Supporting Information for "Observing the Local Emergence of the Southern Ocean Residual-Mean Circulation"

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Introduction

This supporting information provides Text S1, Text S2, Text S3, Text S4, and Table S1 cited in the main article.

Text S1: Data

The observational datasets are those used in *Brearley et al.* [2013] and *Sévellec et al.* [2015]. We used the central mooring of the DIMES array (see Tab. S1 for a list of mooring instruments), which was deployed from 12 December 2009 to 5 March 2012 over a topographic feature leeward of the Drake Passage at 56°S, 57°50′W (Fig. 1a). The mooring site is within the ACC, and equatorward of its maximum flow [Fig. 1a and see *Brearley et al.*, 2013, for further details on the location]. A detailed quality control of the CTD and current meter data is given in *Brearley et al.* [2013]. (5 other moorings were deployed around the same time in close-by locations; as their vertical resolution is coarser, we do not consider them in this analysis.) The mooring provides time series of pressure, potential temperature, salinity, and magnitude and direction of the horizontal velocity. The data were linearly interpolated onto constant pressure levels spaced by 100 dbar (other interpolating methods for mooring motion correction were tested, and found to induce quantitative changes in our diagnostics of only less than 5%) with a 15-minute time resolution. We refer the reader to *Sévellec et al.* [2015] for a thorough estimation of measurement uncertainty. Finally, density and depth are computed using the Gibbs - SeaWater Oceanographic toolbox [*McDougall and Barker*, 2011].

Text S2: Computation of Vertical Velocity

To evaluate vertical velocity, we consider the buoyancy conservation equation in the limit of weak diffusion (Pe≫1, where Pe is the Péclet number and measures the ratio of advective to diffusive terms). This reads:

$$\partial_t b + u\partial_x b + v\partial_y b + w\partial_z b = 0, \tag{1}$$

where t is time; x, y, and z are the along, across, and vertical coordinates (along and across directions are relative to the time- and depth-mean direction of the flow, as indicated in Fig. 1a); u, v, and w are the along, across, and vertical velocities; b is the buoyancy [$=-g\rho/\rho_0$, where g is the acceleration of gravity, $\rho_{(0)}$ is the (reference) density].

To estimate the balance in (1), mooring measurements give most of the needed quantities: $\partial_t b$, u, v, $\partial_z b$. However, three required variables need to be estimated indirectly: $\partial_x b$, $\partial_y b$, and w. The horizontal buoyancy gradients are obtained using the three momentum equations under the assumption of low viscosity (Re≫1), small inertial terms (Ro≪1), and hydrostatic balance. This set of equations reads:

$$\partial_t u - fv = -\frac{1}{\rho_0}\partial_x P, \tag{2a}$$

$$\partial_t v + fu = -\frac{1}{\rho_0}\partial_y P, \tag{2b}$$

$$\partial_z P = \rho_0 b, \tag{2c}$$

where P is the pressure; f is the Coriolis parameter; Re is the Reynolds number, which measures the ratio of inertial to viscous forces; and Ro is the Rossby number, which measures the ratio of inertial forces to the Coriolis acceleration. After algebraic manipulation, we obtain:

$$\partial_x b = -\partial_t \partial_z u + f \partial_z v, \tag{3a}$$

$$\partial_y b = -\partial_t \partial_z v - f \partial_z u. \tag{3b}$$

Since u and v are known at all times and in the vertical, both along and across buoyancy gradients can be evaluated from the mooring.

Finally, vertical velocity is obtained with (1) and (3), such as:

$$w = -\frac{\partial_t b}{\partial_z b} + \frac{f}{N^2}\left(v\partial_z u - u\partial_z v\right) + \frac{1}{N^2}\left(u\partial_t\partial_z u + v\partial_t\partial_z v\right), \tag{4}$$

where $N^2{=}\partial_z b$ is the squared Brunt-Väisälä frequency; and where we have assumed a stable stratification ($N^2{>}0$).

This method closely follows the study of *Sévellec et al.* [2015]. We refer the reader to this article for further details. In particular, *Sévellec et al.* [2015] show the validity of the assumptions adopted here (in particular Ro$\ll$1 along the vertical), and the consistency of the leading-order balance of the equations with quasi-geostrophy, which is a widely used approximation in studying mesoscale turbulence. Errors in derived variables were also estimated by those authors, and confirm the accuracy of the method.

Text S3: Computation of Buoyancy Budget

We commence this analysis by decomposing all variables into a time mean and an anomaly, such as:

$$X = \overline{X} + X', \text{ where } \overline{X} = \frac{1}{\tau}\int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} X(s,z)\, ds, \tag{5}$$

where X is any variable and τ is the averaging timescale.

We take advantage of this formulation to diagnose the buoyancy balances occurring for different timescales of the flow. This is done through the time integral of (1), which becomes:

$$\overline{\partial_t b} + \overline{u}\partial_x \overline{b} + \overline{v}\partial_y \overline{b} + \overline{w}\partial_z \overline{b} = -\overline{u'\partial_x b'} - \overline{v'\partial_y b'} - \overline{w'\partial_z b'}. \tag{6}$$

All these terms can be reassessed as time averages over the entire length of the time series to diagnose the statistical equilibrium for different averaging times:

$$\text{Trend}(\tau, z) = \frac{1}{t_2 - t_1 - \tau} \int_{t_1+\frac{\tau}{2}}^{t_2-\frac{\tau}{2}} \overline{\partial_t b}\, dt, \tag{7a}$$

$$\text{Mean}_{\text{hor}}(\tau, z) = \frac{1}{t_2 - t_1 - \tau} \int_{t_1+\frac{\tau}{2}}^{t_2-\frac{\tau}{2}} \left(\overline{u}\partial_x \overline{b} + \overline{v}\partial_y \overline{b}\right)\, dt, \tag{7b}$$

$$\text{Mean}_{\text{ver}}(\tau, z) = \frac{1}{t_2 - t_1 - \tau} \int_{t_1+\frac{\tau}{2}}^{t_2-\frac{\tau}{2}} \overline{w}\partial_z \overline{b}\, dt, \tag{7c}$$

$$\text{Turb}_{\text{hor}}(\tau, z) = -\frac{1}{t_2 - t_1 - \tau} \int_{t_1+\frac{\tau}{2}}^{t_2-\frac{\tau}{2}} \left(\overline{u'\partial_x b'} + \overline{v'\partial_y b'}\right)\, dt, \tag{7d}$$

$$\text{Turb}_{\text{ver}}(\tau, z) = -\frac{1}{t_2 - t_1 - \tau} \int_{t_1+\frac{\tau}{2}}^{t_2-\frac{\tau}{2}} \overline{w'\partial_z b'}\, dt, \tag{7e}$$

where t_1 and t_2 are the initial and final times of the observations.

We can then re-write the buoyancy conservation equation by distinguishing between mean and turbulent advection as:

$$\text{Trend} + \text{Mean}_{\text{hor}} + \text{Mean}_{\text{ver}} = \text{Turb}_{\text{hor}} + \text{Turb}_{\text{ver}}, \tag{8}$$

Finally, it is useful to note that Trend=$[b(t_2){-}b(t_1)]/(t_2{-}t_1)$. Since variations in buoyancy are bounded, for long enough observational records ($t_2{-}t_1{\to}\infty$) we have Trend$\simeq$0.

This suggests that with a sufficiently long measurement time series the buoyancy budget can be assessed, regardless of the temporal variations of buoyancy.

Text S4: Temporal-Residual-Mean Framework

Here the application of the Temporal-Residual-Mean (TRM) framework formulated by *McDougall and McIntosh* [2001] to our observations is described. The TRM framework enables the computation of eddy-induced velocities consistently with time-averaging at constant density. Hence, it takes care of the adiabatic conservation of buoyancy and, in particular, it explicitly avoids misinterpreting averaged variables as mixed variables.

Through the TRM-framework, the along and across eddy-induced velocities ($\tilde{u}$ and $\tilde{v}$, respectively) are defined as:

$$\tilde{u} = \partial_z \frac{-\overline{u'b'} + \frac{\overline{\phi}}{\overline{N^2}}\partial_z \overline{u}}{\overline{N^2}}, \tag{9a}$$

$$\tilde{v} = \partial_z \frac{-\overline{v'b'} + \frac{\overline{\phi}}{\overline{N^2}}\partial_z \overline{v}}{\overline{N^2}}, \tag{9b}$$

where $\overline{\phi}=\overline{b'^2}/2$. These definitions can be used in a conservation equation for modified buoyancy:

$$\partial_t \hat{b} + \hat{u}\partial_x \hat{b} + \hat{v}\partial_y \hat{b} + \hat{w}\partial_z \hat{b} = 0, \tag{10}$$

where $\hat{b}=\overline{b}+\tilde{b}$ is the modified buoyancy, with $\tilde{b}=-\partial_z(\overline{\phi}/\overline{N^2})$ being a metric of buoyancy variance (i.e., rescaled buoyancy variance); and $\hat{u}=\overline{u}+\tilde{u}$, $\hat{v}=\overline{v}+\tilde{v}$, and $\hat{w}=\overline{w}+\tilde{w}$ are the residual along, across, and vertical velocities, respectively. The vertical eddy-induced velocity can also be diagnosed as:

$$\tilde{w} = -\overline{w} - \frac{\partial_t \hat{b}}{\hat{N}^2} - \frac{\hat{u}\partial_x \hat{b} + \hat{v}\partial_y \hat{b}}{\hat{N}^2}, \tag{11}$$

where $\hat{N}^2 = \partial_z \hat{b}$ is the modified stratification.

Now that all the terms are known, we re-write the buoyancy balance in the TRM framework as:

$$\text{Trnd} + \text{Hor} + \text{Ver} = 0, \tag{12}$$

where $\text{Trnd}=\partial_t \hat{b}$ is the trend, and $\text{Hor}=\hat{u}\partial_x \hat{b}+\hat{v}\partial_y \hat{b}$ and $\text{Ver}=\hat{w}\partial_z \hat{b}$ are the horizontal and vertical advective terms, respectively. These advective terms are further decomposed into four components: the mean advection of mean buoyancy (MAM), the turbulent advection of mean buoyancy (TAM), the mean advection of rescaled buoyancy variance (MAV), and the turbulent advection of rescaled buoyancy variance (TAV); such that $\text{MAM}_{\text{ver}}=\overline{w}\partial_z \overline{b}$, $\text{TAM}_{\text{ver}}=\tilde{w}\partial_z \overline{b}$, $\text{MAV}_{\text{ver}}=\overline{w}\partial_z \tilde{b}$, and $\text{TAV}_{\text{ver}}=\tilde{w}\partial_z \tilde{b}$ (and equivalently for the horizontal terms).

Finally, we may use the eddy-induced velocities computed within the TRM framework to apply a turbulent closure that relates the impact of turbulent advection to mean properties. Here, we will describe the *Gent and McWilliams* [1990] closure, which leads to an adiabatic redistribution of water. It relates the previously diagnosed eddy-induced velocities to the mean isopycnal slopes of the modified buoyancy as:

$$\tilde{u} = \partial_z \left(\kappa_x \frac{\partial_x \hat{b}}{\hat{N}^2} \right), \tag{13a}$$

$$\tilde{v} = \partial_z \left(\kappa_y \frac{\partial_y \hat{b}}{\hat{N}^2} \right), \tag{13b}$$

where κ_x and κ_y are the along and across eddy-induced velocity coefficients, respectively. Note that our formulation did not strictly followed the original derivation of *Gent and McWilliams* [1990]. Also, we compute raw eddy-induced velocity coefficients (i.e., we assume that rotational eddy fluxes are zero) rather than optimized eddy-induced velocity coefficients (i.e., acknowledging the effects of rotational eddy fluxes). We caution that optimized values could be significantly different from raw ones [*Eden et al.*, 2007; *Colas et al.*, 2013]. However, our set of observations does not allow us to determine them unambiguously. Note also that we account for advection of rescaled buoyancy variance through the TRM framework [not acknowledged in *Eden et al.*, 2007; *Colas et al.*, 2013, for instance].

Using (9), these coefficients read:

$$\kappa_x = \frac{-\overline{u'b'} + \frac{\overline{\phi}}{\overline{N^2}} \partial_z \overline{u}}{\partial_x \hat{b}} \frac{\hat{N}^2}{\overline{\overline{N^2}}}, \tag{14a}$$

$$\kappa_y = \frac{-\overline{v'b'} + \frac{\overline{\phi}}{\overline{N^2}} \partial_z \overline{v}}{\partial_y \hat{b}} \frac{\hat{N}^2}{\overline{\overline{N^2}}}. \tag{14b}$$

It is important to remark that, by definition, the across velocity is small, and hence the along buoyancy gradient is negligible by geostrophy. This leads to κ_x not being well defined, since it is divided by the modified buoyancy gradient for the along direction. In this context, we will ignore coefficients for the along direction (which exhibit large vertical variability, and magnitudes that may exceed 10,000 m^2 s^{-1} and are often negative) and concentrate on the across direction.

This set of diagnostics (i.e., κ_x, κ_y, $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$) has been computed for different time averaging (τ) and at all depths (z).

References

Brearley, J. A., K. L. Sheen, A. C. Naveira Garabato, D. A. Smeed, and S. Waterman (2013), Eddy-induced modulation of turbulent dissipation over rough topography in the southern ocean, *J. Phys. Oceanogr.*, *43*, 2288–2308.

Colas, F., X. Capet, J. C. McWilliams, and Z. Li (2013), Mesoscale eddy buoyancy flux and eddy-induced circulation in eastern boundary currents, *J. Phys. Oceanogr.*, *43*, 1073–1095.

Eden, C., R. J. Greatbach, and J. Willebrand (2007), A diagnosis of thickness fluxes in an eddy-resolving model, *J. Phys. Oceanogr.*, *37*, 727–742.

Gent, P. R., and J. C. McWilliams (1990), Isopycnal mixing in ocean circulation model, *J. Phys. Oceanogr.*, *20*, 150–155.

McDougall, T. J., and P. M. Barker (2011), Getting started with teos-10 and the gibbs seawater (gsw) oceanographic toolbox, *Tech. rep.*, SCOR/IAPSO WG127.

McDougall, T. J., and P. C. McIntosh (2001), The temporal-residual-mean velocity. part ii: Isopycnal interpretation and the tracer and momentum equations, *J. Phys. Oceanogr.*, *31*, 1222–1246.

Meredith, M. P. (2011), Cruise report: Rrs james cook jc054 (dimes uk2) 30 nov 2010 to 8 jan 2011., *Tech. rep.*, British Antarctic Survey Cruise Rep. 206pp.

Naveira Garabato, A. C. (2010), Cruise report rrs james cook jc041 (dimes uk1) 5 dec 2009 to 21 dec 2009, *Tech. rep.*, National Oceanography Centre Southampton Cruise Rep. 164pp.

Sévellec, F., A. C. Naveira Garrabato, J. A. Brearley, and K. L. Sheen (2015), Vertical flow in the southern ocean estimated from individual moorings, *J. Phys. Oceanogr.*, *45*, 2209–2220.

Table S1. Current meter and moored CTD nominal depths used in this study and extracted from the DIMES C-mooring located at 56°S and 57°50′W. Data were returned between 12 December 2009 and 6 December 2010, and between 20 December 2010 and 5 March 2012. Full details of the instruments used in each year can be found in the cruise reports [*Naveira Garabato*, 2010; *Meredith*, 2011].

Instrument type	Nominal depths (m)
Nortek Acoustic Current Meter	1200, 1299, 1853, 1951, 2049, 2152, 3400, 3600
Seabird Microcat (SMP)	1200, 1299, 1853, 1951, 2049, 2152, 3400, 3600