
Interaction of an Upwelling Front with External Vortices: Impact on Cross-shore Particle Exchange

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Abstract :

Coastal upwellings, due to offshore Ekman transport, are more energetic at the western boundaries of the oceans, where they are intensified by incoming Rossby waves, than at the eastern boundaries. Western boundary upwellings are often accompanied by a local vortex field. The instability of a developed upwelling front and its interaction with an external vortex field is studied here with a three-dimensional numerical model of the hydrostatic rotating Navier-Stokes equations (the primitive equations). The baroclinic instability of the front leads to the growth of meanders with 100-200 km wavelength, in the absence of external vortex. On the f-plane, these waves can break into a row of vortices when the instability is intense. The beta-effect is stabilizing and strongly decreases the amplitude of meanders. Simulations are then performed with a front initially accompanied by one or several external vortices. The evolutions in this case are compared with those of the unstable jet alone. On the f-plane, when an external vortex is close to the front, this latter sheds a long filament which wraps up around the vortex. This occurs over a period similar to that of the instability of the isolated front. Cyclones are more efficient in tearing such filaments offshore than anticyclones. On the beta-plane, the filaments are short and turbulence is confined to the vicinity of the front. At long times, waves propagate along the front, thus extending turbulence alongshore. The initial presence of a vortex alley leads to a stronger destabilization of the front and to a larger cross-shore flux than for a single vortex, with many filaments and small vortices pushed far offshore. In the ocean, this cross-shore exchange has important consequences on the local biological activity.

Keywords : coastal upwelling front, vortices, filaments, baroclinic instability, frontal waves, particle motion and tracking

1. Introduction

The Arabian Sea, in the North-Western Indian Ocean, is the seat of oceanographic processes which interact dynamically (see Fig. 1). These physical processes in turn, influence the regional biological activity of the ocean. This sea evolves under the influence of monsoon winds (north-easterlies in winter and south-westerlies in summer; Schott & Fischer, 2000) and of a strong evaporation rate (Privett, 1959). The strong winds induce a cyclonic regional circulation in winter and an anticyclonic regional one in summer. Moreover, these winds generate Rossby waves near the western coast of India (Brandt et al., 2002). These waves propagate westward and they can strengthen large, long-lived eddies, like the Great Whirl near the coast of Somalia (Vic et al., 2015) and the Ras al Hadd dipolar eddy South of Oman (Ayouche et al., 2021) (eddies are oceanic vortices). The Arabian Sea is populated with many eddies (Bruce et al., 1994; Al Saafani et al., 2007; Trott et al., 2019). Such eddies are formed in particular by the instability of coastal currents and of the regional gyre currents.

A second main feature of the Arabian Sea are two western boundary upwellings (near Somalia and South of Oman), as the wind blows parallel to the coast and northwestward in summer (Sastry & d'Souza, 1972; Bruce, 1974; Elliott & Savidge, 1990; Currie, 1992; Shi et al., 2000; Fischer et al., 2002; Piontkovski & Al-Jufaili, 2013; Vic et al., 2017). In upwellings, the Ekman drift due to the wind pushes the surface water offshore leading to an outcropping of deeper and colder water from depths of 150 m. This cold water (which can be 5°C cooler than the Arabian Sea surface water) is rich with nutrients, favoring the local biological activity. And indeed, this upwelling region is an intense fishing area. When the upwelling is developed (in August/September), the sea surface height near the coast can lie 30 cm below the offshore sea surface and a northeastward current, the Oman Coastal Current, flows offshore. Western boundary upwellings are more energetic than their eastern boundary counterparts because they absorb energy from incoming Rossby waves. The temperature and salinity fronts which bound the upwelling region offshore are not steady. They often meander and they may form cold filaments. Capes play an important role in the offshore growth and protrusion of cold filaments from upwellings (Currie, 1992; Shi et al., 2000; Meunier et al., 2010). These filaments can extend offshore over more than 250 km and bring nutrients into

31 the oligotrophic surface ocean (Manghnani et al., 1998). South of Oman, the upwelling lasts
32 for the whole summer and interacts with the local eddy field (see Fig. 5 of Shi et al., 2000).
33 The upwelling front and current can form meanders and eddies, either by instability or *via*
34 the interaction with the eddies (see Fig. 5 of Sastry & d’Souza, 1972). The evolution of an
35 upwelling front and coastal current in the presence of external vortices, and its comparison
36 with the evolution of an unstable front in the absence of such vortices, is the subject of the
37 present paper, with application to the Arabian Sea (again, see Fig. 1).

38 The structure and stability of upwelling systems, with fronts and currents, have been the
39 subject of many previous studies (Yoshida, 1955; Hidaka, 1972; Pedlosky, 1974; McCreary,
40 1981; Csanady, 1982, to name a few). The first studies aimed at understanding the spin-up
41 of the upwelling as the wind starts to blow and before it becomes a well established front
42 associated with a geostrophic current. In particular, the role of the frictional boundary
43 layers in the formation of the upwelling was investigated (Pedlosky, 1974). The importance
44 of the topographic constraint on the deep boundary layer was analyzed (Pedlosky, 1978a,b).
45 Analytical solutions of the density and velocity structure were provided for a time varying
46 wind stress forcing the upwelling. In particular, the formation of a sub-surface counter-
47 current was studied with regard to the vertical mixing of heat and momentum (McCreary,
48 1981). For an upwelling south of a zonal coast, an eastward baroclinic current builds up as
49 the offshore front steepens. When the wind stress has a longshore variation, Kelvin waves
50 propagate along the coast, away from the upwelling region. The alongshore flow deviates
51 from geostrophy. Ageostrophic velocities perpendicular to the coast then form a two-cell
52 vertical circulation (Suginohara, 1977).

53 In our study, we investigate the instability of the geostrophic current associated with
54 the offshore upwelling front, or how it interacts with an external eddy field; we assess the
55 consequences of these interactions on cross-shore transport. The interaction between a zonal
56 current and a single external eddy has been the subject of several studies. Pratt & Stern
57 (1986) studied the growth and detachment of an eddy from a potential vorticity front in
58 a one-layer model. Stern & Flierl (1987) considered the interaction between a single eddy
59 and a zonal potential vorticity front in a one-layer model (with finite or infinite radius of
60 deformation). They showed that, in the linear stage of the interaction, the surface integral

61 of vorticity in the meander of the front is equal and opposite to the area integral of the
62 eddy vorticity. This leads to a dipolar effect which can advect the eddy along the front.
63 Vandermeirsch et al. (2003a,b) determined the conditions under which an eddy can cross
64 a potential vorticity front meridionally, in one and two-layer models. The existence of a
65 stagnation point (a hyperbolic point in the flow field) was found as a necessary condition for
66 this crossing. This condition is identical for the detachment of an eddy from an unstable front
67 (Capet & Carton, 2004). These latter authors found that the circulation of the detached eddy
68 was proportional to that enclosed within a wavelength of the meandering front, extending
69 offshore to the stagnation point.

70 This paper is organized as follows. Section 2 describes the numerical model and the
71 analysis tools. In particular, the novelty of this paper lies in the use of a three-dimensional
72 model (see supplementary information for the presentation of the 3D evolution of some
73 simulations discussed in this study). Section 3 details the numerical results of the upwelling
74 current instability, in the absence of vortices. The results are analyzed with respect to the
75 characteristic wavelengths of the meanders growing on the current and of the inshore and
76 offshore transports. Section 4 considers the nonlinear evolution of the front in the presence
77 of a single vortex offshore. Cyclone-anticyclone asymmetry is considered since the flow is not
78 quasi-geostrophic. Section 5 considers the situation where multiple vortices exist South of
79 the front, as observed in the Arabian Sea. Section 6 characterizes the cross-shore exchanges
80 with particle tracking. Conclusions follow.

81 **2. Methods**

82 *2.1. The numerical simulations*

83 In this section, we present the idealized model runs performed for this study. The aim of
84 these runs is to study the instability of an upwelling front and to simulate the interaction of
85 an upwelling front with one or several vortices.

86 *2.1.1. Numerical setup and domain*

87 The simulations are carried out in a 3D hydrostatic primitive equation framework. They
88 are performed using the Coastal and Regional Ocean COmmunity (CROCO) model (Shchep-

89 etkin & McWilliams, 2005). This model solves the hydrostatic primitive equations for the
 90 velocity, temperature, and salinity, using a full equation of state for seawater (Shchepetkin
 91 & McWilliams, 2011). The simulations performed integrate the primitive equations for 1
 92 year. The numerical settings are similar to previous simulations performed in an idealized
 93 context (see *e.g.* Ménesguen et al., 2018; de Marez et al., 2020b): horizontal advection terms
 94 for tracers and momentum are discretized with fifth-order upwind advection schemes (UP5);
 95 the explicit horizontal viscosity and diffusivity are set to zero, since the UP5 scheme damps
 96 dispersive errors; the vertical advection is discretized with a fourth-order centered parabolic
 97 spline reconstruction (Splines scheme). Further discussion about these parameterizations
 98 can be found in Klein et al. (2008) or Ménesguen et al. (2018). Vertical mixing of tracers
 99 and momentum is done using a K-profile parametrization (KPP, Large et al., 1994), and
 100 there is no bottom friction. Simulations have 32 terrain-following vertical levels, which are
 101 stretched such that the vertical resolution is $\Delta z \sim 8$ m at the surface, and $\Delta z \sim 120$ m
 102 at the bottom. The horizontal resolution is 5 km. The domain is square, with a length
 103 $L_{\text{domain}} = 2000$ km or $L_{\text{domain}} = 3000$ km in vortex alley simulations (see details below). The
 104 bottom is flat and 2000 m deep. We set the Coriolis parameter to $f = f_0 = 10^{-4} \text{ s}^{-1}$ or
 105 $f = f_0 + \beta y$, with $\beta = 2 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$ and y the meridional coordinate. The northern
 106 and southern boundaries are closed, with a 10 km wide sponge layer to avoid the generation
 107 of spurious boundary dynamics. A zonal periodic condition is chosen at the eastern and
 108 western boundaries such that the domain is a zonal channel.

109 We initialize an analytical background stratification $N(z)$, which fits the average ambient
 110 stratification in the five major oceanic basins, similarly as in de Marez et al. (2020a,c):

$$N(z) = N_0 + N_1 e^{z/z_h}, \quad (1)$$

111 with $z < 0$ the vertical coordinate, $N_1 = 9 \times 10^{-3} \text{ s}^{-1}$, $z_h = 150$ m, and $N_0 = 7 \times 10^{-3} \text{ s}^{-1}$.
 112 Integrating this stratification from the surface (where $\rho(z = 0) = 1030 \text{ kg m}^{-3}$), gives the
 113 ambient density background $\rho(z)$; the temperature background $T(z)$ is obtained by inverting
 114 the TEOS-10 equation of state for seawater McDougall & Barker (2011) and assuming a
 115 constant salinity background $S(z) = 35$ psu. The model is initialized with these temperature
 116 and salinity background profiles.

117 *2.1.2. Components of simulations*

118 *Upwelling front.* We chose to study the evolution of the upwelling front once it is established
119 by the wind forcing. This established front is the initial condition of our simulations, which
120 are then run in free decay. These simulations determine the further, unforced, evolution of
121 this front. Not adding surface forcing in the simulations allows us to analyze specifically
122 the front instability or the front-vortex interaction as a classical initial-value problem. Our
123 simulations have an application to the ocean when the time scales of the evolution are shorter
124 than those of the forcing. For other cases, simulations with forcing, or with a relaxation
125 towards a prescribed state, will be performed in a follow-up of this study. It must be noted
126 that free-decay simulations have often been used in studies of oceanic flow stability, since
127 they produce instability waves which depend only on the current and not on the forcing.
128 We simulate the presence of an upwelling front by initializing a zonal jet of a given velocity
129 profile, similarly as in Barth (1994); this velocity profile has the form:

$$u_{up} = V_0^{up} e^{z/H} e^{-\left(\frac{y-y_0^{up}}{L}\right)^2}, \quad (2)$$

130 with V_0^{up} the initial velocity of the upwelling jet, $y_0^{up} = L_{\text{domain}}/2$ the latitude of the upwelling
131 jet, and H and L the vertical and zonal extent of the upwelling jet, respectively. This current
132 is then geostrophically adjusted with the density field and the sea surface height. This leads
133 to a temperature front separating cooler waters North from warmer waters South.

134 In the simulations discussed in this study, we set $V_0^{up} = 0.2 \text{ m s}^{-1}$, $H = 200 \text{ m}$, and
135 $L = 50 \text{ km}$. These values correspond to those chosen in previous studies (*e.g.* Barth, 1994;
136 Vic et al., 2017) for the study of upwellings.

137 *Sinusoidal perturbation.* To trigger the upwelling front instability, we add a sinusoidal pertur-
138 bation along the front, directly in the initialization of a few experiments. This disturbance
139 idealizes the observed perturbations due to the surrounding flow, or to the atmospheric
140 influence, on the upwelling front. It is usual to choose a single wave as a perturbation in
141 linear, normal-mode theory of flow stability. This disturbance is here a temperature anomaly
142 centered around y_0 , and of the form:

$$T_{\text{pert}} = 0.5 e^{-z/500} e^{-\left(\frac{y-y_0^{\text{up}}}{50 \cdot 10^3}\right)^2} \cos\left(\frac{2\pi}{\lambda_{\text{pert}}} x\right), \quad (3)$$

143 with x the zonal direction, and $\lambda_{\text{pert}} = [50, 100, 250, 500, 1000]$ km. Varying the wavelength
 144 allows the determination of the fastest growing disturbance. The choice of a perturbation
 145 deeper than the mean flow is motivated by the likeliness that it is triggered by neighboring
 146 currents which are deeper than the upwelling: vortices, or marginal sea outflow currents, or
 147 baroclinic Rossby waves. A density anomaly is then computed assuming that the perturba-
 148 tion has no signature in salinity. The velocity field associated with this density anomaly is
 149 then obtained *via* the thermal wind balance.

150 *Vortex.* To address the main objective of this article (the interaction of an upwelling front
 151 with a pre-existing vortex field), we initialize one or several surface-intensified vortices, South
 152 of the upwelling front (offshore), in the vortex-front interaction simulations. When several
 153 vortices are initialized, they are aligned along the coast offshore of the front. These vortices
 154 are initialized in azimuthal velocity (or in vorticity) as in de Marez et al. (2020a,c). For each
 155 vortex, we set the initial profile of surface vertical vorticity:

$$\omega(r) = \pm\omega_0 e^{-\left(\frac{r}{R}\right)^\alpha}, \quad (4)$$

156 with the sign depending on the vortex polarity, $r = \sqrt{(x - x_c)^2 + (y - y_c)^2}$ the radial coordi-
 157 nate referenced at the center of the vortex (x_c, y_c) , and α (usually an integer) the steepness
 158 parameter.

159 The surface azimuthal velocity of the vortex can be computed using $v_\theta(r, 0) = \frac{1}{r} \int r dr \omega r$.
 160 In general it takes the complicated form $v_\theta(r) = (\omega_0 R^2 / (\alpha r)) \gamma((2/\alpha), (r/R)^\alpha)$, where $\gamma(s, x)$
 161 is the incomplete gamma function. In the well-known case of a Gaussian vertical vorticity,
 162 with $\alpha = 2$, the azimuthal velocity is $v_\theta(r) = \omega_0 \frac{R^2}{r} [1 - e^{-r^2/R^2}]$. In this case, this velocity
 163 grows linearly with r , for $r \ll R$, similarly to Rankine vortices. Far from the center, it
 164 decays at best as $1/r$ (that is, slower than for a Rankine vortex). To avoid the presence of
 165 spurious velocity at the edges of the domain, we apply a Hanning window on v_θ to make
 166 it smoothly tend to zero at $r > 3R$. The horizontal velocity decreases at depth such that
 167 $v_\theta(x, y, z) = v_\theta(x, y, 0) e^{-z/H_{\text{vortex}}}$, thus defining the height of the vortex H_{vortex} . We de-

168 note (u, v) the horizontal Cartesian components of the velocity of the vortex. The pressure
 169 anomaly field $P'(x, y, z)$ corresponding to this velocity field is computed *via* the gradient
 170 wind equation:

$$2J(u, v) + f(\partial_x v - \partial_y u) = \frac{1}{\rho} \Delta_h P', \quad (5)$$

171 with $J(u, v) = \partial_x u \partial_y v - \partial_y u \partial_x v$ the Jacobian operator, and Δ_h the horizontal Laplacian
 172 operator. From P' we determine the density and the temperature anomalies of the vortex.
 173 These anomalies are computed for as many vortices as we want, at positions x_c , and y_c . Note
 174 that such vortices are robust during the whole simulation.

175 We set $H_{\text{vortex}} = 1000$ m, as surface-intensified vortices are mostly about 1000 m deep in
 176 this part of the ocean (Chaigneau et al., 2011; Pegliasco et al., 2015; Keppler et al., 2018;
 177 de Marez et al., 2019), and $\alpha = 24$, so that the vortex has a profile close to a Rankine
 178 vortex (also called a top-hat vortex, as in *e.g.* de Marez et al. (2020c)). For our study,
 179 we vary other initial parameters as $R = [50, 100]$ km, and $V_0^{\text{vortex}} = [0.2, 0.4, 0.6] \text{ m s}^{-1}$
 180 for both cyclonic and anticyclonic vortices. For each set of parameters, we also vary the
 181 initial distance between the vortices and the upwelling front as $d/R = [2.0, 3.0, 4.0, 5.0]$,
 182 thus setting y_c .

183 In simulations with an isolated vortex, we set $x_c = L_{\text{domain}}/2$. For simulations with a
 184 zonal vortex alley (*i.e.* an alley of vortices of alternated polarities, at a given latitude, with
 185 a constant spacing between their centers) we also vary the distance between each vortex
 186 $d_{\text{btw}}/R = [2.0, 3.0, 4.0]$ and the number of vortices $N = [2, 4, 6]$. In these cases, the center
 187 (in the zonal direction) of the vortex alley is at the center of the domain $L_{\text{domain}}/2$. Therefore
 188 $L_{\text{domain}} = N \times d_{\text{btw}}$.

189 A summary of the three kinds of simulations we ran, with the different parameters we
 190 varied, is shown in Fig. 2.

191 2.2. Diagnostics

192 2.2.1. Modal decomposition

193 To analyse the growth of perturbations on the upwelling front, we decompose its deviation
 194 from a straight front in zonal Fourier series. We apply this analysis to the surface temperature

195 field at each time step

$$T(x, y) - \langle T(y) \rangle = \sum_{j=0}^{j=\infty} [c_j(y) \cos(jx) + s_j(y) \sin(jx)] \quad (6)$$

196 where $\langle T(y) \rangle$ is the straight temperature front corresponding to $u^{up}(y, z = 0)$. Then,
 197 to examine only the modes of the upwelling front, we average c_j and s_j in a 100 km wide
 198 meridional band. This gives the time varying amplitude of each mode $\langle c_j^2 + s_j^2 \rangle_y(t)$. In the
 199 following, we present and discuss the wavelength of each mode $\lambda = L_{\text{domain}}/j$, with L_{domain}
 200 the size of the domain (2000 km).

201 2.3. The frontogenesis function

202 To characterize the evolution of buoyancy gradients, we calculate the frontogenesis func-
 203 tion F associated with the buoyancy. It is defined as follows (Hoskins, 1982):

$$F(\mathbf{u}, b) = \partial_x u (\partial_x b)^2 + (\partial_x v + \partial_y u) \partial_x b \partial_y b + \partial_y v (\partial_y b)^2.$$

204 The opposite of the function F indicates the tendency of the buoyancy gradients to steepen:

$$-F(\mathbf{u}, b) = \frac{d}{dt} |\nabla b|^2.$$

205 2.4. Particle advection

206 We ran particle advection simulations, using the outputs of CROCO simulations, with
 207 the set of python classes Parcels (Parcels stands for ‘‘Probably A Really Computationally
 208 Efficient Lagrangian Simulator’’). This software simulates the advection of an ensemble of
 209 particles, using a given 2D or 3D velocity field. This tool has been widely used in the past
 210 few years, and it is fully described in Lange & van Sebille (2017); Delandmeter & van Sebille
 211 (2019) and in references therein.

212 The surface velocity field is used here, and the forward advection is computed with a
 213 fourth order Runge-Kutta scheme (time spacing $dt = 5$ minutes). Particles are initially set
 214 at $t = 0$ days on a regular grid, at $100 \leq x \leq 1900$ km and $500 \leq y \leq 1500$ km, with a spacing
 215 between particles of 25 km. Thereby, a total of 2993 particles are advected, throughout the
 216 year of simulation.

217 3. Analysis of the upwelling front instability

218 3.1. Modal analysis of growing perturbations

219 Firstly we studied the instability of the straight upwelling front alone (in the absence
220 of any initial vortex). Only a sinusoidal perturbation was added along the front, to trigger
221 its instability (see section 2.1.2 and Fig. 2(left)). The modal amplitude of the growing
222 perturbations (the unstable waves on the front) for various perturbation wavelengths, when
223 one neglects the β -effect (*i.e.* on the f -plane), is shown in Fig. 3(a). We distinguish the
224 time series of amplitudes for a mode with a given wavelength initially (λ_{pert}), from those for
225 the mode which is finally the most unstable one (λ_{max}), and has this wavelength (see also
226 figure caption). If the initial wave grows the fastest, then $\lambda_{\text{pert}} = \lambda_{\text{max}}$.

227 When they are forced initially, waves shorter than the most unstable one also grow but
228 decay rapidly; thus they do not reach large amplitudes. They dominate at short time for
229 fast jets and may thus be related to horizontal shear flow instability. On the contrary,
230 longer waves reach larger amplitudes with time, either within a month or within 8 months.
231 In particular, waves with 200-250 km wavelength are the most unstable over a 1-3 months
232 period; they are followed in amplitude by 500 km long waves. With an internal radius of
233 deformation close to 50-60 km in this oceanic region (Chelton et al., 1998), a wavelength of
234 $2\pi R_d = 300\text{-}350$ km is characteristic of baroclinic instability. Therefore, the growth of these
235 long waves in the first months of simulation can be related to the development of baroclinic
236 instability along the jet. When long waves grow after a longer period (*e.g.* 8 months), their
237 origin is rather to be searched in wave-wave interaction, since the flow is not forced.

238 A similar modal analysis but now including the β -effect is shown in Fig. 4(a). The β -effect
239 renders flows zonal so that long waves are damped (see *e.g.* Flierl et al. (1999), or the well-
240 known Phillips model of baroclinic instability (Vallis, 2017)). As a result, the most unstable
241 waves in our simulations lie near ~ 100 km wavelength. In fact, the dimensionless value of β is
242 $\beta L^2/U = L^2/L_{\text{Rhines}}^2$. For the same geophysical value of $\beta = 2 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$, this damping
243 effect will be weak for a fast jet such as the Gulf Stream ($U = 1 \text{ m s}^{-1}$), but stronger for the
244 jet associated with a typical upwelling front (in our simulations, $U = 0.2 \text{ m s}^{-1}$). Therefore
245 meanders on the upwelling front are expected to have a shorter wavelength than those of
246 intense jets such as western boundary currents.

247 *3.2. Spatial structure of the front with growing perturbations*

248 *3.2.1. In the absence of β -effect*

249 When short waves are forced initially, they can grow even if they are shorter than the
250 linearly most unstable wave. This growth will only be transient. These short waves saturate
251 in amplitude without breaking and then slowly decay. Longer waves break and form vortices
252 of about 200 km diameter (see Fig. 3(b)). We note that northward extending (anticyclonic)
253 meanders do not form vortices while southward (cyclonic) meanders do. This North-South
254 asymmetry of meanders on a front can be explained by (1) the difference in local deformation
255 radius (smaller to the North than to the South, if the upper ocean layer is thinner to the
256 North); for baroclinic instability, a smaller deformation radius favors the growth of shorter
257 waves (and conversely); and (2) the curvature vorticity $V/r \partial_r V$, which contrary to the
258 geostrophic vorticity is parity biased (here V is the jet velocity magnitude in the meander,
259 and r is the radius of curvature of the meander). These arguments can explain the growth
260 of longer waves than those initialized, South of the jet, thus leading to the breaking of
261 meanders into vortices. Such asymmetries do not appear in quasi-geostrophic simulations of
262 jet instability.

263 The most unstable wave is determined by computing its growth during the early stage of
264 the evolution. When this wave is initialized (*e.g.* $\lambda_{\text{pert}} = 250$ km), it grows up to breaking
265 the front, generating vortices of ~ 200 km diameter. Since the most unstable perturbation
266 is initialized everywhere, vortices form both north and south of the jet. Along the central
267 latitude, the jet comes back to a quasi zonal state once the vortices have detached. This
268 effect has been explained in previous studies (Baey et al., 1999) and it can be clearly seen
269 on Fig. 3(c).

270 If a longer wave is initialized, shorter waves emerge *via* nonlinear wave-wave interactions,
271 and finally vortices of ~ 200 km diameter form. These nonlinear wave interactions also
272 generate other features such as filaments and smaller vortices (see Fig. 3(d)).

273 *3.2.2. In the presence of β -effect*

274 When β -effect is added, long waves do not grow and shorter waves dominate the evolution
275 of the flow. This leads to the generation of filaments and of small vortices (of ~ 50 km

276 diameter). The most unstable wave form vortices of ~ 100 km diameter. Since the β -effect
277 renders flows zonal, meanders grow less north and south, and the perturbations remain
278 confined near the front axis, see Fig. 4(b,c,d). Even long waves imposed initially on the
279 front bifurcate towards smaller meanders. Still, they do not form very small scale features
280 and thus the flow pattern is spatially more regular.

281 4. Upwelling front evolution in the presence of a single vortex

282 Now that the instability of the front, without external vortex, has been studied, we
283 investigate the influence of a single vortex on this same front. The vortex is initialized south
284 of the front and it replaces the perturbation added to the front in the previous section (see
285 section 2.1.2 and Fig. 2(middle)). This vortex deforms the front along its own spinning
286 motion. Here, we do not perform a Fourier analysis of the front perturbation; indeed,
287 the wavelength which is the most unstable on the straight front in the absence of external
288 vortex, is now perturbed by the presence of the vortex. Performing this analysis here would
289 in particular reflect the vortex size. For the simulations discussed here, the vortex position,
290 polarity, radius, and velocity are varied. The evolution of the front on the f -plane and on the
291 β -plane is now presented. The position of the front is chosen as the isotherm corresponding
292 to the average of the northern and southern temperature after 100 (resp. 200) days of
293 simulated time in Fig. 5 (resp. Fig. 6).

294 4.1. The reference cases

295 To further study the influence of the vortex parameters on the front stability, we define
296 reference cases, for which the vortex can have either polarity, a radius $R = 100$ km (corre-
297 sponding to the radius of vortices formed by the instability of the front previously described),
298 a distance to the front $d = 2R$, and an azimuthal velocity $V_0^{\text{vortex}} = \pm 0.2 \text{ m s}^{-1}$ initially. The
299 front positions in these cases are shown by solid color lines in Figs. 5 and 6.

300 In this case, on the f -plane, the presence of a vortex south of the front induces a merid-
301 ional deformation of the front in its vicinity, but not over the whole length of the domain,
302 during the first three months of simulation. At later times (200 days), this filament extrudes

303 offshore and detaches from the front. Also, the most unstable wave of the front grows,
 304 leading to small scale perturbations along the whole front.

After 3 months, we observe that if the vortex is cyclonic, the front is more deformed than if the vortex were anticyclonic. This can be explained by the following. Though their relative vorticities are initially antisymmetric, their Ertel potential vorticities are asymmetric. Indeed, even at first order beyond quasi-geostrophy the absolute vorticity multiplies the density anomaly, and we can write this first-order Ertel potential vorticity anomaly as

$$\delta Q = \omega + (f_0 + \omega)b/N^2,$$

305 where b is the buoyancy anomaly associated with the vortex, and N the Brunt-Väisälä fre-
 306 quency $N^2 = -\frac{\partial b}{\partial z}$. δQ characterizes the vortex ability to remotely influence its environment.
 307 Since ω and b are antisymmetric at first order for cyclones and anticyclones, δQ is larger
 308 in modulus for cyclones. Again at first order, this potential vorticity anomaly is conserved
 309 in time. Stern & Flierl (1987) have shown that in two-dimensional flows, a vorticity front
 310 develops a meander with equal and opposite circulation to that of the vortex deforming it.
 311 Applying this principle here explains why fronts extrude longer and/or larger filaments when
 312 nearing a cyclone than an anticyclone; this can be seen after 100 simulated days. After 200
 313 days, the perturbation breaks down non-linearly and becomes spatially more convoluted and
 314 intricate. These nonlinear effects can also lead to the production of long offshore filaments
 315 by fronts under the influence of anticyclones, but this effect is weaker than after 100 days.
 316 Meanders and filaments are much smaller in the presence of β -effect than in its absence.

317 4.2. Sensitivity to the vortex radius

318 The influence of the vortex radius R on the flow evolution is studied for a vortex lying at
 319 a distance $d = 2R$ from the front, with an azimuthal velocity $V_0^{\text{vortex}} = \pm 0.2 \text{ m s}^{-1}$ initially,
 320 see Figs. 5(a,b) and 6(a,b). Under these conditions, wider vortices lead to longer filaments
 321 offshore both after 100 and 200 days. On the f -plane, smaller vortices yield more wave
 322 amplification downstream, and in particular more coastal intrusion of open ocean fluid (*i.e.*
 323 South of the front). The growth of a longer filament in the presence of a larger vortex is also
 324 associated with a weaker growth of the 200 km long wave on the front; the evolution of the
 325 front under the influence of the vortex supersedes the unstable evolution of the front (alone).

326 At long times, the long frontal wave recovers its strength of the jet instability case, for both
327 small and large vortices. In the presence of β -effect, cyclonic vortices drift northwestward
328 and thus come closer to the front while anticyclonic vortices drift away from the front on
329 the β -plane, explaining the difference in the generated filament lengths. Again, the zonal
330 straightening effect of β on streamlines prevails, so that shorter filaments are generated on
331 the β -plane than on the f -plane.

332 4.3. Sensitivity to the vortex intensity

333 The influence of the vortex intensity V_0^{vortex} is studied for a vortex lying at a distance
334 $d = 2R$ from the front, with a radius $R = 100$ km initially, see Figs. 5(c,d) and 6(c,d). When
335 V_0^{vortex} increases, so does the vortex dynamical influence on the front. Thus, the filaments
336 growing from the front increase in size.

337 On the f -plane, after 100 days, the filaments wrap up around the vortex. Though
338 the perturbation is concentrated around the quasi-steady vortex, this interaction leads to
339 much offshore export of coastal fluid. Again, cyclones disturb the front more deeply than
340 anticyclones do. After 200 days, filaments still protrude far offshore on the f -plane, but
341 forced waves have now propagated along the front axis so that much offshore-coastal fluid
342 exchange occurs, all along the front axis.

343 On the β -plane, short waves develop on the front, with smaller amplitude than on the f -
344 plane. The most intense external vortices induce more small scale turbulent features. Again,
345 more small-scale features are induced by cyclones than by anticyclones. After 100 days, these
346 small-scale features remain trapped between the front and the vortex and do not drift far
347 offshore. This can be related to the flow straightening by the β -effect, and to the westward
348 vortex drift which, for the front and its meanders, displaces the external deformation field.
349 After 200 days, turbulence has developed from its state at 100 days. It extends all along the
350 front and the inshore-offshore fluid exchange is amplified. Still, no coastal fluid is advected
351 offshore beyond the vortex. Here, the meridional gradient of upper layer vortex stretching
352 (across the front) is positive, which increases the westward Rossby wave speed so that frontal
353 meanders are located more to the West than on the f -plane.

354 4.4. Sensitivity to the vortex distance to the front

355 The influence of the distance between the vortex and the front d/R is studied for a
356 vortex with a radius $R = 100$ km, and an azimuthal velocity $V_0^{\text{vortex}} = \pm 0.2 \text{ m s}^{-1}$ initially,
357 see Figs. 5(e,f) and 6(e,f). The same observations as in the previous subsection hold here.
358 On the f -plane, after 100 days, long waves grow and large meanders break into offshore
359 filaments when the vortex is close to the front ($d/R = 2$ or 3). After 200 days, shorter
360 waves develop and affect the whole length of the front. Here again, cyclones induce more
361 pronounced deformations and longer filaments than anticyclones. When $d/R = 4$ or 5 , very
362 little deformation of the front occurs, both on the f - and on the β -plane. As a consequence,
363 vortices lying 400 km away from the upwelling front influence it very little, as could be
364 anticipated from the calculation of the velocity field. On the β -plane, the stabilizing effect
365 of the planetary vorticity gradient confines the waves and the turbulence to the vicinity of
366 the front.

367 5. Interaction of an upwelling front with a vortex alley

368 Though the interaction of a single vortex with an upwelling front is an important step
369 in our study, and though analyzing this case provides important and general conclusions,
370 sea surface measurements in the Arabian Sea show that the southern Omani upwelling is
371 rarely bordered by only one vortex; multiple vortices surround the front (see *e.g.* Fig. 1).
372 Therefore, we generalize the previous case to that of a vortex alley along the front (see section
373 2.1.2 and Fig. 2(right)). We use the physical vortex parameters of the most significant case
374 previously analyzed: $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, $R = 100$ km, and $d/R = 2$. Here we vary the
375 alongshore distance between the vortices d_{btw} , the number of vortices N , and the presence
376 or absence of β -effect.

377 5.1. Sensitivity to the number of vortices

378 We present the case with $d_{\text{btw}}/R = 4$, which shows the highest efficiency for filament
379 production, for $N = 2 - 6$ vortices, see Figs. 7(a,b) and 8(a,b). On the f -plane, at $t = 100$
380 days, the effect for each vortex is comparable to that of a single vortex, *i.e.* long filaments
381 protrude offshore from the front. The velocity field field resulting from the addition of the

382 vortices is stronger at the center of the domain, and thus the filaments are longer there.
 383 Filaments are advected southward between an anticyclone (to the West) and a cyclone (to
 384 the East). On the contrary, the front is pushed northwards and forms a wide meander to
 385 the North, between a cyclone (to the West) and an anticyclone (to the East). At $t = 200$
 386 days, the filaments have been cut, and some of them are brought closer to the front by
 387 the velocity field of the zonal jet. We can also see (as in the reference cases, see section
 388 4.1), the development of shorter waves, which now grow on the wide meanders. This state is
 389 comparable to that of the nonlinear evolution of the unstable front in the absence of external
 390 vortices initially. Note a difference between the present case and that with a single external
 391 vortex: here, the front is shifted North, which did not occur with only one vortex. On the
 392 β -plane, again, shorter and less prominent meanders and filaments are produced after 100
 393 days. These waves break into a turbulent field, in the vicinity of the front, after 200 days.

394 5.2. Sensitivity to the distance between vortices

395 The influence of the distance between vortices within the vortex alley d_{btw}/R is studied
 396 for a vortex alley of 6 vortices (*i.e.* $N=6$, this corresponds to cases with the most intense
 397 deformation of the front), see Figs. 7(c,d) and 8(c,d). We do not detail each figure individ-
 398 ually, but we note that for short times, the larger the inter-vortex distance is, the stronger
 399 the offshore mass transport is. Indeed, the alongshore extent of the front deformation and
 400 the filament widths increase with d_{btw} . At longer time, the northward displacement of the
 401 front is larger for small d_{btw} because the vorticity dipoles which are generated are more
 402 intense (the distance between the centers of the two vortices in the dipole being smaller).
 403 On the contrary, on the β -plane, the front show more long wave (200 km length) instability
 404 when d_{btw} is larger (the vortices triggering longer waves). Again, the β -effect reduces the
 405 amplitude of meanders and intensifies turbulence in the vicinity of the front.

406 6. Discussion: cross-shore exchange of particles

407 We discuss here the impact of vortices and of upwelling destabilization on the transport
 408 of fluid particles in the domain. In particular, we present the export and import of particles
 409 across the y -position of the upwelling, to assess the role of the different dynamical elements in

410 the particle transport off-shore (from North to South) and on-shore (from South to North).
411 This point is key to the local dynamics of upwelling rich regions because this can trigger
412 phytoplankton blooms (Shi et al., 2000; Liao et al., 2016).

413 To do so, we ran particle advection simulations (see details in section 2.4) in 4 particular
414 CROCO simulations, which are the most representative of the upwelling-vortex interactions
415 behavior. For further use, we define onshore as shorewards of the front (*i.e.* North of the
416 front), and offshore conversely. The particle evolution with time is presented for an upwelling
417 front with a sinusoidal perturbation (a), in the presence of a cyclone (b), of an anticyclone
418 (c), or of a vortex alley (d), on the f -plane (Fig. 9) or on the β -plane (Fig. 10).

419 For the perturbed front, the particles drift onshore and offshore nearly equally on the
420 f -plane, while the cross-shore exchange is strongly reduced on the β -plane (as expected).
421 When the front faces a single cyclone, on the f -plane, offshore particles are trapped in a
422 meander which wraps counterclockwise and shore-ward, after 100 days. After 200 days,
423 this meander undergoes instability and produces shorter scale meanders; it breaks and
424 gives birth to many small vortices inshore of the front. The offshore meander at 100
425 days produces a long-wave perturbation on the front, leading to the offshore motion of
426 onshore particles. This meander also breaks into smaller fragments (small vortices and fil-
427 aments). Such fragments are seen on Chlorophyll-a images of the Oman upwelling front
428 (such an image can be found in lecture 17 of Lisa Beal’s oceanography course: [https://beal-
429 agulhas.rsmas.miami.edu/teaching/courses/lecture-seventeen/index.html](https://beal-agulhas.rsmas.miami.edu/teaching/courses/lecture-seventeen/index.html)). On the β -plane,
430 the cross-shore displacement of particles, and their meridional flux, are strongly reduced, but
431 small-scale patches of displaced particles are still present on both sides of the front. When
432 the front faces an anticyclone on the f -plane, a large meander of coastal water is pulled
433 offshore and carries passive particles across the front after 100 days. This meander breaks
434 into a coherent, medium scale cyclone, next to the anticyclone. After 360 days, both the
435 vortex interactions and the front breaking lead to the production of small-scale filaments
436 and vortices offshore. This phenomenon is considerably weakened on the β -plane where only
437 three small vortices containing coastal water are found offshore after 200 days. After 360
438 days, particles have been displaced inside the offshore region, but little cross-front exchange
439 has taken place. In the presence of a vortex row, on the f -plane, the intrusion and extrusion

440 of fluid *via* the front meanders and the subsequent filaments lead to substantial cross-front
441 exchange, in particular *via* small-scale features. On the β -plane, the cross-front flux is still
442 the largest of all simulations, visually.

443 The median particle export distance for each case, in the onshore and offshore direction,
444 and the percentage of exported particles are shown in Fig. 11. On the f -plane, the front
445 with a vortex alley or with a cyclone displace more particles than the unstable front or the
446 front with an anticyclone, but the front with a cyclone advects particles only close to the
447 front. On the β -plane, much fewer particles are advected across the front and the export
448 distance is strongly reduced. Again, the front with a cyclone or with a vortex alley advect
449 more particles. The latter case is the most efficient to horizontally stir particles. Finally,
450 the frontogenesis function computed in the front-vortex alley case (Fig. 12) indicates that
451 frontogenetic tendency is large around the small vortices and formed filaments.

452 For a comparison of these results with the upwelling and vortices off Oman (see Fig. 1),
453 we can see that the cyclone lying offshore of the front (at 17°N , 57°E) pulls a long filament
454 away from the front. On the SST map (Fig. 1(a)) cold water extends offshore at least
455 over 300 km. South of the main cyclone, a smaller cyclone, containing cold water, is found.
456 Another filament, shorter than the former, is torn away from the upwelling front by the
457 anticyclone lying at 20°N and 60°E . On the SST and SSH maps, contrary to our model,
458 more vortices lie farther offshore of those close to the front. They also carry cold water away
459 from the coast and make it recirculate.

460

461 These results suggest that the impacts of upwelling front instability or front interaction
462 with a vortex field on biological activity are expected to be important. Firstly, nutrient rich
463 onshore waters will be exported offshore by the meanders which then break into filaments
464 and into small vortices. These flow structures will be the seat of intense biological activity
465 (blooms) favored by the further vertical uplift of nutrients to the surface, related to fron-
466 togenesis. Secondly, exchange of coastal and deep water species will be achieved by these
467 horizontal exchanges, leading to a possible modification of local ecosystems, and to possible
468 competitive exclusion of species, in an otherwise protected environment for them.

469 **7. Conclusions**

470 We have studied the evolution of an unstable upwelling front and of a front in the presence
471 of vortices offshore. We have not considered a stable front in the presence of external
472 vortices, because upwelling fronts are naturally unstable. The characteristic wavelengths
473 and coherent structures produced by these evolutions have been determined. Their impact
474 in terms of horizontal fluxes of particles has been studied. In particular, wavelengths of 100
475 or 200 km appear on the front, and small-scale filaments and vortices are finally produced.
476 Frontogenesis is strong around them. Mesoscale vortices and smaller scale features advect
477 particles across the front. The timescales of 3-6 months considered here can be made shorter
478 by considering faster frontal currents (*e.g.* in the linear approximation, a $0.4\text{-}1\text{ m s}^{-1}$ frontal
479 jet will reduce the timescales by factors of 2 to 5, leading to periods of about 1 month for
480 the formation of meanders and filaments). Numerical simulations using particles advection
481 have shown that a vortex alley is the most efficient perturbation to the upwelling front, in
482 terms of cross-front transport (compared with a single vortex, or with the instability of the
483 front alone).

484 Still, this study remains idealized in terms of flow conditions. First and foremost, it has
485 only considered the inertial (unforced) evolution of the front. We will extend these results in
486 a following study by adding wind stress and/or the front relaxation to a prescribed state, to
487 assess the energetic balance between atmospheric forcing and mean flow instability. Bottom
488 topography and coastal irregularities should also be added as they can alter or favor the
489 formation of filaments and of vortices from upwelling fronts (Meunier et al., 2010). The
490 wind variability is also essential in the evolution of upwelling fronts, by either weakening
491 them or amplifying them. Further studies with nested models of the Arabian Sea and of
492 the region south of Oman will be done in the near future. They will benefit from *in situ*
493 measurements performed during the Physindien 2019 experiment South of the Sultanate of
494 Oman, for their validation.

495 **Participation in the study**

496 The authors declare their participation in the study, CdM for conception, numerical work
497 and contribution to the paper writing, XC for conception, participation in the analyses and

498 paper writing. The authors declare no conflict of interest in the realization of this work.

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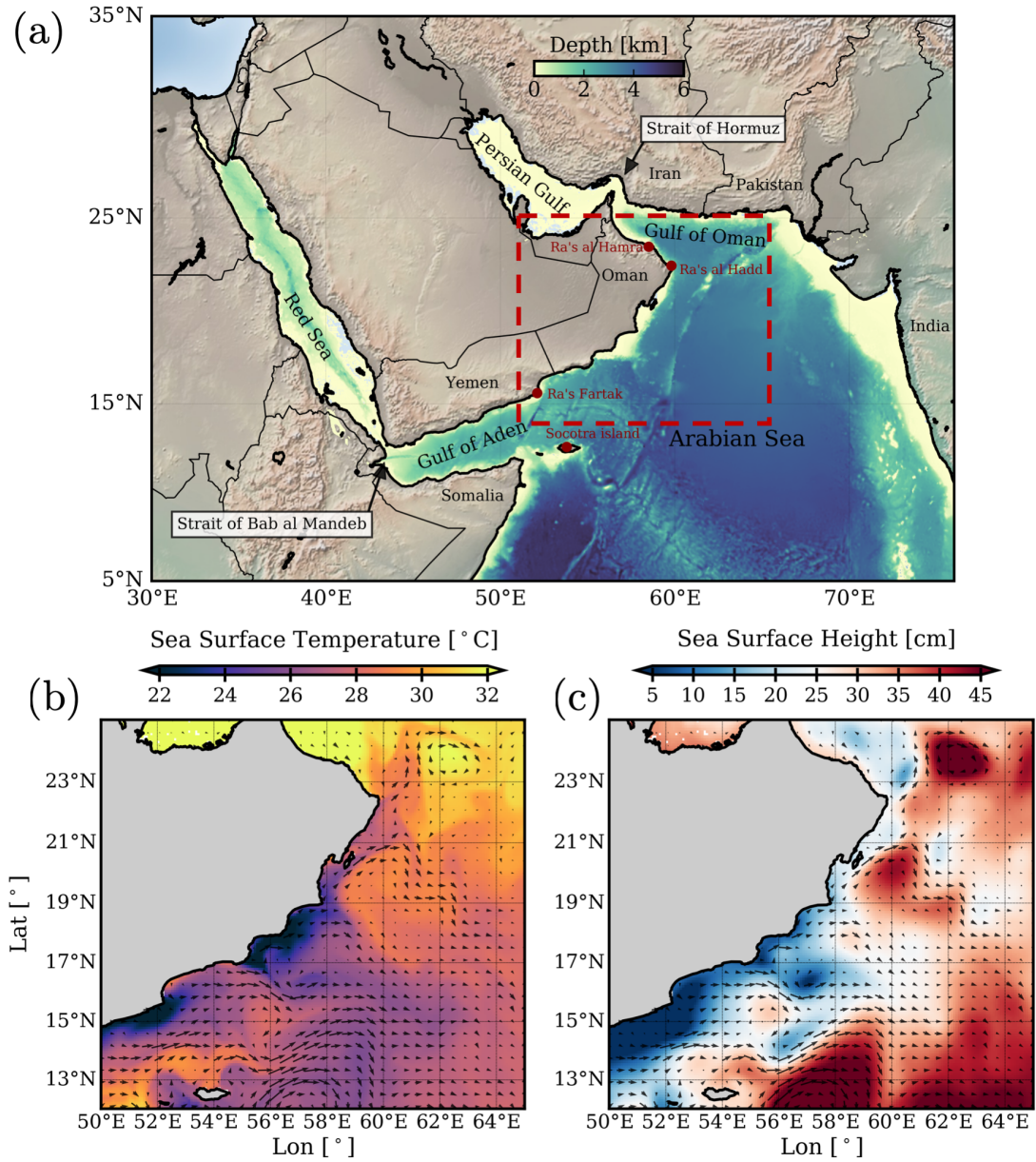


Figure 1: (a) Bathymetry in the Arabian Sea and adjacent gulfs from ETOPO2 (Smith & Sandwell, 1997), dashed rectangle shows the geographic position of panels (b,c). (b) Sea Surface Temperature and (c) Sea Surface Height (SSH) along the Omani coast in the Arabian Sea on 05/08/2020. Arrows show the geostrophic velocity derived from SSH. Cold filaments resulting from the steering of the upwelling by mesoscale eddies are seen near 56°E 16°N. Data are from Operational Mercator global ocean analysis and forecast system (downloaded on Copernicus website).

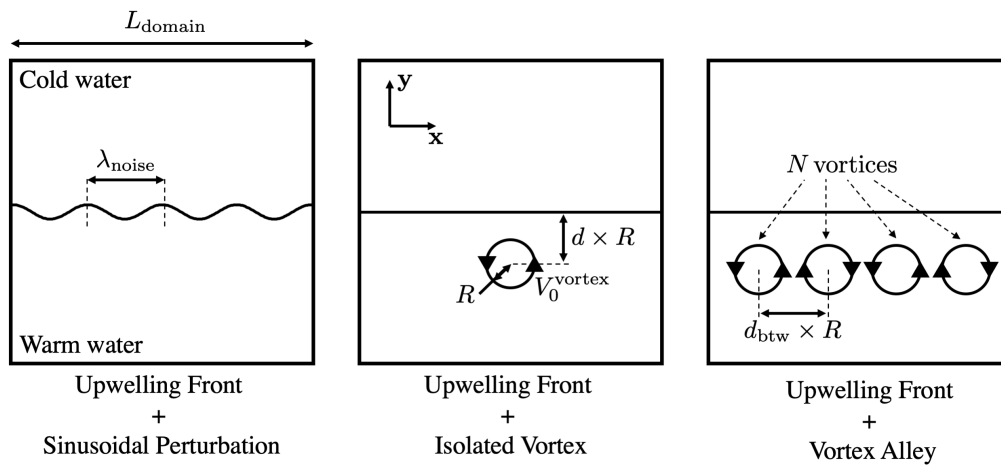


Figure 2: Scheme of the three kinds of simulations we performed and the different physical parameters we varied.

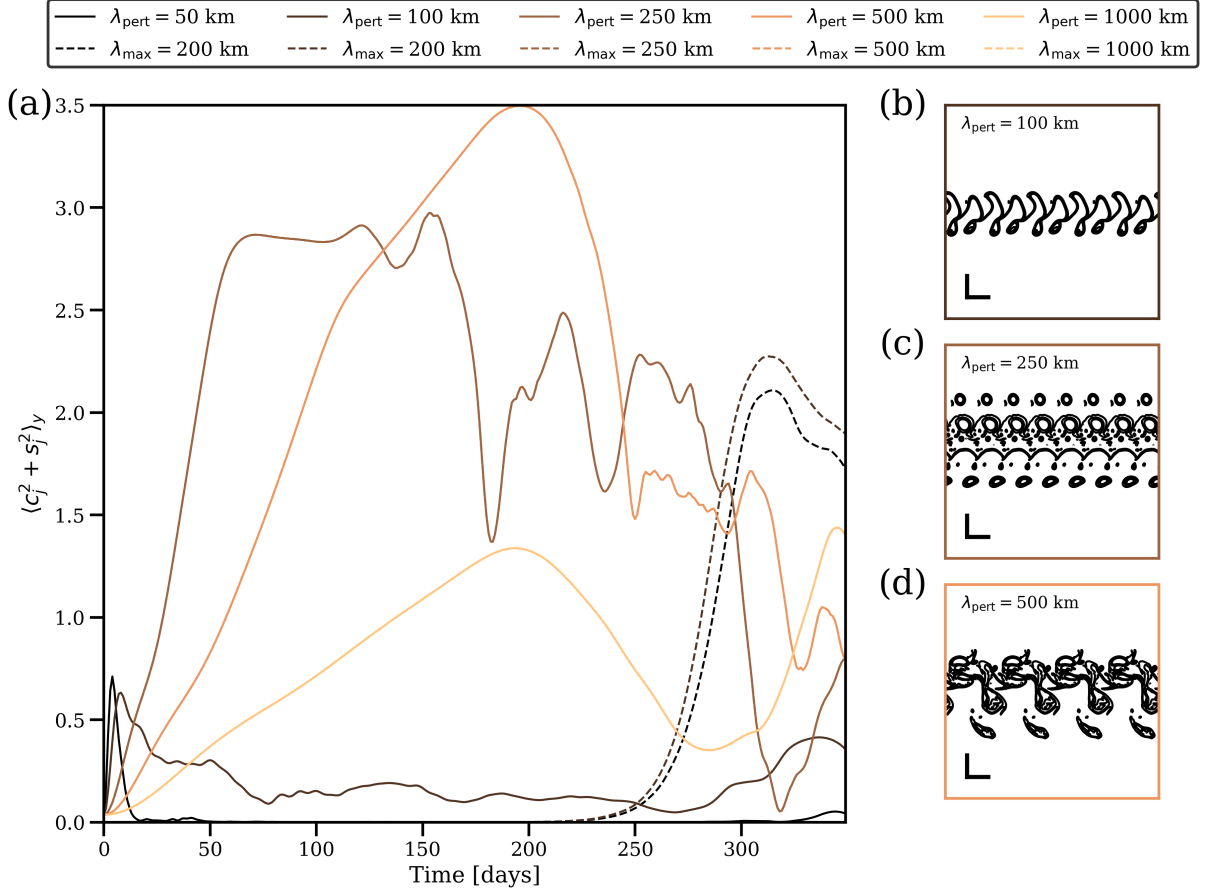


Figure 3: a) Time evolution of the amplitude of modes from the modal decomposition of the surface temperature, in simulations with an upwelling front and an initial sinusoidal perturbation, for $L = 50$ km, $H = 200$ m, and $V_0^{up} = 0.2 \text{ m s}^{-1}$. The color of the line indicates the wavelength of the wave at initialization (λ_{pert}), such that one color designate one simulation. Solid lines show the evolution of the mode with the same wavelength as the initial perturbation (λ_{pert}), while dashed lines show the evolution of the mode that reaches the largest amplitude (with a wavelength λ_{max}). If the solid and dashed lines are superposed, the mode that reaches the largest amplitude has the same wavelength as the initial perturbation. b), c), and d) Surface temperature contours after one year in the simulations with $L = 50$ km, $H = 200$ m, and $V_0^{up} = 0.2 \text{ m s}^{-1}$, and $\lambda_{\text{pert}} = 100, 250, \text{ and } 500$ km. Sizebars in the bottom left of each panel show a distance of 200×200 km.

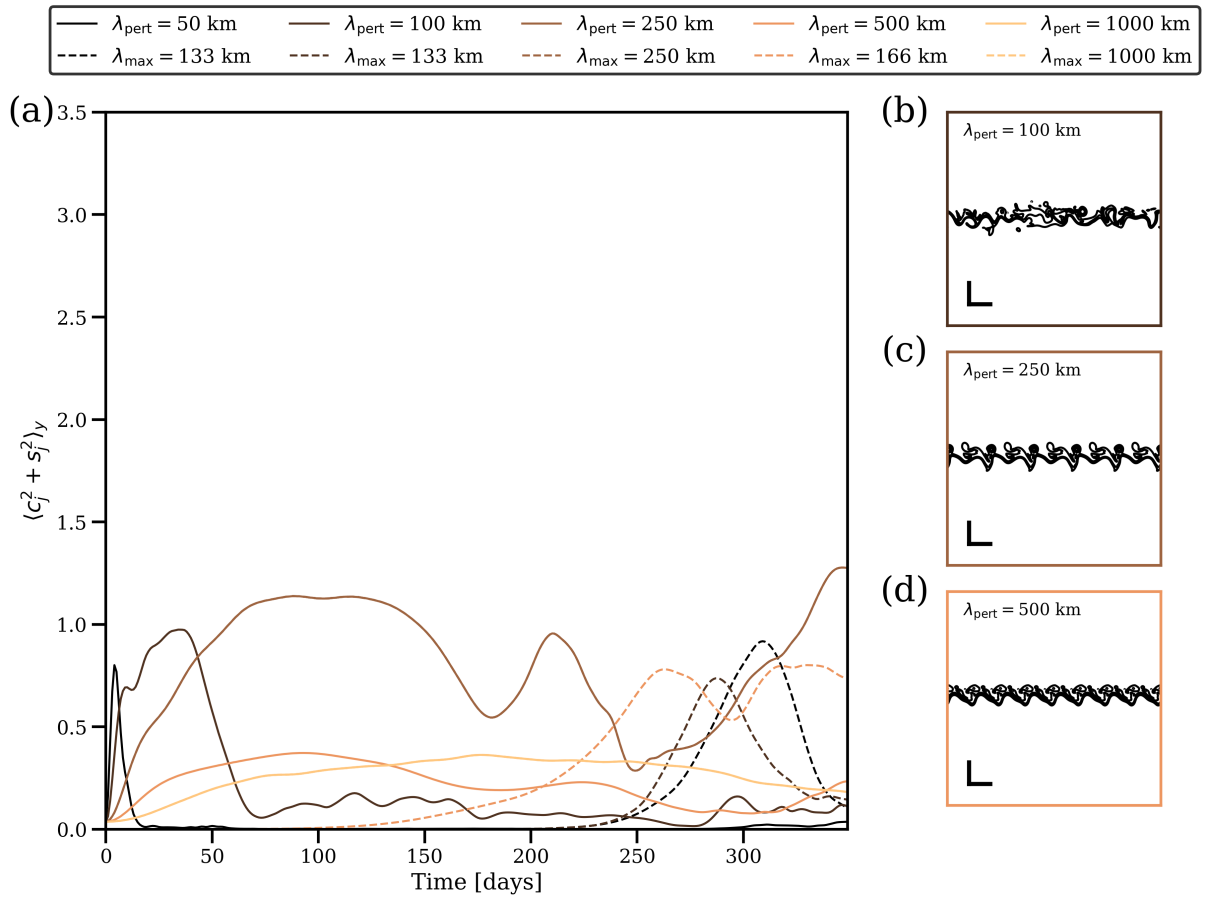


Figure 4: Same as Fig. 3, but with a β -effect added in the simulation.

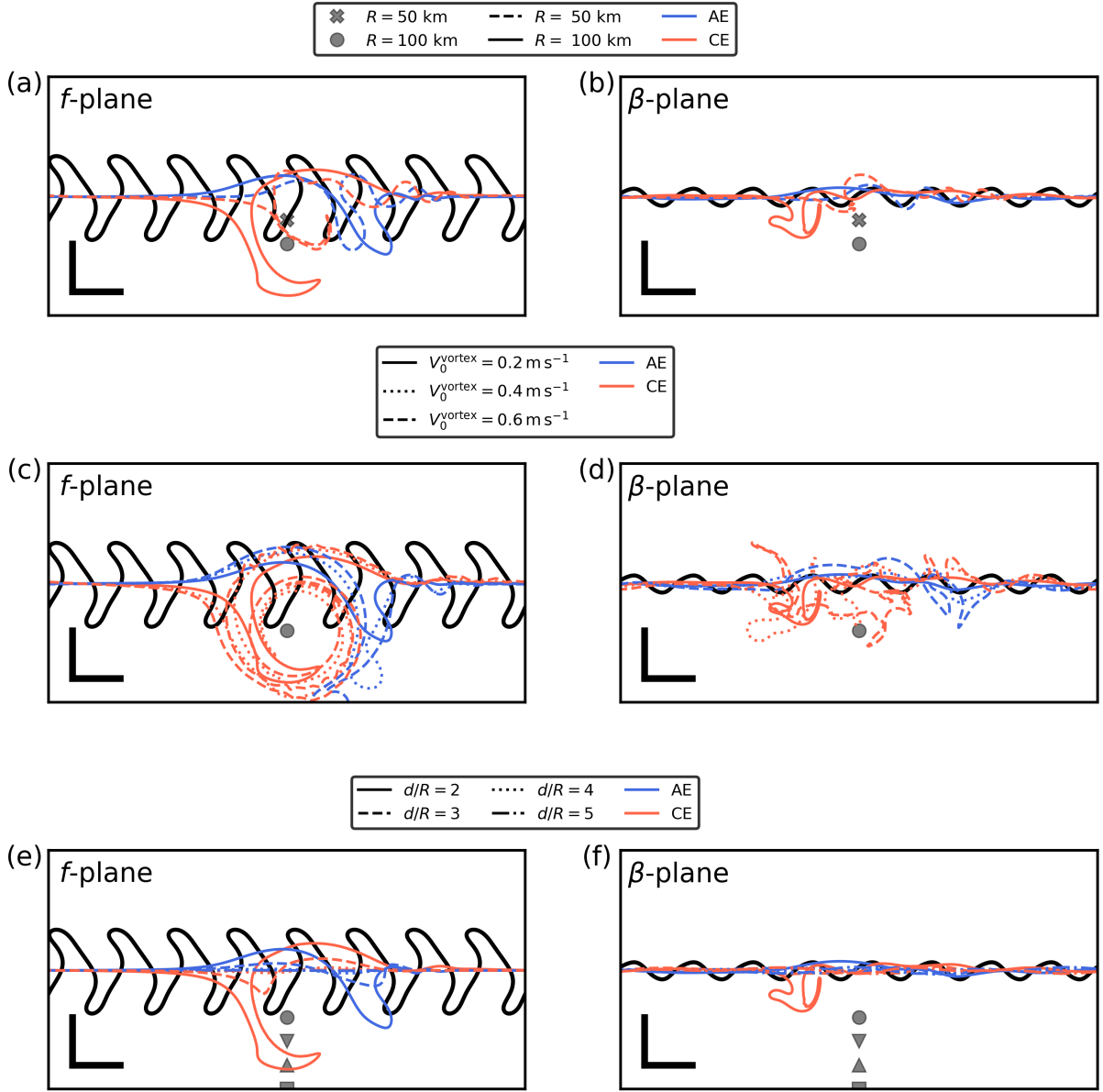


Figure 5: Position of the upwelling front (initially $L = 50$ km, $H = 200$ m, and $V_0^{\text{up}} = 0.2 \text{ m s}^{-1}$) at $t = 100$ days, in simulations with an isolated vortex of parameters (a,b) $d/R=2$, $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, and different values of vortex radius R ; (c,d) $d/R = 2$, $R = 100$ km, and different values of azimuthal velocity V_0^{vortex} ; (e,f) $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, $R = 100$ km, and different values of upwelling-vortex distance $dR/$. The left (resp. right) column shows simulations without (resp. with) β -effect. Note that for each column, solid lines of a given color show the same simulation. In all panels, black line shows the upwelling front initialized along with a sinusoidal perturbation ($\lambda_{\text{pert}}=250$ km), red (resp. blue) lines show the upwelling front initialized along with a cyclonic (resp. anticyclonic) vortex, markers show the initial position of the vortex, and sizebars in the bottom left show a distance of 200×200 km.

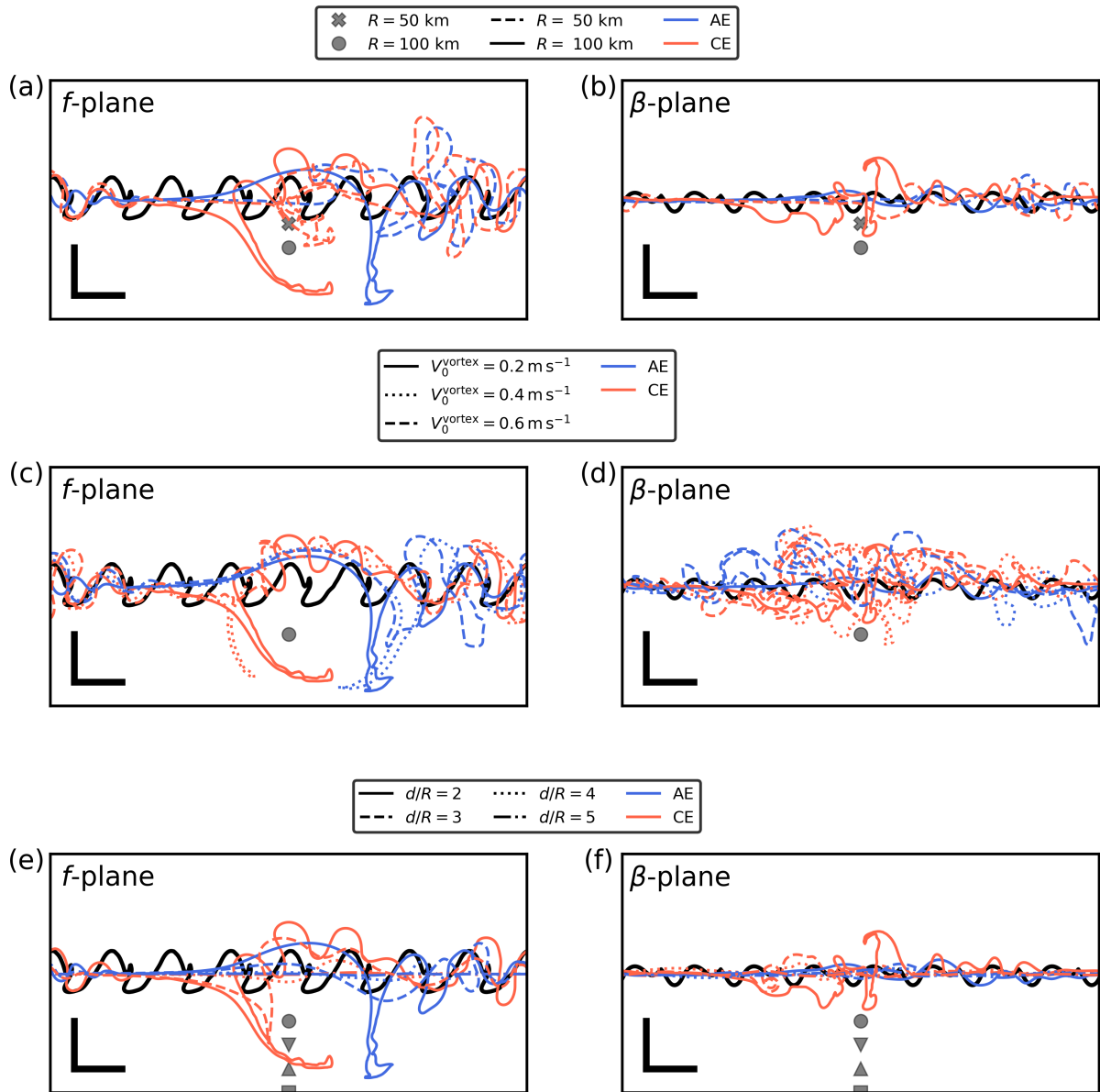


Figure 6: Same as Fig. 5 at $t = 200$ days

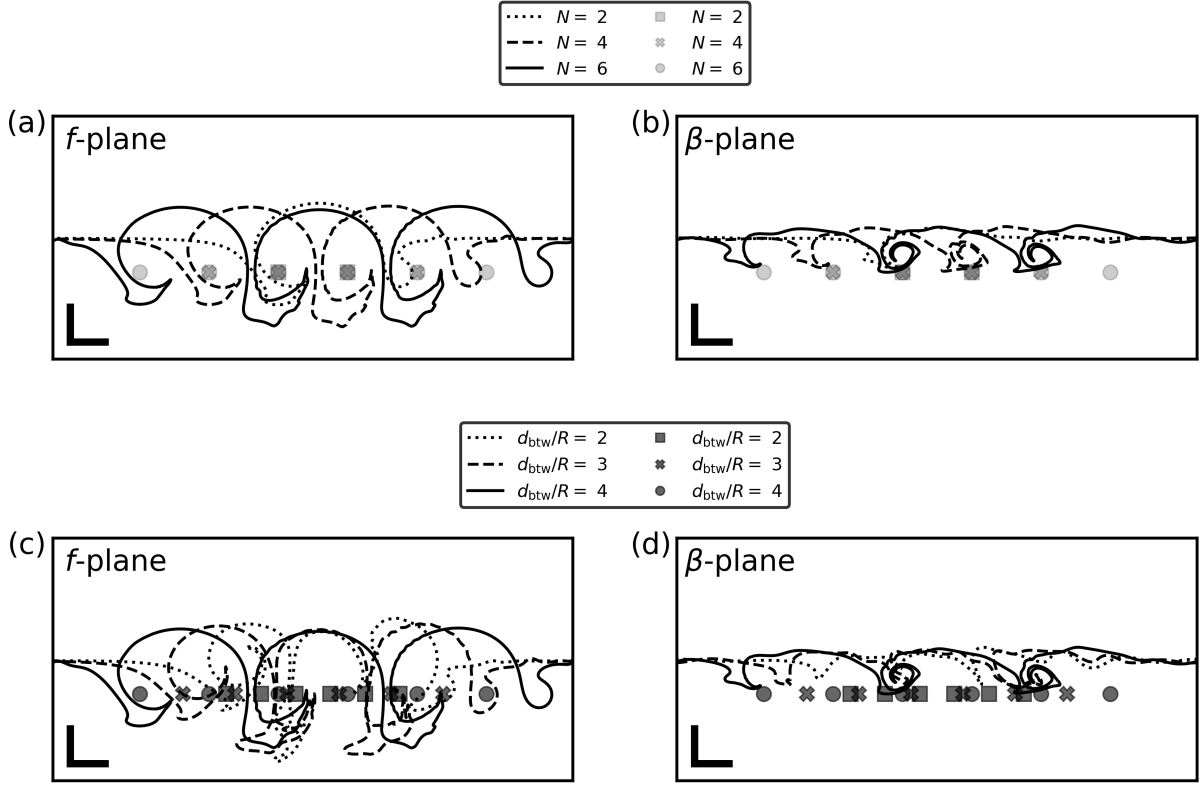


Figure 7: Position of the upwelling front (initially $L = 50 \text{ km}$, $H = 200 \text{ m}$, and $V_0^{\text{up}} = 0.2 \text{ m s}^{-1}$) at $t = 100$ days, in simulations with a vortex alley of parameters (a,b) $d/R=2$, $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, $R = 100 \text{ km}$, $d_{\text{btw}}/R = 4$, and different number N of vortices; (c,d) $d/R=2$, $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, $R = 100 \text{ km}$, $N = 6$, and different values of distance between vortices d_{btw}/R . The left (resp. right) column shows simulations without (resp. with) β -effect. Note that for each column, solid lines show the same simulation. Markers show the initial position of vortices, and sizebars in the bottom left show a distance of $200 \times 200 \text{ km}$.

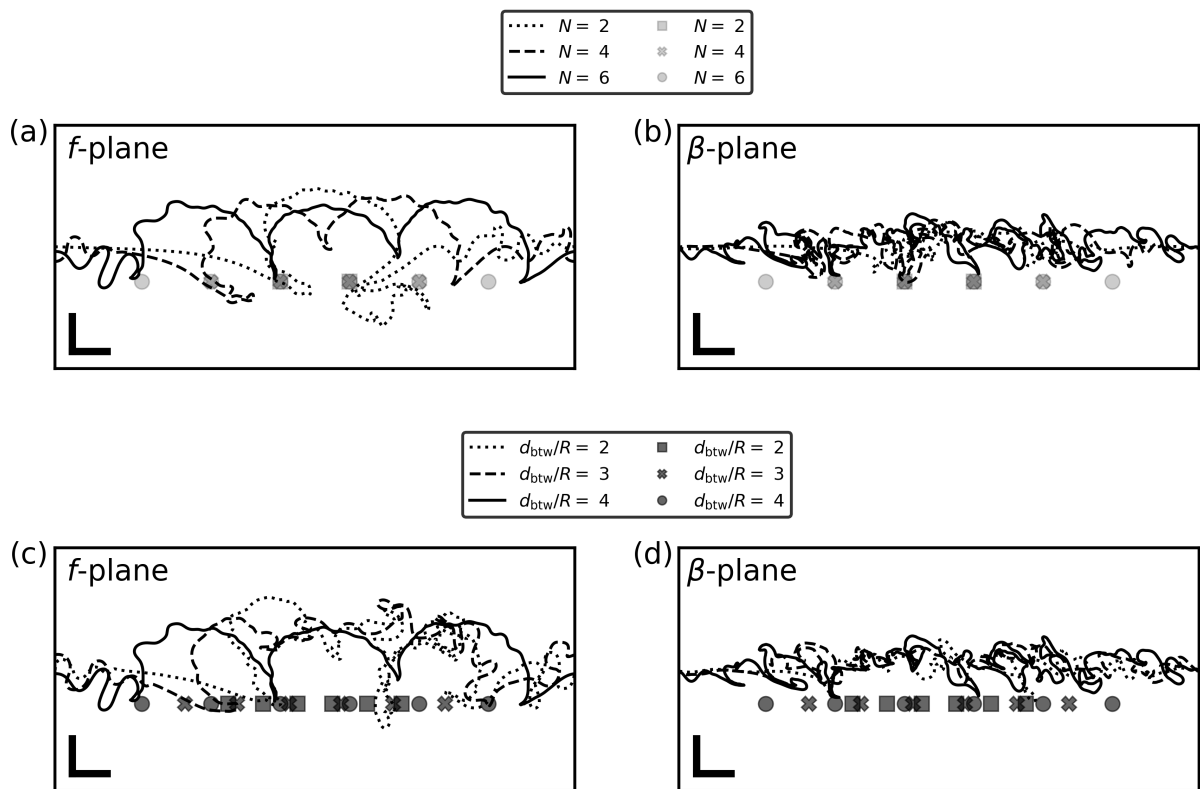


Figure 8: Same as Fig. 8 at $t = 200$ days

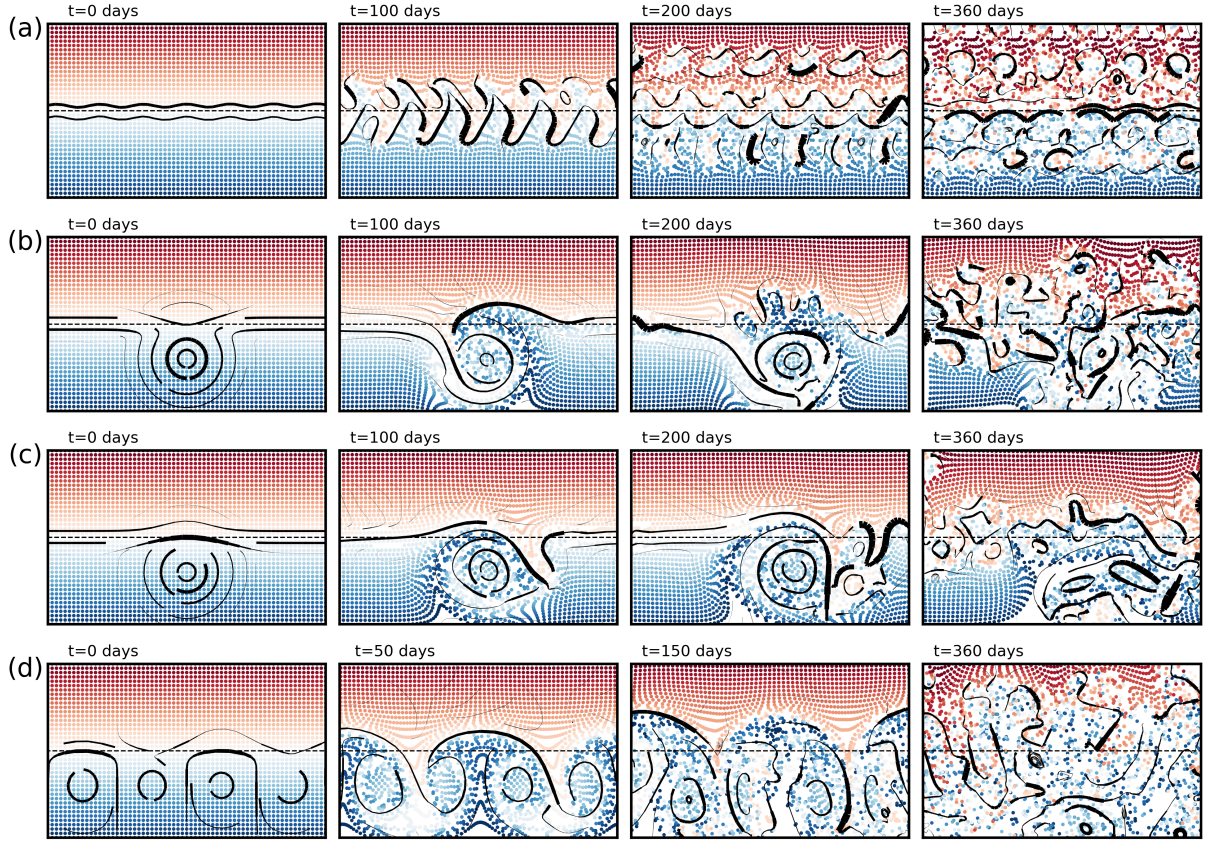


Figure 9: Snapshots of particle advection simulations with an upwelling front of parameters $L = 50$ km, $H = 200$ m, and $V_0^{\text{up}} = 0.2 \text{ m s}^{-1}$, and (a) a sinusoidal perturbation ($\lambda_{\text{pert}} = 250$ km), (b, resp. c) an isolated cyclonic (resp. anticyclonic) vortex ($R = 100$ km, $H_{\text{vortex}} = 1000$ m, $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, and $d/R = 2$), and (d) a vortex alley ($R = 100$ km, $H_{\text{vortex}} = 1000$ m, $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, $d/R = 2$, $N = 6$, and $d_{\text{btw}}/R = 4$). The particle color indicates the initial y -position of particles. Black solid lines show streamlines of the surface velocity field from the CROCO simulations used for the particle advection. Black dashed lines show the initial y -position of the upwelling front, that defines the frontier between off-shore (South) and on-shore (North) regions. Each panel is centered at the center of the domain and is 1600×1000 km wide.

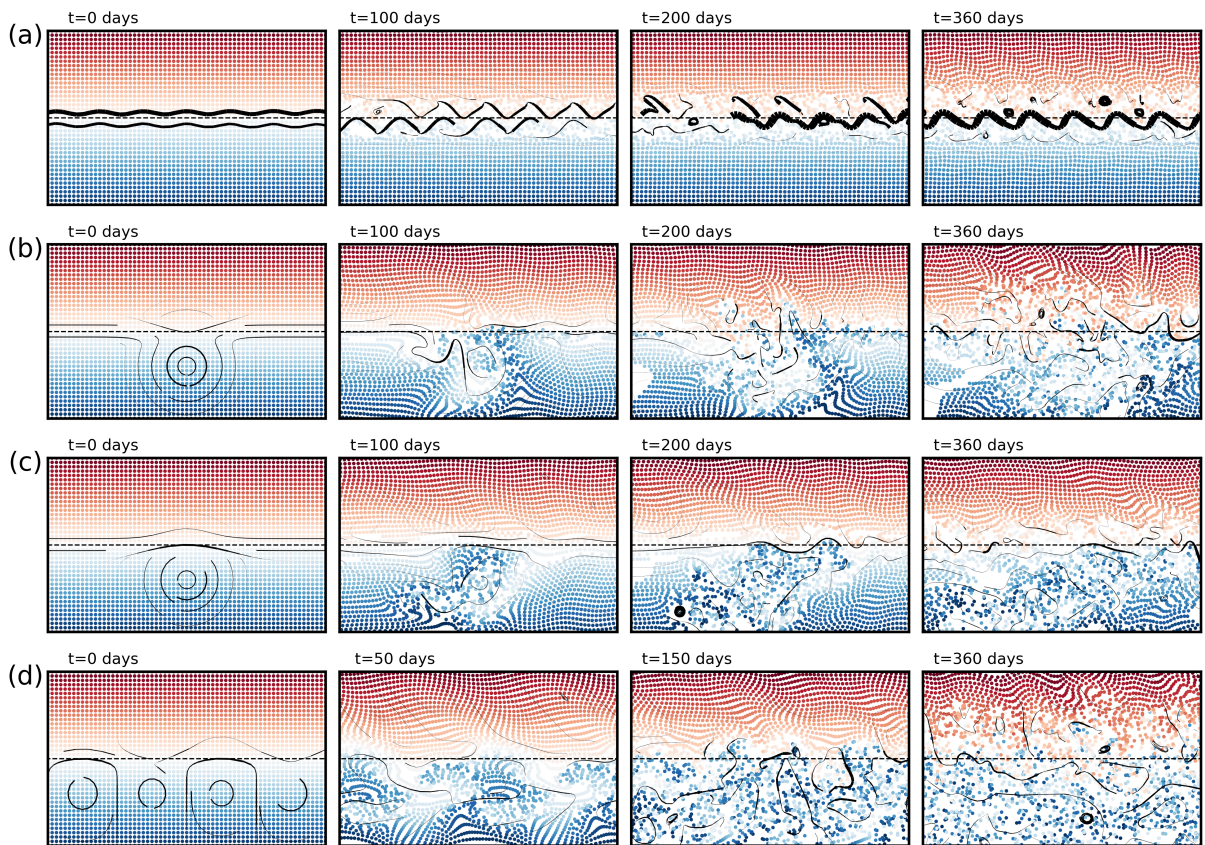


Figure 10: Same as Fig. 9 but with β -effect added in simulations.

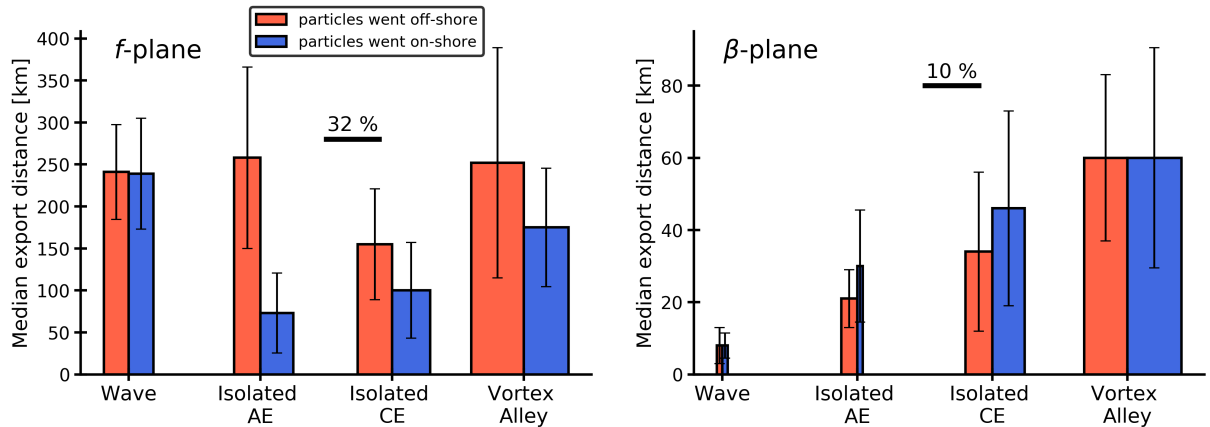


Figure 11: Median export distance of particles after one year of advection in different simulations on the f -plane (left) or the β -plane (right). Errorbars show the standard deviation of export distance, and width of boxes indicate the percentage of exported particles (see the size bar in each panel). Note that scales are different in each panel. All simulation have initially an upwelling front with $L = 50$ km, $H = 200$ m, and $V_0^{\text{up}} = 0.2 \text{ m s}^{-1}$; "wave" corresponds to simulations with a sinusoidal perturbation ($\lambda_{\text{pert}} = 250$ km), "isolated CE (resp. AE)" corresponds to simulations with an isolated cyclonic (resp. anticyclonic) vortex ($R = 100$ km, $H_{\text{vortex}} = 1000$ m, $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, and $d/R = 2$), and "vortex alley" corresponds to simulations with a vortex alley ($R = 100$ km, $H_{\text{vortex}} = 1000$ m, $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, $d/R = 2$, $N = 6$, and $d_{\text{btw}}/R = 4$). Note that simulations discussed in this figure are the same as simulations shown in Fig. 9 and in Fig. 10.

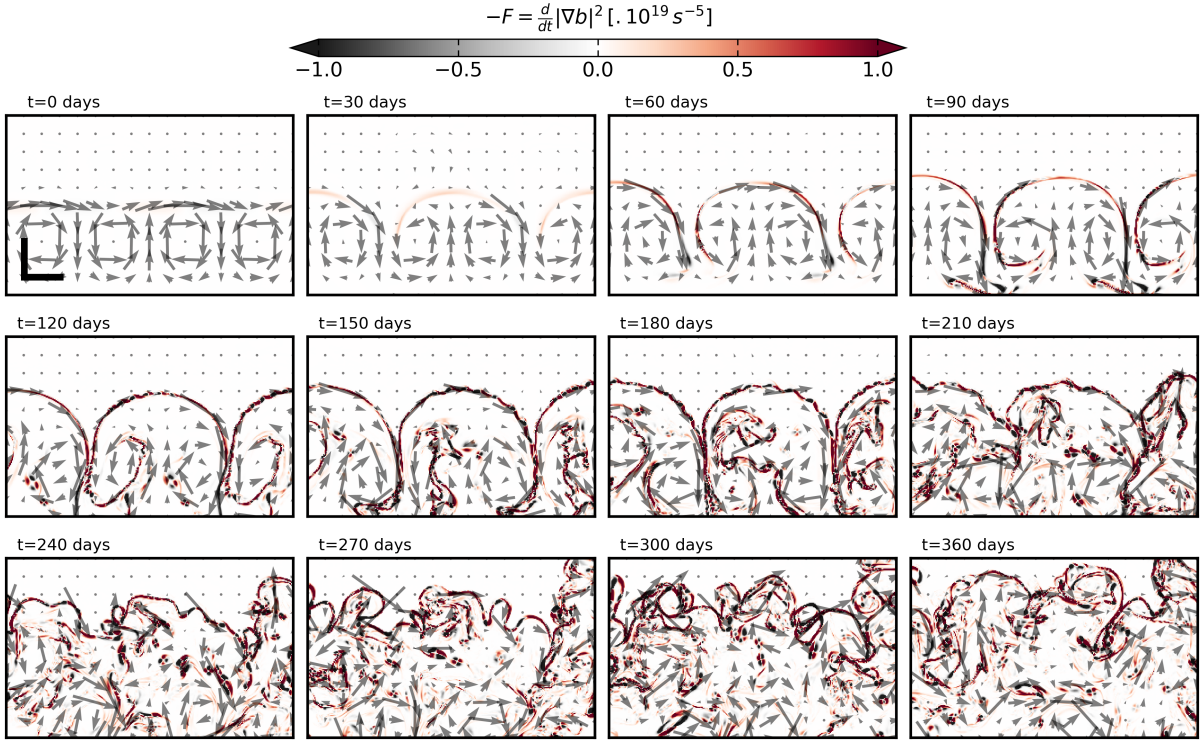


Figure 12: Time evolution of $-F$ in the simulation with an upwelling front of parameters $L = 50$ km, $H = 200$ m, and $V_0^{\text{up}} = 0.2 \text{ m s}^{-1}$, and a vortex alley ($R = 100$ km, $H_{\text{vortex}} = 1000$ m, $V_0^{\text{vortex}} = 0.2 \text{ m s}^{-1}$, $d/R = 2$, $N = 6$, and $d_{\text{btw}}/R = 4$), *i.e.* the "vortex alley" case in Fig. 11. Grey arrows show the surface velocity field. Sizebars in the bottom left show a distance of 200×200 km. Each panel is centered at the center of the domain and is 1600×1000 km wide.