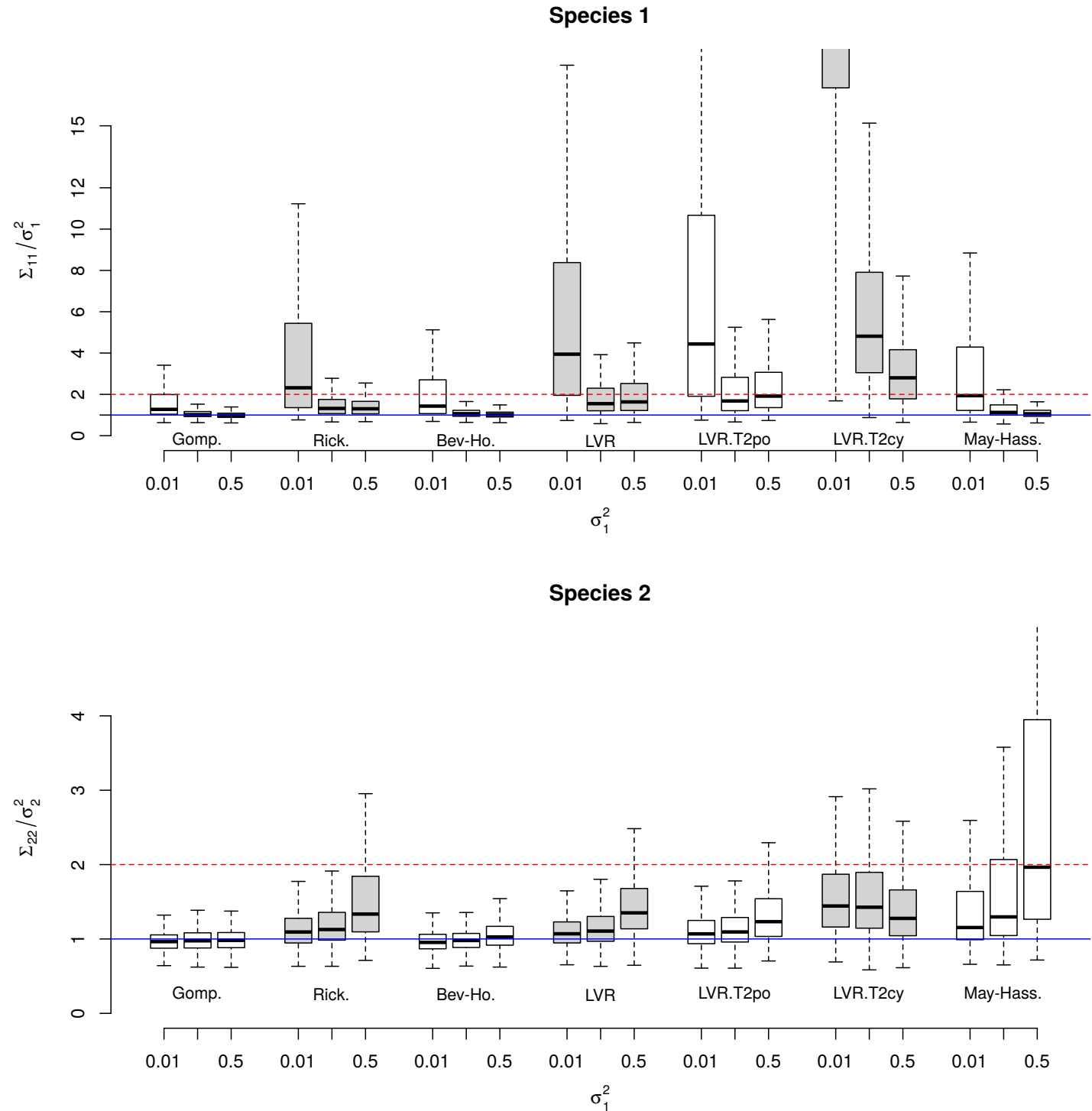


Figure S4.7: These boxplots display, for each data-generating model and each intensity of simulated process error on species 1 ( $\sigma_1^2$ , x-axis), the distribution range across 1000 simulations of the ratio of fitted versus simulated process error ( $\Sigma_{xx}/\sigma_x^2$ ). The blue horizontal line corresponds to a ratio of 1, that is, perfect estimate. The red dashed line corresponds to a ratio of 2, that is, when fitted process error is twice as high as the one used to simulate the data. Inflated ( $>1$ ) ratio is observed when (i) the simulated dynamics is other than Gompertz and (ii) the focal species has lower simulated process error than the non-focal species (recall that in our simulation setting,  $\sigma_1^2$  varies from 0.01 to 0.5 but  $\sigma_2^2$  is kept constant at 0.1).

# 5 Sensitivity analysis with respect to Time Series Length

		Time Series Length = 25																										
Data-generating model:		Gompertz			Ricker			Beverton-Holt			LVR-simple			LVR-T2-point			LVR-T2-cycle			May-Hassell								
process error on species 1:		0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5						
evaluation criterion																												
Corr. between $b_{11}$ and $j_{11}$		0.98	0.95	0.93	0.91	0.88	0.76	0.89	0.84	0.74	0.78	0.73	0.47	0.73	0.69	0.41	-0.03	-0.18	-0.21	constant Jacobian value								
Corr. between $b_{12}$ and $j_{12}$		0.87	0.60	0.32	0.91	0.78	0.56	0.89	0.71	0.47	0.78	0.59	0.24	0.66	0.53	0.27	0.51	0.38	0.16	0.89	0.77	0.61						
Corr. between $b_{21}$ and $j_{21}$		0.48	0.66	0.86	0.66	0.80	0.85	0.48	0.69	0.82	0.45	0.60	0.60	0.40	0.55	0.66	0.48	0.52	0.69	constant Jacobian value								
Corr. between $b_{22}$ and $j_{22}$		0.93	0.93	0.95	0.84	0.85	0.80	0.62	0.67	0.67	constant Jacobian value															0.66	0.69	0.68
median of $b_{11} - j_{11}$		0.02	0.00	-0.03	0.04	0.01	0.00	0.03	-0.02	-0.05	0.09	0.06	0.10	0.00	-0.05	-0.16	0.03	-0.06	-0.76	-0.01	-0.04	-0.07						
median of $b_{12} - j_{12}$		0.00	0.02	0.06	0.02	0.07	0.17	-0.01	0.00	0.01	0.05	-0.02	-0.15	0.09	0.06	0.03	0.32	0.36	0.47	0.02	0.05	0.06						
median of $b_{21} - j_{21}$		0.02	0.00	0.01	0.07	0.06	0.05	0.03	0.02	0.00	-0.06	-0.07	-0.07	-0.04	-0.05	-0.06	-0.17	-0.13	-0.07	0.01	-0.01	-0.02						
median of $b_{22} - j_{22}$		-0.07	-0.04	-0.04	-0.03	0.01	0.05	-0.10	-0.09	-0.06	-0.11	-0.13	-0.12	-0.14	-0.13	-0.12	-0.05	-0.04	-0.09	0.03	0.04	0.06						
% of $j_{11}$ out of c.i.95% of $b_{11}$		5.20	5.90	8.80	30.20	23.20	21.00	19.80	15.20	15.40	37.60	26.70	27.60	40.70	31.50	39.60	38.90	40.90	78.60	3.20	6.40	7.90						
% of $j_{12}$ out of c.i.95% of $b_{12}$		3.30	7.00	8.20	10.40	9.40	14.60	6.20	8.40	10.30	41.00	22.70	19.00	48.10	27.90	18.60	71.60	63.00	43.40	17.30	19.40	18.70						
% of $j_{21}$ out of c.i.95% of $b_{21}$		7.50	7.70	5.90	13.40	14.60	20.30	12.40	12.50	16.60	15.10	15.70	24.10	11.60	14.30	22.30	53.90	47.80	29.20	5.30	5.20	3.00						
% of $j_{22}$ out of c.i.95% of $b_{22}$		8.80	8.00	8.00	12.20	10.80	21.00	12.30	10.90	11.40	15.10	13.80	17.70	21.60	19.70	16.40	8.10	7.60	16.70	18.00	22.00	29.20						
% of $j_{11}$ correctly ranked		77.90	75.50	67.50	72.20	70.10	65.10	79.50	75.20	69.60	60.10	61.40	47.70	55.50	57.60	48.10	69.10	67.60	38.10	Ties in the Jacobian								
% of $j_{12}$ correctly ranked		67.00	61.70	47.40	68.70	66.20	49.90	77.00	69.30	59.10	82.30	78.50	64.40	71.70	72.90	72.90	95.20	92.10	78.30	96.80	97.90	98.50						
% of $j_{21}$ correctly ranked		61.40	62.70	60.00	61.40	62.50	60.50	53.90	56.80	56.10	64.60	76.10	65.00	63.50	70.60	60.80	83.20	80.60	60.00	Ties in the Jacobian								
% of $j_{22}$ correctly ranked		72.00	72.90	69.50	63.40	66.20	54.40	58.10	57.00	52.20	91.20	96.70	96.50	91.90	96.50	96.80	83.20	83.50	64.80	96.30	97.10	98.10						
% of $j_{11}$ correctly signed		95.10	92.60	89.90	91.70	86.60	81.70	95.70	89.20	87.00	80.20	77.90	68.20	80.30	77.10	65.60	96.20	90.60	67.50	100.00	100.00	100.00						
% of $j_{12}$ correctly signed		93.90	83.70	67.80	92.60	82.80	66.10	93.60	82.20	68.70	92.50	92.70	84.10	88.30	84.20	82.20	98.30	95.60	83.30	99.80	98.30	97.80						
% of $j_{21}$ correctly signed		83.80	90.00	94.70	68.40	77.90	86.90	62.00	69.00	80.00	93.00	95.70	95.80	85.70	89.80	94.80	96.70	95.90	93.40	100.00	100.00	100.00						
% of $j_{22}$ correctly signed		88.50	88.90	91.30	83.60	84.00	82.60	69.80	67.40	69.60	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.90	83.40	89.00	93.70						
% fitted $B$ having a stable equilibrium		99.20	99.30	99.30	99.60	99.10	99.40	98.50	99.00	99.50	97.00	97.20	97.30	95.40	96.70	96.90	85.70	88.10	93.70	99.50	99.60	99.60						
Corr. between max. eigen. of $B$ and $J$		0.78	0.75	0.71	0.61	0.55	0.49	0.70	0.70	0.62	0.24	0.21	0.04	0.11	0.15	0.07	0.13	0.03	-0.05	0.81	0.69	0.59						

Subjective color scale:	Best performing cases	Fairly Good	Some Issues	Less performing cases	irrelevant
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Table S4.1: Same as Table 3 in the main manuscript, but with Time Series Length = 25 points



Time Series Length = 50

Data-generating model: process error on species 1:	Gompertz			Ricker			Beverton-Holt			LVR-simple			LVR-T2-point			LVR-T2-cycle			May-Hassell					
	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5			
evaluation criterion																								
Corr. between $b_{21}$ and $j_{11}$	0.99	0.98	0.97	0.96	0.92	0.81	0.94	0.90	0.84	0.83	0.78	0.45	0.73	0.69	0.36	-0.13	-0.24	-0.27	constant Jacobian value					
Corr. between $b_{12}$ and $j_{12}$	0.95	0.76	0.50	0.95	0.89	0.66	0.94	0.84	0.58	0.85	0.77	0.41	0.70	0.67	0.28	0.53	0.55	0.25	0.88	0.81	0.70			
Corr. between $b_{21}$ and $j_{21}$	0.72	0.84	0.94	0.80	0.87	0.87	0.64	0.82	0.91	0.62	0.74	0.72	0.51	0.68	0.76	0.53	0.53	0.67	constant Jacobian value					
Corr. between $b_{22}$ and $j_{22}$	0.97	0.97	0.97	0.91	0.93	0.85	0.79	0.82	0.81	constant Jacobian value												0.68	0.73	0.76
median of $b_{11} - j_{11}$	0.01	0.00	-0.02	0.06	0.05	0.07	0.03	0.02	0.01	0.15	0.13	0.21	0.08	0.02	-0.04	0.08	0.04	-0.55	-0.01	-0.03	-0.05			
median of $b_{12} - j_{12}$	0.00	0.00	0.02	0.03	0.06	0.16	-0.01	0.01	0.04	0.05	0.03	-0.05	0.09	0.10	0.14	0.42	0.43	0.57	0.03	0.05	0.08			
median of $b_{21} - j_{21}$	0.01	0.00	0.00	0.05	0.04	0.06	0.01	0.01	0.00	-0.08	-0.06	-0.07	-0.06	-0.05	-0.06	-0.19	-0.18	-0.06	-0.02	-0.02	-0.03			
median of $b_{22} - j_{22}$	-0.02	-0.02	-0.02	0.01	0.03	0.12	-0.04	-0.03	-0.01	-0.07	-0.06	-0.06	-0.07	-0.06	-0.06	-0.03	-0.03	-0.05	0.05	0.06	0.08			
% of $j_{11}$ out of c.i.95% of $b_{11}$	3.90	5.90	6.80	37.70	27.70	32.10	29.40	23.10	20.40	54.20	39.70	46.70	55.30	45.60	49.20	57.90	51.90	81.10	4.50	7.60	7.80			
% of $j_{12}$ out of c.i.95% of $b_{12}$	2.00	5.20	6.40	11.70	12.00	16.10	7.40	6.50	9.00	50.60	27.60	17.30	61.40	41.00	32.50	93.90	90.50	79.30	28.90	28.90	28.90			
% of $j_{21}$ out of c.i.95% of $b_{21}$	6.90	6.30	6.20	15.60	16.50	32.00	8.80	11.00	15.00	22.70	23.10	35.20	17.40	19.20	34.90	77.80	75.80	43.90	7.70	7.40	8.00			
% of $j_{22}$ out of c.i.95% of $b_{22}$	7.70	6.20	6.50	12.30	15.90	33.30	9.60	9.40	12.70	10.60	13.40	14.70	17.70	17.00	15.00	9.60	7.60	12.60	28.70	33.90	44.60			
% of $j_{11}$ correctly ranked	81.90	84.50	77.90	80.60	78.40	68.10	86.40	83.80	77.90	61.30	62.90	45.20	58.50	56.70	53.90	74.40	77.00	45.00	Ties in the Jacobian					
% of $j_{12}$ correctly ranked	77.90	70.10	58.50	77.20	75.20	57.20	84.00	79.30	67.50	86.50	85.70	67.10	76.50	75.70	79.80	97.30	95.70	85.90	99.40	99.30	99.50			
% of $j_{21}$ correctly ranked	72.10	72.10	65.60	70.50	73.20	66.70	66.80	70.90	66.30	69.00	75.20	63.70	67.00	72.20	61.30	83.90	87.70	69.70	Ties in the Jacobian					
% of $j_{22}$ correctly ranked	81.50	82.10	78.80	73.40	74.50	62.10	70.60	72.50	66.80	98.00	99.60	99.30	98.10	98.80	99.30	88.70	86.40	62.00	98.40	99.00	98.30			
% of $j_{11}$ correctly signed	96.80	94.90	93.00	93.20	91.30	83.20	96.60	95.50	93.70	83.00	82.90	63.10	80.50	78.10	67.80	97.60	95.50	81.00	100.00	100.00	100.00			
% of $j_{12}$ correctly signed	97.60	90.10	76.30	95.00	87.80	69.60	96.50	87.60	73.00	95.00	93.80	87.50	89.20	88.30	84.90	98.70	98.80	89.30	99.90	99.10	98.80			
% of $j_{21}$ correctly signed	87.70	92.60	96.20	80.30	83.80	89.10	70.70	78.50	86.40	96.10	98.90	98.80	89.80	96.20	98.10	98.40	99.00	97.90	100.00	100.00	100.00			
% of $j_{22}$ correctly signed	92.90	93.30	93.50	88.60	91.00	85.10	79.60	80.90	82.90	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	90.80	93.70	96.60			
% fitted $B$ having a stable equilibrium	99.70	99.80	99.80	99.30	99.80	99.90	99.30	99.70	99.50	97.90	98.10	98.30	96.80	98.10	97.80	94.70	95.90	96.00	100.00	99.80	100.00			
Corr. between max. eigen. of $B$ and $J$	0.90	0.88	0.84	0.71	0.68	0.54	0.83	0.83	0.76	0.46	0.46	0.25	0.30	0.33	0.20	0.01	-0.05	0.09	0.87	0.83	0.76			

Subjective color scale:	Best performing cases	Fairly Good	Some Issues	Less performing cases	irrelevant
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Table S4.2: Same as Table 3 in the main manuscript, but with Time Series Length = 50 points

		Time Series Length = 200																										
Data-generating model:		Gompertz			Ricker			Beverton-Holt			LVR-simple			LVR-T2-point			LVR-T2-cycle			May-Hassell								
process error on species 1:		0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5			
evaluation criterion																												
Corr. between $b_{11}$ and $j_{11}$		1.00	1.00	0.99	0.98	0.96	0.86	0.97	0.96	0.95	0.81	0.79	0.47	0.77	0.72	0.46	0.11	-0.22	-0.29	constant Jacobian value								
		0.99	0.93	0.78	0.98	0.95	0.85	0.97	0.94	0.82	0.91	0.87	0.71	0.82	0.77	0.49	0.73	0.65	0.28	0.87	0.84	0.73						
Corr. between $b_{21}$ and $j_{21}$		0.87	0.96	0.99	0.91	0.96	0.92	0.84	0.96	0.97	0.74	0.81	0.67	0.70	0.79	0.78	0.71	0.64	0.64	constant Jacobian value								
		0.99	0.99	0.99	0.97	0.96	0.92	0.93	0.93	0.92	constant Jacobian value															0.77	0.81	0.80
median of $b_{11} - j_{11}$		0.00	0.00	0.00	0.08	0.09	0.14	0.04	0.04	0.07	0.31	0.24	0.38	0.26	0.15	0.14	0.13	0.10	-0.26	-0.01	-0.02	-0.04						
		0.00	0.00	0.01	0.03	0.05	0.14	0.00	0.01	0.02	0.09	0.07	0.03	0.14	0.14	0.23	0.50	0.55	0.69	0.03	0.05	0.08						
		0.00	0.00	0.00	0.05	0.05	0.06	0.01	0.01	0.00	-0.11	-0.09	-0.09	-0.07	-0.06	-0.08	-0.24	-0.23	-0.09	-0.03	-0.03	-0.05						
		-0.01	0.00	0.00	0.05	0.06	0.14	0.01	0.01	0.03	-0.03	-0.02	-0.01	-0.02	-0.02	-0.02	-0.03	-0.02	-0.01	0.05	0.07	0.13						
% of $j_{11}$ out of c.i.95% of $b_{11}$		6.20	4.40	5.10	59.00	51.20	61.30	49.40	38.10	38.70	85.00	73.60	82.50	81.70	70.60	68.10	85.10	82.20	84.20	22.60	21.20	23.10						
		3.40	5.70	5.80	29.70	21.70	30.10	14.80	8.40	8.10	79.80	59.00	30.50	87.00	79.70	74.30	100.00	100.00	99.70	51.30	48.20	51.60						
		7.10	4.80	5.60	29.80	32.90	56.50	11.10	12.80	21.40	48.90	50.90	72.20	39.90	42.20	72.30	98.70	95.90	75.70	22.40	27.90	44.40						
		5.30	5.00	5.90	25.80	33.10	57.60	8.90	11.20	21.40	19.70	14.80	12.50	16.60	14.70	13.10	35.10	28.90	10.20	44.60	59.00	75.70						
% of $j_{11}$ correctly ranked		91.00	91.60	88.60	87.90	87.00	79.80	93.90	92.80	92.40	53.60	57.80	35.00	58.00	58.70	54.80	78.30	81.30	59.60	Ties in the Jacobian								
		85.40	83.50	76.30	87.90	84.40	72.20	92.20	90.10	84.10	88.70	88.70	65.80	85.70	84.60	87.70	99.30	98.20	95.00	100.00	99.90	100.00						
		82.50	84.70	80.00	83.70	84.10	73.90	87.20	85.50	82.60	63.00	68.00	49.70	65.50	71.70	59.70	87.50	90.60	90.80	Ties in the Jacobian								
		90.90	90.70	88.50	85.70	83.40	78.30	89.30	88.10	89.70	97.80	98.90	99.90	98.00	99.60	100.00	90.20	89.60	63.30	98.50	98.70	97.00						
% of $j_{11}$ correctly signed		98.70	98.40	97.70	95.10	93.80	79.60	99.70	98.00	98.30	75.90	78.10	51.80	77.60	81.10	73.40	99.30	98.40	96.00	100.00	100.00	100.00						
		99.50	95.60	89.90	97.50	92.20	74.10	98.40	95.50	87.80	97.90	97.80	92.30	94.30	92.30	88.30	99.70	99.00	94.00	100.00	99.90	99.90						
		94.20	96.60	99.00	87.50	93.20	95.10	86.70	89.30	94.50	99.70	99.90	100.00	97.60	99.70	100.00	99.70	99.30	99.80	100.00	100.00	100.00						
		96.10	97.20	96.90	95.10	92.60	89.40	92.60	91.90	93.40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	98.30	98.30	99.50						
% fitted $B$ having a stable equilibrium		99.90	100.00	100.00	100.00	100.00	99.90	99.90	100.00	99.00	99.30	98.30	99.10	98.90	98.60	99.10	99.80	99.20	100.00	100.00	100.00	100.00						
Corr. between max. eigen. of $B$ and $J$		0.97	0.97	0.96	0.71	0.71	0.68	0.93	0.93	0.92	0.68	0.71	0.61	0.63	0.68	0.41	0.07	0.12	0.15	0.96	0.94	0.87						

Subjective color scale: Best performing cases (green), Fairly Good (yellow), Some Issues (grey), Less performing cases (red), irrelevant (black)

Table S4.3: Same as Table 3 in the main manuscript, but with Time Series Length = 200 points

		Time Series Length = 25																					
Data-generating model:		Gompertz			Ricker			Beverton-Holt			LVR-simple			LVR-T2-point			LVR-T2-cycle			May-Hassell			
process error on species 1:		0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	
evaluation criterion																							
% of $q_{1t}$ out of c.i.95% of $c_{1t}$		29.80	20.10	13.10	23.80	15.40	8.90	21.20	15.30	10.10	22.10	11.90	9.40	22.80	13.80	9.40	16.80	12.40	10.50	21.80	15.00	11.80	
% of $q_{2t}$ out of c.i.95% of $c_{2t}$		8.00	5.90	7.10	6.70	6.50	6.90	8.70	7.90	6.50	10.10	8.60	10.60	8.70	10.00	10.90	10.80	10.60	10.10	8.30	8.50	9.40	
% of $q_{1t}$ correctly signed		99.60	96.00	92.60	98.50	94.70	81.20	98.60	93.70	87.30	94.50	89.90	65.20	95.60	87.80	72.90	67.40	69.70	64.30	98.60	95.40	86.80	
median of $c_{1t} - q_{1t}$		-0.02	-0.03	-0.03	-0.02	-0.02	-0.01	-0.01	-0.02	-0.01	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.03	-0.01	-0.02	-0.01	
median of $c_{2t} - q_{2t}$		0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	
% of well predicted next trend, sp1		84.60	78.60	77.40	85.00	76.80	68.50	83.50	71.00	66.50	80.40	76.80	67.70	78.60	73.80	68.70	73.90	69.00	67.10	87.90	80.50	72.60	
% of well predicted next trend, sp2		74.40	77.00	78.60	72.00	73.80	76.30	71.60	67.60	68.60	60.60	61.30	61.90	54.20	58.10	59.60	73.10	67.80	60.50	83.40	85.60	87.50	
% ASE > 1, sp1		10.10	17.50	20.10	12.70	26.30	36.30	15.60	28.40	37.20	19.00	28.90	35.10	19.80	29.50	34.20	33.80	35.60	36.40	11.30	20.70	31.20	
% ASE > 1, sp2		25.30	26.30	20.10	28.70	27.90	26.80	35.20	35.80	35.80	41.20	40.80	42.20	45.40	42.30	41.10	35.00	38.00	42.00	15.30	13.00	6.80	
Correlation between $\Delta x_{1t}^+$ and $\Delta n_{1t}^+$		0.94	0.88	0.68	0.66	0.60	0.49	0.65	0.64	0.56	Species 1 equilibrium is not affected by the press perturbation										0.90	0.82	0.67
Correlation between $\Delta x_{2t}^+$ and $\Delta n_{2t}^+$		0.69	0.72	0.59	0.44	0.46	0.33	0.25	0.32	0.24	0.16	0.23	0.03	0.14	0.16	0.15	0.15	0.07	0.05	0.80	0.71	0.61	
Slope of the linear regression $\Delta x_{1t}^+ - \Delta n_{1t}^+$		0.85	0.76	0.64	4.05	3.48	2.15	2.97	2.47	2.04	Species 1 equilibrium is not affected by the press perturbation										0.74	0.71	0.68
Slope of the linear regression $\Delta x_{2t}^+ - \Delta n_{2t}^+$		0.51	0.50	0.42	0.36	0.38	0.37	0.21	0.21	0.27	0.27	0.37	0.04	0.30	0.34	0.41	1.22	0.66	0.42	0.87	0.84	0.79	
% of $\Delta n_{1t}^+$ out of c.i.95% of $\Delta x_{1t}^+$		7.16	7.35	9.67	30.02	17.15	7.85	20.10	13.74	9.95	19.18	10.60	3.70	27.78	10.86	4.64	10.71	6.77	2.98	2.81	4.32	6.43	
% of $\Delta n_{2t}^+$ out of c.i.95% of $\Delta x_{2t}^+$		5.85	6.04	7.15	9.74	8.78	6.64	15.64	13.64	13.27	13.09	11.63	15.31	15.62	10.76	8.88	8.54	7.10	9.57	7.04	7.13	8.33	
% of $\Delta n_{1t}^+$ correctly signed		99.40	95.67	92.55	97.99	93.54	80.08	97.16	93.43	86.53	Species 1 equilibrium is not affected by the press perturbation										97.49	94.38	87.05
% of $\Delta n_{2t}^+$ correctly signed		79.74	78.55	78.45	68.78	68.42	65.90	58.78	64.95	63.22	77.01	71.19	58.99	70.65	68.87	61.40	68.34	63.60	55.96	98.79	94.98	86.25	
Subjective color scale:		Best performing cases			Fairly Good			Some Issues			Less performing cases			Irrelevant									

Table S4.4: Same as Table 4 in the main manuscript, but with Time Series Length = 25 points

		Time Series Length = 50																							
Data-generating model:		Gompertz			Ricker			Beverton-Holt			LVR-simple			LVR-T2-point			LVR-T2-cycle			May-Hassell					
process error on species 1:		0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5			
evaluation criterion																									
% of $q_{1t}$ out of c.i.95% of $c_{1t}$		24.40	14.90	8.40	18.50	10.20	7.10	19.60	10.80	7.60	22.30	10.90	8.20	22.30	11.10	8.50	18.10	15.10	8.60	19.40	12.40	8.60			
% of $q_{2t}$ out of c.i.95% of $c_{2t}$		5.70	6.60	5.00	5.60	6.70	5.30	5.20	6.30	7.20	8.10	7.30	8.70	8.70	7.50	8.40	10.20	8.00	7.70	7.20	7.30	7.60			
% of $q_{1t}$ correctly signed		99.60	98.90	94.80	99.50	96.60	85.50	99.70	97.00	89.20	96.60	92.90	72.60	96.90	92.10	73.80	72.50	72.10	71.30	99.50	97.20	91.70			
median of $c_{1t} - q_{1t}$		-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.02	-0.02	-0.01	-0.01	-0.01			
median of $c_{2t} - q_{2t}$		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00			
% of well predicted next trend, sp1		90.40	83.60	78.70	84.70	76.10	69.70	82.80	77.30	69.30	79.10	75.00	73.70	78.10	73.30	68.80	75.20	69.10	66.20	86.90	78.00	73.20			
% of well predicted next trend, sp2		74.90	77.40	78.90	73.20	74.00	74.20	69.60	71.00	70.90	62.00	65.20	63.50	57.60	60.20	62.50	72.90	70.40	63.20	82.20	85.10	87.80			
% ASE >1, sp1		3.10	11.80	18.40	11.10	22.50	30.70	10.40	26.60	36.20	18.20	26.90	33.10	17.30	26.30	32.40	28.60	31.10	33.00	6.80	20.30	28.20			
% ASE >1, sp2		25.40	19.20	17.80	25.20	23.40	23.70	33.40	31.30	30.20	39.00	38.90	38.20	42.50	40.00	39.00	31.50	32.90	39.70	14.00	9.40	6.30			
Correlation between $\Delta x_{1t}^*$ and $\Delta n_{1t}^*$		0.97	0.95	0.86	0.59	0.65	0.64	0.55	0.62	0.60	Species 1 equilibrium is not affected by the press perturbation										0.97	0.93	0.82		
Correlation between $\Delta x_{2t}^*$ and $\Delta n_{2t}^*$		0.83	0.80	0.79	0.54	0.45	0.45	0.42	0.46	0.31	0.41	0.40	0.21	0.26	0.24	0.20	0.23	0.29	0.17	0.84	0.83	0.62			
Slope of the linear regression $\Delta x_{1t}^* \sim \Delta n_{1t}^*$		0.88	0.90	0.73	6.83	5.08	3.34	5.52	4.22	3.41	Species 1 equilibrium is not affected by the press perturbation										0.86	0.84	0.81		
Slope of the linear regression $\Delta x_{2t}^* \sim \Delta n_{2t}^*$		0.62	0.69	0.60	0.39	0.36	0.37	0.26	0.32	0.34	1.13	0.96	0.43	0.70	0.75	0.80	1.96	2.47	1.73	1.11	1.03	0.93			
% of $\Delta n_{1t}^*$ out of c.i.95% of $\Delta x_{1t}^*$		6.52	4.31	4.81	45.02	29.06	12.51	34.84	24.07	15.68	17.88	9.99	5.19	25.52	14.99	3.58	10.67	8.86	3.54	1.60	2.00	2.80			
% of $\Delta n_{2t}^*$ out of c.i.95% of $\Delta x_{2t}^*$		4.61	4.41	4.01	8.56	7.42	5.31	15.81	15.45	13.57	10.62	7.54	8.44	11.67	10.81	6.95	10.98	8.97	7.50	14.30	9.82	9.30			
% of $\Delta n_{1t}^*$ correctly signed		99.30	98.60	94.69	99.09	96.19	84.38	98.99	96.59	89.05	Species 1 equilibrium is not affected by the press perturbation										98.20	97.19	91.70		
% of $\Delta n_{2t}^*$ correctly signed		85.46	86.17	85.57	76.54	75.85	74.17	70.19	71.52	73.27	85.90	81.65	63.38	77.38	75.03	66.67	71.81	70.80	59.90	99.40	97.09	92.10			

Subjective color scale:	Best performing cases	Fairly Good	Some Issues	Less performing cases	Irrelevant
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Table S4.5: Same as Table 4 in the main manuscript, but with Time Series Length = 50 points

Time Series Length = 200

Data-generating model:	Gompertz			Ricker			Beverton-Holt			LVR-simple			LVR-T2-point			LVR-T2-cycle			May-Hassell			
process error on species 1:	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	
evaluation criterion																						
% of $q_{1t}$ out of c.i.95% of $c_{1t}$	18.40	9.00	7.10	30.10	16.10	9.60	26.10	10.90	7.10	45.80	19.10	9.90	45.20	21.30	12.70	27.10	21.70	14.30	10.60	8.90	8.60	
% of $q_{2t}$ out of c.i.95% of $c_{2t}$	5.30	5.50	4.60	8.00	7.60	6.70	5.00	5.20	6.10	9.90	10.60	6.70	8.50	8.10	8.20	10.50	10.10	6.90	5.70	6.30	8.30	
% of $q_{1t}$ correctly signed	100.00	99.60	98.30	100.00	98.50	91.50	100.00	98.20	96.20	98.10	96.40	81.10	98.30	95.60	83.00	78.60	77.30	78.10	100.00	99.40	96.50	
median of $c_{1t} - q_{1t}$	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.01	-0.02	-0.02	-0.02	-0.02	-0.02	-0.03	0.00	0.00	0.00	
median of $c_{2t} - q_{2t}$	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	
% of well predicted next trend, sp1	92.20	84.50	79.80	85.90	77.40	69.80	84.50	77.10	70.20	80.30	73.30	67.60	76.70	73.50	68.20	70.70	70.50	64.60	90.30	81.80	74.90	
% of well predicted next trend, sp2	80.70	77.90	81.10	72.20	78.40	78.10	69.40	72.70	73.20	62.60	63.90	64.70	59.60	61.60	67.40	75.90	74.60	68.10	84.60	83.70	87.80	
% ASE >1, sp1	2.80	13.70	18.60	8.10	20.20	32.70	9.70	23.50	33.20	17.00	23.00	32.40	20.80	25.20	30.90	28.70	30.40	34.90	5.10	16.60	27.10	
% ASE >1, sp2	20.30	20.20	15.70	25.80	21.50	20.70	33.30	30.40	29.70	35.60	34.70	37.50	39.30	37.70	36.20	29.40	30.30	34.00	13.50	7.80	5.00	
Correlation between $\Delta x_{1t}^+$ and $\Delta n_{1t}^+$	0.99	0.99	0.97	0.64	0.65	0.70	0.54	0.58	0.61	Species 1 equilibrium is not affected by the press perturbation										1.00	0.99	0.95
Correlation between $\Delta x_{2t}^+$ and $\Delta n_{2t}^+$	0.97	0.96	0.95	0.72	0.77	0.67	0.63	0.61	0.54	0.58	0.54	0.37	0.42	0.49	0.37	0.60	0.51	0.31	0.90	0.88	0.83	
Slope of the linear regression $\Delta x_{1t}^+ \sim \Delta n_{1t}^+$	0.96	0.99	0.93	9.56	7.15	5.56	6.34	7.20	6.38	Species 1 equilibrium is not affected by the press perturbation										0.93	0.92	0.91
Slope of the linear regression $\Delta x_{2t}^+ \sim \Delta n_{2t}^+$	0.95	0.97	0.91	0.34	0.41	0.61	0.27	0.29	0.35	3.51	3.51	1.21	2.08	2.73	2.25	4.77	4.49	4.39	1.13	1.10	1.18	
% of $\Delta n_{1t}^+$ out of c.i.95% of $\Delta x_{1t}^+$	3.80	2.50	3.70	68.60	53.40	29.80	58.86	49.35	36.90	11.41	10.37	4.88	20.79	14.46	6.29	12.01	11.12	5.34	0.60	0.90	2.70	
% of $\Delta n_{2t}^+$ out of c.i.95% of $\Delta x_{2t}^+$	2.10	2.50	4.10	14.60	12.70	5.00	25.93	23.32	17.70	28.18	22.16	4.88	24.12	20.73	9.84	19.07	13.93	10.28	37.90	26.90	20.30	
% of $\Delta n_{1t}^+$ correctly signed	100.00	99.60	98.20	99.90	98.30	90.50	99.90	98.50	96.00	Species 1 equilibrium is not affected by the press perturbation										99.80	99.20	96.70
% of $\Delta n_{2t}^+$ correctly signed	91.19	93.30	91.50	85.70	87.80	83.30	84.79	86.59	85.70	94.95	91.74	77.21	90.01	86.75	75.56	80.73	78.26	70.67	100.00	99.40	96.60	

Subjective color scale:	Best performing cases	Fairly Good	Some Issues	Less performing cases	Irrelevant
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Table S4.6: Same as Table 4 in the main manuscript, but with Time Series Length = 200 points