Supporting Information for "Arctic Ocean freshwater content and its decadal memory of sea-level pressure"

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Text S1. Methodology for constraining the impulse and step-response functions of Arctic FWC

Following *Hasselmann et al.* [1993], we can represent the anomaly in Arctic FWC, $FWC(t)$ $[m^3]$, as a linear convolution of a lagged principal component index of sea-level pressure, $PC(t)$, with an impulse response function (a Green's function), $G(\tau)$ $[m^3]$:

$$FWC(t) = \int_0^{\tau_{max}} G(\tau)PC(t-\tau)d\tau + \varepsilon(t), \tag{1}$$

where τ [months] denotes the time lag after an impulse perturbation of magnitude one standard inter-monthly deviation (1σ) of the PC. The term $\varepsilon(t)$ denotes residual noise at each time step.

In theory, the system may have infinitely long memory of past forcing. However, for practical purposes, in our calculation we impose a maximum cut-off lag τ_{max} (35 years) beyond which we assume that the FWC has no memory of past atmospheric variability. The choice of τ_{max} is a compromise between learning as much as possible about the time-evolution of the response and ensuring that the response functon is well-constrained by the available data from the 130 year control run.

In discretized form equation (1) becomes

$$FWC(t) = \sum_{j=0}^{J} \left[G(\tau_j)PC(t-\tau_j)\Delta\tau\right] + \varepsilon(t), \text{ with } \tau_J = \tau_{max}, \tag{2}$$

with time increments $\Delta\tau$ of length 1 month. Following *Kostov et al.* [2017], we perform a multiple linear least-squares regression of $FWC(t)$ against the lagged PC index, $PC(t)$, and estimate the impulse response function, $G(\tau)$ $[m^3]$. We seek the least-squares solution to the system of equations $M * G = FWC$, where G and FWC are column vectors and

M contains the forcing. We use the matrix left division operator \ in Matlab to obtain G.

We obtain the corresponding FWC step-response function, $FWC_{step}(\tau)$ $[m^3]$ by integrating $G(\tau)$ in time t, :

$$FWC_{step}(\tau) \approx \sum_{j=0}^{J} G(\tau_j'), \text{ with } \tau_J' = \tau, \tag{3}$$

where we estimate $G(\tau)$ and $FWC_{step}(\tau)$ separately for each PC.

In order to obtain a range of estimates for $FWC_{step}(\tau)$, we vary the cutoff lag τ_{max} and consider subsets of the full FWC and PC timeseries. This provides an ensemble of fits. At each lag, we calculate the standard deviation of the ensemble of fits, $\sigma_{Spread}(\tau)$, which reflects the uncertainty associated with sub-sampling the timeseries and the choice of τ_{max}. In addition, the residual ϵ of the fit gives us a different uncertainty estimate, $\sigma_{Resid}(\tau)$. We combine $\sigma_{Spread}(\tau)$ and the ensemble-mean $\sigma_{Resid}(\tau)$ in quadrature to obtain a more comprehensive uncertainty estimate on FWC_{step}: $\sigma_{step}(\tau) = (\sigma_{Spread}^2 + \overline{\sigma_{Resid}}^2)^{1/2}$

We can use our response functions to estimate the contribution of a given PC, $PC_n(t)$, to the evolution of Arctic FWC, $FWC(t)$:

$$FWC_n(t) \approx \int_{t-\tau_{max}}^{t} G(t-\tau) PC_n(\tau) d\tau. \tag{4}$$

We repeat the above procedure with the full range of $G(\tau)$ estimates to obtain an ensemble of $FWC_n(t)$ calculations with a standard deviation $\sigma_{Std}(t)$. In addition, we use the variance-covariance matrix of the mean $G(\tau)$ to propagate the uncertainty of the impulse-response fits forward. We add σ_{Std} and the ensemble-mean $\sigma_{Fit}(t)$ in quadrature to obtain a combined uncertainty estimate $\sigma_n(t)$ for the contribution of PC_n to the FWC evolution: $\sigma_n(t) = (\sigma_{Std}^2 + \overline{\sigma_{Fit}}^2)^{1/2}$.

Finally, we reconstruct the timeseries of Arctic FWC by combining $FWC_n(t)$ for multiple PCs, $PC_n(t), n = 1, 2, 3$.

Note that we remove the mean seasonal cycle from the FWC and PC timeseries before estimating the impulse and step response functions. It is likely that the relationship between sea-level pressure and FWC is seasonally dependent due to the seasonal cycle in ice cover. In principle we could construct a different impulse response function for each month of the year, and then use these in our FWC reconstruction. However, in practice the control run is not long enough for us to be able to estimate such monthly impulse response functions reliably.

References

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