Supplementary Information to Thingstad T.F. :

How trophic cascades and photic zone nutrient content interact to generate basin-scale differences in the microbial food web.

Linear NFC (nutrients – flagellates – ciliates) food chain

Assuming food uptake to be proportional to food concentration (Holling Type I functional relationships), the mass flows of the limiting element in the model of Fig. 2A can be described with the set of Lotka-Volterra type differential equations:

Autotrophic flagellates: $\frac{dF}{dt} = \alpha_F NF - \alpha_C FC,$ (1)

Ciliates: $\frac{dC}{dt} = Y_C \alpha_C FC - \delta_C C,$ (2)

Mass balance: $N_T = N + F + C.$ (3)

Where the α-parameters are the affinities/clearance rates, Y_C is the fraction of limiting element transferred from flagellates to ciliates, and δ_C is the loss rate of ciliates to higher predators. Eqn. 3 is the assumption that all of the limiting element released during grazing is immediately recycled to the free inorganic form *N*.

At steady state ($\frac{dN}{dt} = \frac{dF}{dt} = \frac{dC}{dt} = 0$) the solution of Eqns (1) and (2) and insertion in Eqn. 3 gives

$$N^* = \frac{N_T - \frac{\delta_C}{Y_C \alpha_C}}{1 + \frac{\alpha_F}{\alpha_C}},\ F^* = \frac{\delta_C}{Y_C \alpha_C},\ \text{and}\ C^* = \frac{N_T - \frac{\delta_C}{Y_C \alpha_C}}{1 + \frac{\alpha_C}{\alpha_F}}.$$

The steady state [N^*, F^*,C^*] thus depends on the parameters (α and Y values) and the two external drivers total nutrient N_T and loss rate to higher predators δ_C as illustrated in Fig. 2B.