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Supporting Information for

The Glacial Mid-Depth Radiocarbon Bulge and Its Implications for the Overturning Circulation

Andrea Burke[1,2], Andrew L. Stewart[2,3], Jess F. Adkins[2], Raffaele Ferrari[4], Malte F. Jansen[5], and Andrew F. Thompson[2]

[1]Department of Earth and Environmental Sciences, University of St Andrews, St Andrews, UK

[2]Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, USA

[3]Department of Atmospheric and Oceanic Sciences, UCLA, Los Angeles, USA

[4]Department of Earth, Atmospheric and Planetary Sciences, MIT, Cambridge, USA

[5]Department of the Geophysical Sciences, University of Chicago, Chicago, USA

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Introduction

The supplementary text below describes details of the two-dimensional model used in the paper.

Text S1.

In Section 3 we provided an overview of the residual-mean model, which is sketched in Figure 4. This Appendix provides additional details that are not strictly necessary for the interpretation of our results, but are included in the interest of clarity and reproducibility.

In the ocean interior, three physical processes modify all zonally averaged tracer distributions: advection by the ocean circulation, mixing along isopycnal surfaces by mesoscale eddy stirring, and mixing across isopycnal surfaces by small-scale mixing processes. This combination of effects is summarized by the following conservation equation for some property ϕ [*Plumb and Ferrari*, 2005; *Lund et al.*, 2011]

$$\overline{\phi}_t + J\left(\psi_{\mathrm{trc}}, \overline{\phi}\right) = \nabla\cdot\left(\kappa_{\mathrm{iso}}\nabla_{//}\overline{\phi}\right) + \nabla\cdot\left(\kappa_{\mathrm{dia}}\nabla\overline{\phi}\right) + \Phi\,. \tag{S1}$$

Here the overbar $\overline{}$ denotes an average in time and longitude, so all quantities are functions of the Cartesian latitude y and depth z. For convenience we align $y=0$ with $45^{\circ}\mathrm{S}$ and $z=0$ with the ocean surface. The Jacobian $J(\psi_{\mathrm{trc}}, \overline{\phi}) = \mathbf{u}_{\mathrm{trc}}\cdot\nabla\overline{\phi}$ describes advection by the tracer streamfunction $\psi_{trc}(y,z)$. The isopycnal diffusivity κ_{iso} quantifies stirring by mesoscale eddies along isopycnal surfaces, where $\nabla_{//}\overline{\phi}$ denotes the gradient of $\overline{\phi}$ parallel to the local isopycnal slope. The isopycnal diffusivity κ_{iso} should not be confused with *K,* which is the buoyancy diffusivity that describes the slumping of isopycnals and the release of available potential energy by eddies. Isopycnal diffusion does not mix density because there is no density gradient along isopycnals. The diapycnal diffusivity κ_{dia} quantifies small-scale mixing across isopycnals, due, for example, to breaking of internal waves. We have agglomerated any additional sources and sinks of the tracer, such as fluxes at the ocean surface, into a forcing term Φ.

In order to determine the density distribution, and thus the ocean's isopycnal surfaces, we replace the abstract tracer ϕ with neutral density γ in equation (S1). By definition the gradient of $\overline{\gamma}$ along mean isopycnal surfaces is zero, $\nabla_{//}\overline{\gamma}=0$, so (S1) reduces to

$$\overline{\gamma}_t + J\left(\psi_{\mathrm{trc}}, \overline{\gamma}\right) = \left(\kappa_{\mathrm{dia}}\overline{\gamma}_z\right)_z + \frac{\partial\Gamma}{\partial z}. \tag{S2}$$

Here $\partial\Gamma/\partial z$ represents sources and sinks of density due to thermodynamic fluxes at the ocean surface, and has been written in the form of a vertical divergence of a downward density flux Γ. We have also reduced the diapycnal mixing term to its vertical component, which is typically larger than the lateral component by around 6 orders of magnitude. The streamfunction ψ_{trc} is related to the surface wind stress and local isopycnal slope via equation (5). We obtain steady solutions to (S1) by discretizing the Southern Ocean portion of our model domain on a grid of 50×50 points. The depth of the ocean is $H = 5000\,\mathrm{m}$ and the width of the ACC channel is $L_{\mathrm{channel}} = 2500\,\mathrm{km}$, so the vertical and horizontal grid spacings are $\Delta_z = 100\,\mathrm{m}$ and $\Delta_y = 50\,\mathrm{km}$ respectively. The northern basin is represented as an additional column of grid cells as long as the basin ($L_{\mathrm{basin}} = 10,000\,\mathrm{km}$). The computational domain excludes the northern and southern convection regions, which are instead treated as boundary conditions as described below. Equation (S1) is then integrated forward in time until it reaches steady state, defined as a root-mean-square time derivative smaller than $10^{-16}\,\mathrm{kg\,m^{-3}\,s^{-1}}$. The numerical approach follows that of [*Stewart et al.*, 2014].

At the ocean bed we require that there be no normal flux of mass or density through the solid boundary, setting $\psi_{\mathrm{trc}} = 0$ and $\bar{\gamma}_z = 0$ on $z = -H$. We similarly insist that there should be no mass flux through the ocean surface, $\psi_{\mathrm{trc}} = 0$ on $z = 0$. However we permit thermodynamic fluxes through the surface by setting Γ on $z = 0$ as described in Section 3; Γ is set to zero everywhere in the ocean interior.

In the AABW convective region $-(L_{\mathrm{channel}} + L_{\mathrm{AABW}}) \le y \le -L_{\mathrm{channel}}$ we assume for simplicity that strong convection results in vertical isopycnals, in such a way that the surface meridional density gradient and deep vertical density gradient are proportional. That is, we assume that the densities at the surface and at the edge of the AABW convective region are matched via

$$\bar{\gamma}\left(-L_{\mathrm{channel}}, z_0\right) = \bar{\gamma}\left(-L_{\mathrm{channel}} + z_0 \cdot L_{\mathrm{AABW}}/H, 0\right), \tag{S3}$$

for any depth z_0. This implies that the density gradients are related via

$$H \cdot \bar{\gamma}_z\left(-L_{\mathrm{channel}}, z_0\right) = L_{\mathrm{AABW}} \cdot \bar{\gamma}_y\left(-L_{\mathrm{channel}} + z_0 \cdot L_{\mathrm{AABW}}/H, 0\right). \tag{S4}$$

At the surface we prescribe a fixed flux of the form

$$\Gamma(y,0) = \Gamma_{\mathrm{ice}} \cdot \frac{\exp\left[\left(y + L_{\mathrm{channel}} + L_{\mathrm{AABW}}\right)/L_{\mathrm{AABW}}\right] - 1}{\exp(1) - 2}, \tag{S5}$$

which has been constructed such that average density input into the surface of the convective region remains equal to Γ_{ice}. Our results are not sensitive to the exact structure of this forcing, but it is important that $\Gamma|_{z=0}$ should rapidly approach zero at the southern wall $y = -(L_{\mathrm{channel}} + L_{\mathrm{AABW}})$, as otherwise the weak density stratification will produce a spuriously strong overturning circulation. Assuming that $\Gamma = 0$ at the base of each vertical isopycnal, and neglecting diapycnal mixing, we integrate the steady ($\bar{\gamma}_t = 0$) form of equation (S2) along each isopycnal to obtain a boundary condition for ψ_{trc} at $y = -L_{\text{channel}}$,

$$\psi_{\text{trc}}(-L_{\text{channel}}, z_0) = \frac{L_{\text{AABW}}}{H\bar{\gamma}_z} \cdot \Gamma(-L_{\text{channel}} + z_0 \cdot L_{\text{AABW}}/H, 0). \tag{S6}$$

We apply an identical procedure to the NADW convective region ($L_{\text{basin}} \le y \le L_{\text{basin}} + L_{\text{NADW}}$), except only isopycnals less dense than γ_{NADW} are matched to the surface, *i.e.*

$$\bar{\gamma}\left(L_{\text{basin}}, z_0\right) = \bar{\gamma}\left(L_{\text{basin}} - z_0 \cdot L_{\text{NADW}}/H_{\text{NADW}}, 0\right), \qquad z_0 \ge -H_{\text{NADW}}. \tag{S7}$$

Here H_{NADW} is the depth of the isopycnal $\bar{\gamma} = \gamma_{\text{NADW}}$ at $y = L_{\text{basin}}$. The density gradients at the surface and in the basin are related similarly. We impose a surface density forcing that restores the water column above $z = -H_{\text{NADW}}$ towards $\bar{\gamma} = \gamma_{\text{NADW}}$ with a timescale T_{NADW},

$$\Gamma(y, 0) = H_{\text{NADW}} \cdot \frac{\gamma_{\text{NADW}} - \bar{\gamma}(y, 0)}{T_{\text{NADW}}}. \tag{S8}$$

Thus the boundary condition for ψ_{trc} at $y = L_{\text{basin}}$ may be written as

$$\psi_{\text{trc}}(L_{\text{basin}}, z_0) = -\frac{L_{\text{NADW}}}{\bar{\gamma}_z} \cdot \frac{\gamma_{\text{NADW}} - \bar{\gamma}(L_{\text{basin}}, z_0)}{T_{\text{NADW}}}, \tag{S9}$$

for $z_0 \ge -H_{\text{NADW}}$, and $\psi_{\text{trc}}(L_{\text{basin}}, z_0) = 0$ for $z_0 \le -H_{\text{NADW}}$.

In the case of radiocarbon concentration, the source/sink term Φ in equation (S1) encompasses both radioactive decay and restoration by the atmosphere,

$$\overline{C}_t + J\left(\psi_{\text{trc}}, \overline{C}\right) = \nabla \cdot \left(\kappa_{\text{iso}} \nabla_{//} \overline{C}\right) + \left(\kappa_{\text{dia}} \overline{C}_z\right)_z - rC + \dot{C}_{\text{atm}}. \tag{S10}$$

Here r is the exponential decay constant for radiocarbon, and is equivalent to a half-life of 5730 yr. The atmospheric forcing term $\dot{C}_{\text{atm}}$ restores $\overline{C}$ to prescribed radiocarbon value at the ocean surface and in the convective regions, as described in Section 3. We solve (S10) in a similar fashion to equation (S2), but for the radiocarbon distribution over the entire ocean, including the northern basin and convective regions. For the purpose of this calculation the isopycnals are assumed to be flat everywhere in the convective region, because vertical isopycnals invalidate the assumption of a small isopycnal slope, and the radiocarbon evolution in these regions is dominated by the rapid restoring to the surface concentration. We use the same numerical grid spacing as for (S2), which yields a grid of 255×50 points. We impose boundary conditions of zero normal tracer flux at the ocean bed and the northern and southern walls, and permit only the atmospheric restoring flux at the ocean surface. The numerical approach is qualitatively similar to that of *Stewart et al.* [2014]: the advection-diffusion equation (S10) is stepped forward in time using the total variation-diminishing finite volume scheme of [*Kurganov and Tadmor*, 2000]. The computation is halted once the solution has reached a steady state, defined as a root-mean-square time derivative smaller than 10^{-13}‰ s^{-1}.

Supporting Information References

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