APPENDIX

**Adaptation of the von Bertalanffy nonlinear growth curve for models of functional dissimilarities along spatial or environmental gradients**

We propose that the von Bertalanffy growth model (VBGM) may be adapted directly to create a nonlinear model that relates mean pair-wise (or nearest-neighbour) functional dissimilarities (or distances) between two samples *versus* the absolute differences in the positions of those two samples along a spatial or environmental gradient. The typical von Bertalanffy growth model is designed to model the length of fish as they grow over time, and its equation may be written as follows (Cailliet *et al*. 2006; Ogle 2016):

$E\left[L|t\right]=L\_{\infty }-\left(L\_{\infty }-L\_{0}\right)e^{-Kt}$ (1)

where $E\left[L|t\right]$ is the expected length at time $t$, $L\_{\infty }$ is the asymptotic average length, $K$ is the growth-rate coefficient, and $L\_{0}$ is the mean length at time zero. The essential idea here is that a fish grows quickly at a young age, but the size (length) of the fish begins to level off over time, eventually approaching an asymptote for the model of average length-at-age ($L\_{\infty }$).

Next, consider a measure such as the mean nearest-neighbour distance (MNND.*beta*) or the mean pairwise functional distance (MPFD.*beta*), calculated in a chosen multivariate functional space, between the species occurring in two samples along a gradient of interest. We expect functional differences among species to increase with increasing differences in their (modal) positions along the gradient. Furthermore, presuming that we remain within a given clade (e.g., Actinopterygii), we expect eventually to approach an asymptote, where no additional distances in functional space are observed or accrued, even for samples occurring at very large distances apart along the gradient.

Our response variable (either MNND.*beta* or MPFD.*beta*, to be plotted along the y-axis) is a distance, $d\_{Y}$. Our predictor variable (the absolute difference in position along the gradient of interest) is also a distance, $d\_{X}$. As an example, consider the plot of mean nearest-neighbour distances in functional trait space based on eight morphological measurements of ray-finned fishes (MNND.*beta*) versus absolute differences in depth (Fig. A1).

We recognise that individual points in Fig. A1 are not independent of one another, as both $d\_{Y}$ and $d\_{X}$ arise from structured inter-point distance matrices. However, we are not able to fit a “distance-decay” type of model here (e.g., Nekola & White 1999; Millar *et al*. 2011), as there is no upper bound on the functional distances; transforming $d\_{Y}$ values to a 0 – 1 scale (or a 0 – 100 % scale) is not really sensible or warranted for unbounded functional multivariate spaces.

We may, however, fit a von Bertalanffy nonlinear growth curve, treating $d\_{Y}$ as “length” ($L$) and treating $d\_{X}$ as “time” ($t$). Nonlinear least squares may be used to estimate the model parameters. Variances and confidence intervals on the model parameters may then be obtained using a jack-knife procedure (see Millar *et al*. 2011, for details), followed by the delta method to obtain prediction intervals. The jack-knife approach explicitly and empirically acknowledges that the correct sampling units (and degrees of freedom) in the system being modelled here arise from the original ($N$) sampling units occurring along the gradient, and not the ($N\left(N-1\right)/2$) distances we have calculated among them.

Generally, it is clear that our proposed nonlinear model may well provide a better fit than a linear model, depending on the nature of the data and the relative importance, steepness and length of the sampled gradient, relative to biotic functional change. The model also has greater biological and ecological credence, compared to some other potential nonlinear models. For example, adding a quadratic term to a linear model of $d\_{Y}$ versus $d\_{X}$ would generate a parabolic response curve. A parabolic model would predict that, for samples occurring at increasingly large distances apart along the gradient, the functional distances will eventually decrease and return to zero, or even become negative, which is clearly biologically unrealistic.



**Fig. A1.** Mean nearest-neighbour distance in functional multivariate space (MNND.*beta*), based on eight normalized continuous morphological trait variables for marine ray-finned fishes at the Three Kings Islands, New Zealand *vs*. absolute differences in depth (in metres). The black line shows the adapted von Bertalanffy growth model fit to these data; the 1-standard-error prediction region (grey lines) was obtained using jack-knife estimates of variances in parameters, followed by the delta method.

**References**

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