
On the Use of Dual Co-polarized Radar Data to Derive a Sea Surface Doppler Model—Part 1: Approach

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Abstract :

This paper proposes a Doppler velocity (DV) model based on dual co-polarized (co-pol) decomposition of a normalized radar cross-section of an ocean surface on polarized Bragg scattering and nonpolarized (NP) radar returns from breaking wave components. The dual co-pol decomposition provides a quantitative description of resonant and NP scattering, as well as their dependence on the incident angle, azimuth, and wind speed. Subsequently, the contributions of the facet (resonant Bragg waves and breakers) velocities, tilt, and hydrodynamic modulations due to long waves to the resulting DV can be quantified. The tilt modulation contributions to DV are estimated using the measured/empirical tilt modulation transfer function (MTF). The hydrodynamic modulations are mostly dominated by wave breaking and are estimated using a semiempirical model based on in situ measurements. In addition to the VV and HH radar data, which are required for dual co-pol decomposition and tilt MTF estimates, the surface wave spectrum is required in the DV determination for a given radar observation geometry. In this paper, qualitative and quantitative consistencies are presented between the model simulations and the empirical CDOP model. In a companion paper, DV analysis is presented to analyze the Sentinel-1 synthetic aperture radar measurements and collocated in situ measurements of surface wind and wave spectra.

Keywords : Sea surface, Radar, Surface waves, Scattering, Modulation, Doppler radar, Radar scattering, Dual co-polarized decomposition, radar Doppler velocity (DV), radar geophysical model functions (GMFs), tilt and hydrodynamic modulations

I. Introduction

22
23 Through the Doppler shift contained in complex signals (magnitude and phase),
24 spaceborne synthetic aperture radar (SAR) data can be considered potentially powerful
25 tool for monitoring and investigation of ocean surface currents on regional and global
26 scales [1], [2]. Owing to the precise knowledge of satellite orbit and attitude, the
27 difference between the measured and predicted Doppler shifts represents a geophysical
28 quantity that comprises the line-of-sight velocity of scatters (wind, waves, and current)
29 caused by ocean surface movements. The most interested contribution of surface
30 current to the Doppler shift derived from SAR measurements is significantly
31 “contaminated” by orbital velocities of long surface waves [2]-[4]. The wave motion
32 contributions to the Doppler shift must therefore be properly removed to derive reliable
33 ocean surface current estimates from SAR data.

34 This Doppler shift is defined as the power-weighted mean frequency of the power
35 spectrum of a backscattered signal, i.e., the Doppler centroid (DC). DC from the sea
36 surface can be simulated using the following three approaches: (i) numerical solution
37 of Maxwell’s equations for electromagnetic waves backscattered from a random
38 surface with a prescribed spectrum [5]; (ii) use of spatiotemporal autocovariance of the
39 scattered field derived from theoretical scattering models (e.g., KA and SSA [6]-[8] and
40 **generalized curvature ocean surface scattering model [9]**); and (iii) use of tilt and
41 hydrodynamic modulations of radar backscatters due to long surface waves [2], [10]-
42 [12]. The third approach is the most efficient and computationally wise and provides
43 results that are comparable to the numerical solutions of Maxwell’s equations [10], [13].

44 The DC model based on radar backscatter modulations requires adequate simulation
45 of the normalized radar cross-section (NRCS) in terms of the modulation transfer
46 function (MTF). The DC model in [10] was jointly based on a two-scale Bragg
47 scattering model and MTF developed by [14] and [15]. By comparison, Johannessen et
48 al. (2008) and Hansen et al. (2012) proposed the DopRIM model [11], [12], which was
49 based on a semiempirical model developed in [16] and [17] and the radar MTF model
50 [18]. These works explicitly considered the effects of long waves and wave breaking
51 on NRCS. Alternatively, the Ka-band DC model in [19] was also based on the MTF
52 concept but employed the empirically derived VV and HH radar geophysical model
53 function (GMF) [20] and hydrodynamic radar MTF that comprised all scattering
54 mechanisms [21]. The other DC models are empirical types that relate the Doppler shift
55 to the wind field, e.g., CDOP model [22], CDOP3S model [23], and its modification,
56 i.e., CDOP3SiX [24], to best separate the effects of wind waves and swell on the
57 Doppler shift for C- or X-band measurements [25].

58 The present study aims to further investigate the use of dual co-polarized (co-pol)
59 NRCS properties [26] to develop and evaluate a novel consistent Doppler velocity (DV)
60 model. Dual co-pol decomposition has been successfully applied to investigate quad-
61 polarization SAR imaging sensitivities of ocean currents. SAR NRCS variations caused
62 by currents are indeed polarization dependent (VV, HH, HV, and VH) [27], [28]. In
63 particular, the strong sensitivity of wave breaking to the horizontal gradient of surface
64 current results in large NRCS contrasts. The polarization sensitivity is then
65 demonstrated to trace the variable contribution of radar returns from breaking waves to

66 the magnitude of SAR signals. In the present study, this dual co-pol decomposition
67 approach is further applied to model the polarized Doppler shifts. This DV model
68 follows the MTF concept but limits the use of NRCS and/or radar MTF theoretical
69 models through the extraction of maximum information from radar scattering
70 mechanisms obtained from dual co-pol measurements. In this context, the model can
71 be considered as a semiempirical model, i.e., between fully empirical (e.g., CDOP [22],
72 X-band DV [25], and KaDOP [19]) and physical models [10]-[12] that utilize the
73 scattering and statistical properties of sea surface described at certain physical
74 approximations.

75 The decomposition of the VV and HH data on resonant Bragg scatters and
76 nonpolarized (NP) radar returns from breaking wave scattering enables us to determine
77 the empirical dependence of different scattering mechanisms on the radar geometry
78 (radar wavelength, incident angle, and azimuth **angle**) and environmental conditions.
79 In the DV context, such decomposition enables a more “direct” estimation of the
80 velocities of scattering facets and the contribution of tilt and hydrodynamic
81 modulations of the facets by long surface waves. Hence, DV is related to a two-
82 dimensional (2D) wave spectrum in which the resonant and NP scatterings are
83 determined from the dual co-pol decomposition.

84 The paper is organized as follows. Section II describes the approach to simulate DV
85 using tilt and hydrodynamic modulations of the scattering facets. Under real conditions,
86 the scattering facets of the ocean surface are represented by Bragg and breaking waves.
87 Further, we describe the approach to discriminate and quantify the contributions of

88 polarized Bragg scattering and nonpolarized radar returns from the breaking waves to
 89 total surface NRCS. In Section III, we describe the modification of the DV model
 90 (denoted as dual co-pol Doppler or DPDop model hereafter), which is primarily based
 91 on the decomposition of radar backscattering into the contributions of short resonant
 92 Bragg waves and wave breaking and their tilt and hydrodynamic modulations due to
 93 long surface waves. Section IV demonstrates the features of the proposed DV model,
 94 the effect of different scattering mechanisms on DV, and a comparison of the proposed
 95 DPDop model with empirical models, namely, CDOP and CDOP3SiX, for different
 96 geometries of radar observations, wind speeds, and sea states, including a mixed sea
 97 case. Section V provides the concluding remarks and our proposal to continue by
 98 moving on to the companion paper [29], which presents the results of the comparison
 99 of the DPDop model simulations with DV measurements from Sentinel-1 SAR over
 100 ocean wave buoys.

101 **II. Approach**

102 *A. Governing equations*

103 Doppler frequency f_D of the radar backscatters from a moving sea surface is
 104 expressed as $\pi f_D = -k_R V_D$, where k_R is the radar wavenumber and V_D is the surface
 105 velocity (assumed positive if directed away from the radar). According to [2] and [10],
 106 sea surface NRCS (σ_0) can be expressed as the sum of surface scattering facets that
 107 experience vertical and horizontal movements due to long surface waves and their
 108 modulation, i.e., $\sigma_0^{pp} = \bar{\sigma}_0^{pp} + \tilde{\sigma}_0^{pp}$, which have a small value of $(\tilde{\sigma}_0^{pp} / \bar{\sigma}_0^{pp} \ll 1)$. In
 109 this case, surface radial velocity U_D (projection of the surface velocity in the radar look

110 direction) measured using V_D , which is weighted over all scattering facets \bar{V}_D , is
 111 computed as follows:

$$112 \quad U_D = \frac{\bar{V}_D}{\sin\theta} = \bar{c}_f + u_s + c_f^{TH} \quad (1)$$

113 where θ is the radar incident angle, subscript “ f ” denotes the facet type (scattering
 114 mechanism, e.g., Bragg scattering), \bar{c}_f is the inherent velocity of the scattering facets
 115 (e.g., phase velocity of Bragg waves), and u_s is the projection of the surface current
 116 velocity in the radar look direction. Further,

$$117 \quad c_f^{TH} = -\cot\theta \frac{\tilde{w}\tilde{\sigma}_0^{pp}}{\tilde{\sigma}_0^{pp}} + \frac{\tilde{u}\tilde{\sigma}_0^{pp}}{\tilde{\sigma}_0^{pp}} \quad (2)$$

118 is an equation that describes the mean effect of long-wave modulations (in the second-
 119 order wave steepness) on DV. Here, \tilde{u} and \tilde{w} refer to the horizontal and vertical
 120 velocities of the long surface waves that carry the scattering facets, respectively, and
 121 $\tilde{\sigma}_0^{pp}$ is the wave-induced modulation of the facet NRCS. Thus, c_f^{TH} , which describes
 122 the modulations of facet NRCS due to variations in the local incident angle and facet
 123 geometrical properties (e.g., spectrum level of Bragg waves), is expressed through tilt
 124 and hydrodynamic MTFs.

$$125 \quad c_f^{TH} = \iint_{k < K_{LF}} [(-\cot\theta \cdot M_f^t + M_{1f}^h) \cos(\varphi_R - \varphi) + \cot\theta \cdot M_{2f}^h] \\
 126 \quad \cdot cB(k, \varphi) d\varphi dl nk \quad (3)$$

127 where φ_R is the radar look direction, k and φ are the wavenumber and direction of the
 128 surface wave components, respectively, c is its phase velocity, $B(k, \varphi)$ is the 2D

129 saturation spectrum of the large-scale surface (modulating facets), K_{LF} is the spectral
 130 cutoff of the large-scale surface linked to the spectral scale of the facets (e.g., to the
 131 Bragg wavenumber), $M_f^t = \partial \ln(\sigma_0) / \partial \theta$ is tilt MTF, and $M_f^h = M_{1f}^h + iM_{2f}^h$ is
 132 hydrodynamic MTF. The latter is a complex number where the real (M_{1f}^h) and
 133 imaginary (M_{2f}^h) parts account for the correlations of the scattering facet modulations
 134 with the surface elevation and slopes, respectively. The first two terms in Eq. (3)
 135 indicate changes in the sign of c_f^{TH} as the radar look direction shifts from downwind to
 136 upwind. **In contrast,** the third term (facet–slope correlation term) is independent of the
 137 radar look direction and therefore **provides (after summing up the first two terms)** the
 138 upwind and downwind asymmetries and nonzero crosswind values in DV. Because the
 139 saturation spectrum is almost constant, the main contribution to c_f^{TH} comes from the
 140 long surface waves. **It should be noted that following [21] (and other data cited therein),**
 141 **the width of the Doppler spectrum shall increase with increasing orbital wave velocities,**
 142 **and is always much larger than the mean Doppler shift.**

143 Yurovsky et al. (2019) found that the DV models, namely, Eqs. (1) and (3),
 144 supplemented with empirical MTFs (that comprise tilt and hydrodynamic modulations)
 145 and the measured long wave spectrum satisfactorily reproduce the observed DV [19].
 146 This finding suggests that conceptually, Eqs. (1) and (3) represent an adequate DV
 147 model. However, remarkable discrepancies are encountered when the DV model only
 148 accounts for the resonant Bragg scattering mechanism [25]. By comparison, [11] and
 149 [12] generalized the DV models (Eqs. (1) and (3)) and demonstrated that radar returns
 150 from breaking waves significantly contributed to sea surface NRCS. Total NRCS σ_0^{pp}

151 can then be robustly represented by the sum of the Bragg scattering (σ_{br}^{pp}) and NP radar
 152 returns (σ_{np}) from the regular (nonbreaking) surface and breaking waves [16]-[18].

153
$$\sigma_0^{pp} = \sigma_{br}^{pp} + \sigma_{np} \quad (4)$$

154 where $\sigma_{np} = \sigma_{sp} + \sigma_{0wb}q$, σ_{sp} is NRCS due to specular reflection from the regular
 155 surface, q ($q \ll 1$) is a fraction of the sea surface covered by breaking zones generated
 156 by waves with wavenumbers in the range $k < k_R/10$ (k_R is the radar wavelength), and
 157 σ_{0wb} is NRCS of the individual breaking zone, which is also considered a quasi-
 158 specular reflection from breaking-wave patches. At small incident angles (below 20°–
 159 25°), the main contribution to NP is provided by the specular reflections from a regular
 160 surface. At larger incident angles, the main contribution is provided by radar returns
 161 from the breaking waves. Study [30] demonstrated that the scattering model (general
 162 curvature model in that study), which incorporated wave breaking effects according to
 163 Eq. (4), well reproduced the polarization ratio measurements (ASAR AP) versus the
 164 wind vector and incident angle.

165 Because both types of scattering mechanisms contribute to DV ([11], denoted as
 166 DopRIM), the DV model, i.e., Eq. (1), is modified as follows:

167
$$V_D = u_s + \sum_f P_f^{ppp} (\bar{c}_f + c_f^{TH}) \quad (5)$$

168 where f represents the Bragg ($f \rightarrow br$) and NP ($f \rightarrow np$) scattering mechanisms and
 169 $P_{br}^{ppp} = \sigma_{br}^{pp} / \sigma_0^{pp}$ and $P_{np}^{ppp} = \sigma_{np} / \sigma_0^{pp}$ denote their relative contributions to the total
 170 NRCS.

171 DopRIM is based on the model description of the sea surface geometry (wave
 172 spectrum in the wavelength range from the millimeter scale to the spectral peak and
 173 wave breaking parameters) and radar returns from the breaking waves. The model
 174 uncertainties are caused by limited knowledge of the dynamics of short wind waves and
 175 wave breaking and the physics of radar scattering from the breaking waves.
 176 Nevertheless, the DopRIM simulations demonstrate its capability to reproduce V_D
 177 observations, as reported by [11] and [12].

178 *B. Dual co-pol decomposition*

179 For symbiosis of the physical modeling and empirical knowledge of radar scattering
 180 from the sea surface, a semiempirical DV model based on the decomposition of radar
 181 scattering into resonant polarized and NP scattering [similar to Eq. (4)] can be presented.
 182 If VV and HH NRCS are known, e.g., from dual co-pol GMF such as C-band C-
 183 SARMOD [31], Ku-band NSCAT-4 [32], [33], and Ka-band KaDPMOD [20], then Eq.
 184 (4) can be solved to derive the NP contribution [26].

$$185 \quad \sigma_{np} = \sigma_0^{vv} - \frac{\Delta\sigma_0}{(1-p_{br})} \quad (6)$$

186 where $\Delta\sigma_0 = \sigma_0^{vv} - \sigma_0^{hh}$ is the polarization difference (PD) and $p_{br} = \sigma_{br}^{hh}/\sigma_{br}^{vv}$ is the
 187 polarization ratio from the two-scale Bragg scattering model (hereafter referred to as
 188 TSM) that accounts for the slope of large-scale waves (see Eq. (A5) in [35]). Once the
 189 NP signal is known, the contribution of the polarized Bragg scattering to NRCS is
 190 estimated as follows:

$$191 \quad \sigma_{br}^{pp} = \sigma_0^{pp} - \sigma_{np}. \quad (7)$$

192 The following analysis is based on the combination of C-SARMOD2 GMF
 193 developed in [34] for VV polarization and NP parametrization derived from
 194 RADARSAT-2 data for VV and HH polarizations [35]. Taking VV and NP as input
 195 scattering components results in HH-polarized NRCS, which is defined as

$$196 \quad \sigma_0^{hh} = (\sigma_0^{vv} - \sigma_{np})p_{br} + \sigma_{np}. \quad (8)$$

197 Fig. 1 shows C-band GMF ([34], C-SARMOD2) for VV NRCS, NP parametrization
 198 [35], and HH NRCS derived from the input scattering components, namely, Eq. (8), as
 199 a function of the incident angle at wind speeds of 5, 10, and 15 m/s in the upwind,
 200 crosswind, and downwind directions. The NP-derived estimates from the combination
 201 of two different VV and HH GMFs, C-SARMOD2 and CMODH [36], are also shown
 202 for comparison.

203 Except for small incident angles ($\theta < 25^\circ$), NP in [35] and that derived from different
 204 VV [34] and HH [36] GMFs exhibit consistent behavior, revealing the dominant
 205 contribution of NP radar returns from the breaking waves to total NRCS in both
 206 polarizations. The differences between the two types of NP parametrization are
 207 considered to be partly due to the different radar data.

208 Fig. 2 shows the relative contribution of NP to total NRCS on the VV and HH
 209 polarizations under various wind speeds and incident angles. Except for the case of low
 210 wind speed (5 m/s), the relative contributions of NP to VV and HH NRCS ($P_{np}^{pp} =$
 211 $\sigma_{np}/\sigma_0^{pp}$) exhibit maximum values at the crosswind directions where the Bragg
 212 scattering is minimal. Moreover, because of the asymmetry of the wave breakers, the
 213 radar returns in the upwind directions are stronger than those in the downwind
 214 directions. These conditions lead to the upwind–downwind asymmetry of P_{np}^{pp} .

215 Fig. 3 shows that the relative NP contributions strongly depend on the incident angle
 216 at VV, and the dependence on the incident angle of the HH polarization is significantly
 217 less. Finally, Fig. 4 shows that P_{np}^{pp} at the upwind and downwind directions and at
 218 incident angles of 24° and 37° are weakly dependent on the wind speed in both
 219 polarizations. This finding implies the important effect of wave breaking on the radar
 220 scattering at any wind speed. The weak wind dependence of P_{np}^{pp} implies that the rate
 221 of NP growth with the increase in wind speed is similar to the Bragg-wave growth with
 222 a wind exponent of approximately one (see [35] for more details).

223 We note that for the crosswind directions at small incident angles and rather strong
 224 wind speeds (Fig. 2 upper-right column, Figs. 3 and 4 right columns), the partial
 225 contribution of NP can be $P_{np}^{pp} > 1$, indicating that $\sigma_0^{hh} > \sigma_0^{vv}$. We consider such cases
 226 as an artifact resulting from the use of VV and NP GMFs derived from different data
 227 sources. To avoid this artifact, we use VV GMF under the $\sigma_0^{vv} = \min(\sigma_0^{vv}, \sigma_{np})$
 228 condition.

229 III. DV Model based on Dual Co-Pol Decomposition

230 The dual co-pol decomposition of NRCS provides direct estimates of the partial
 231 contribution of the NP radar returns ($P_{np}^{pp} = \sigma_{np}/\sigma_0^{pp}$) and resonant Bragg scattering
 232 ($P_{br}^{pp} = \sigma_{br}^{pp}/\sigma_0^{pp} = 1 - P_{np}^{pp}$) to total NRCS. NP shows a large contribution, which
 233 varies from approximately one in the crosswind direction and low incident angle (24°)
 234 to approximately 0.4–0.6 for the VV and HH polarizations at an incident angle of 37° .
 235 Thus, the important parameters in the DV model (Eq. (5)), namely, P_{np}^{pp} and $P_{br}^{pp} = 1 -$
 236 P_{np}^{pp} , can be estimated.

237 Hereafter, we consider incident angles that exceed $\theta = 20^\circ\text{--}25^\circ$ when specular
 238 reflection from regular (nonbreaking surface) can be ignored. As shown in Fig. 10 in
 239 [35] (where the difference between the black solid and dotted lines gives specular
 240 reflection NRCS), the specular reflections at 20° are comparable to the observed NP
 241 values. However, the specular reflections rapidly decrease at 25° and become an order
 242 of magnitude smaller than NP. Hence, by considering the DV model for moderate
 243 incident angles, e.g., $\theta > 24^\circ$, we further treat NP as radar returns from breaking waves.

244 A. Velocity of scattering facets

245 Bragg scattering

246 The velocity of Bragg scattering facets $c_B(\varphi)$ is defined as follows [37]:

$$247 \quad c_B(\varphi) = c_{br} \frac{A_{br}(\varphi) - A_{br}(\varphi + \pi)}{A_{br}(\varphi) + A_{br}(\varphi + \pi)} \quad (9)$$

248 where φ is the angle between the radar look and wind directions, $c_{br} = c(k_{br})$ denotes
 249 the phase velocity of the Bragg waves, and $A_{br}(\varphi)$ refers to the directional distribution
 250 of the Bragg wave spectrum (not directly available from the radar data). Depending on
 251 the input information, directional distribution $A_{br}(\varphi)$ can be reconstructed from either
 252 the Bragg scattering component (Eq. (7)) or PD, in which each serves as a proxy for the
 253 Bragg wave spectrum and provides important information on the dependence of the
 254 Bragg wave spectrum at the azimuth direction and at certain wind speeds [20], [38].
 255 However, the directional distribution of the Bragg waves derived from these data is
 256 related to the angular distribution of the “folded” wave spectrum (see Appendix A for
 257 more details).

$$258 \quad A_{br}^f = 1 + \delta \cos(2\varphi),$$

259 where δ corresponds to a parameter for angular distribution that can be expressed using
 260 the coefficients of truncated Fourier series A_j^{pp} in Eq. (A1) for Bragg scattering. PD
 261 parameter δ is expressed as $\delta = (A_2^{vv} - A_2^{hh}) / (A_0^{vv} - A_0^{hh})$. Moreover, the link
 262 between the angular distributions of the “folded” Bragg waves (A_{br}^f) and directional
 263 spectra ($A_{br}(\varphi)$) is expressed as

$$264 \quad A_{br}^f(\varphi) = \frac{1}{2} [A_{br}(\varphi) + A_{br}(\varphi + \pi)]. \quad (10)$$

265 Therefore, the angular distribution of the directional spectrum reconstructed from the
 266 folded spectra can be expressed as follows (see Appendix for more details):

$$267 \quad A_{br}(\varphi) = 2(1 + \delta) \exp \left[-\ln \left(\frac{2(1+\delta)}{1-\delta} \right) \left(\frac{2\varphi}{\pi} \right)^2 \right]. \quad (11)$$

268 *Non-Bragg scattering*

269 DV of the breaking facets can be associated with the motion of advancing breaking
 270 wave crests (which is close to the phase velocity of the breaking waves), as originally
 271 proposed in [39] and [40] and later used in the DopRIM model [11]. Recent
 272 measurements by [21] and [41] have further revealed that at moderate incident angles,
 273 the DV of the radar returns from the breaking wave is noticeably less than the speed of
 274 the breaker advance (which is identified by the white-cap velocity). Fig. 3 in [41]
 275 showed that, on average, the ratio of DV of the breaking wave to the velocity of the
 276 white cap was approximately 0.5, i.e., $V_d \approx 0.5c_{wb}$. Thus, from [41], we speculate that
 277 at a moderate incident angle, radar returns are provided by the steep roughness elements
 278 located on the crest of the breaking waves, which are embedded in the water surface.
 279 In this case, $V_d \approx 0.5c_{wb}$ corresponds to the orbital velocity of the breaking wave rather

280 than the phase velocity. At larger incident angles, radar returns occur from the forward,
 281 steep, and slope of the breaking wave and thus must equal its phase velocity, similar to
 282 that found in [42]. Therefore, we then assume that the inherent DV of the breaker facet
 283 c_{np} is proportional to the phase velocity of the breaking wave, i.e., $c_{wb}: c_{np} = \varepsilon_{wb} c_{wb}$,
 284 where ε_{wb} is a tuning parameter. In this work, we set ε_{wb} as

$$285 \quad \varepsilon_{wb} = 1 - 0.5 \exp[-(\theta - 20^\circ)/20^\circ]$$

286 which indicates that at small and moderate incident angles, DV is proportional to the
 287 orbital velocity on the crest of the breaking wave (equal to half the phase velocity if the
 288 maximum steep Stokes wave is used as the prototype). At large θ , DV is equal to the
 289 velocity of the breaking wave slope, which varies with the phase velocity.

290 Finally, we assume that the DV of all breaker facets c_{np} is proportional to the mean
 291 phase velocity of the breaking waves weighted over the breaking areas (\bar{c}_{wb}) whose
 292 azimuthal distribution is described by directional spreading A_{np} of the breaker facets.

$$293 \quad c_{np}(\varphi) = \varepsilon_{wb} \bar{c}_{wb} \frac{A_{np}(\varphi) - A_{np}(\varphi + \pi)}{A_{np}(\varphi) + A_{np}(\varphi + \pi)} \quad (12)$$

294 where \bar{c}_{wb} is the mean phase velocity of the breaking waves.

$$295 \quad \bar{c}_{wb} = \frac{\int_{k < k_{np}} c k^{-1} \Lambda(k) dk}{\int_{k < k_{np}} k^{-1} \Lambda(k) dk}$$

$$296 \quad = 2c(k_{np}) \quad (13)$$

297 where $k_{np} = k_R/10$ is the wavenumber of the shortest breaking waves that provide
 298 radar returns [16]. Λ represents the omnidirectional distribution of the breaking crest
 299 length, which is $\Lambda \propto \beta B$ according to [39], where $\beta \propto (u_*/c)^2$ is the wind-wave
 300 growth rate and B is the saturation wave spectrum that is assumed to be constant.

301 To find the angular distribution of the breaker facets in Eq. (12), we assume that
 302 $A_{np}(\varphi)$ can be associated with the angular distribution of the NP scattering. Then,
 303 following the approach suggested for Bragg facets, the NP scattering should be first
 304 expanded into a truncated Fourier series with a corresponding coefficient A_j^{np} . These
 305 coefficients can be found from the NP values at the upwind, downwind, and crosswind
 306 directions (σ_{npU} , σ_{npD} , and σ_{npC} , respectively). The coefficient of azimuthal
 307 anisotropy ($\delta_{np} = A_2^{np}/A_0^{np}$) for NP can then be expressed as follows:

$$308 \quad \delta_{np} = \frac{\sigma_{npU} + \sigma_{npD} - 2\sigma_{npC}}{\sigma_{npU} + \sigma_{npD} + 2\sigma_{npC}}. \quad (14)$$

309 Subsequently, the directional distribution of the breaker facets can be expressed as
 310 follows (similar to Eq. (11)):

$$311 \quad A_{np}(\varphi) = 2(1 + \delta_{np}) \exp \left[-\ln \left(\frac{2(1 + \delta_{np})}{1 - \delta_{np}} \right) \left(\frac{2\varphi}{\pi} \right)^2 \right]. \quad (15)$$

312 Eq. (12) together with (13) to (15) determine the DV of the breaker facets.

313 *B. Tilt modulations*

314 *Resonant Bragg scattering*

315 The Bragg scattering component is determined using Eq. (7), and its tilt MTF is
 316 defined as follows:

$$317 \quad M_{br}^t(\theta, \varphi) = \partial \ln(\sigma_{br}^{pp}) / \partial \theta. \quad (16)$$

318 The tilt modulation contribution to DV from the large-scale waves is then defined as
 319 follows:

$$320 \quad c_B^T = -\cot\theta \cdot M_{br}^t \iint_{k < K_L^B} \cos(\varphi_R - \varphi) cB(k, \varphi) d\varphi d\ln k \quad (17)$$

321 where $K_L^B = d \cdot k_{br}$, with $d = 1/4$ as the upper limit of the large-scale surface that
 322 carries the Bragg waves.

323 *Non-Bragg scalar scattering*

324 NP scattering is provided by the radar returns from the breaking waves with
 325 wavenumbers in the range of $k < k_{np} = k_R/10$ [16]. Thus, long surface waves can tilt
 326 and modulate the breaker density, which provides the tilt and hydrodynamic
 327 contributions to DV. For consistency with TSM, the upper limit of these long waves is
 328 defined as $k < K_L^{np} \equiv d \cdot k_{np}$ (with the same value of $d=1/4$). Consequently, the
 329 contribution of long waves (c_{np}^T) to the tilting of the breakers is defined by a relationship
 330 similar to Eq. (17) where Bragg tilt MTF M_B^T is replaced by the following:

331
$$M_{np}^T = \partial \ln(\sigma_{np}) / \partial \theta \quad (18)$$

332 where the limit of integration over the long waves is set to $k < K_L^{np}$.

333 *Consolidated contribution of Bragg and non-Bragg tilting to DV*

334 The total contribution of the tilt modulations of the Bragg waves and breakers to DV
 335 can be expressed as follows:

336
$$c^T = P_{br}^{pp} c_B^T + P_{np}^{pp} c_{np}^T. \quad (19)$$

337 Because $P_{br}^{pp} + P_{np}^{pp} = 1$, Eq. (19) (with the use of Eq. (17) for c_B^T and a similar
 338 relationship for c_{np}^T) can be reduced to the following:

339
$$c^T = -\cot\theta \cdot M^t \iint_{k < K_L^{np}} \cos(\varphi_R - \varphi) c_B(k, \varphi) d\varphi d\ln k + P_{br}^{pp} \delta c_B^T \quad (20)$$

340 where $M^t = \partial \ln(\sigma_0^{pp}) / \partial \theta$ is total tilt MTF and δc_B^T is the residual part of the Bragg
 341 contribution (c_B^T) supported by the tilting of long waves in the range of $K_L^{np} < k < K_L^B$.

342 For C-band SAR, this range corresponds to wavelengths from 0.24 to 2.4 m, which can
 343 be treated as the equilibrium range of short gravity waves.

344 In situ data of the short-wave spectrum (to calculate δc_B^T) are rarely available. Still,
 345 in the equilibrium range, spectral levels can be constrained. Notably, Phillips's
 346 spectrum $B(k) = \text{const} = 4.6 \times 10^{-3}$ [43] can be used to assess δc_B^T . Assuming a
 347 relatively wide angular distribution of energy in the equilibrium range (but confined
 348 within $\pm\pi/2$ relative to the wind direction), δc_B^T , when introduced into Eq. (17), is
 349 computed as follows:

$$\begin{aligned}
 350 \quad \delta c_B^T &= -\cot\theta \cdot M_{br}^t \iint_{K_L^{np}}^{K_L^B} \cos(\varphi_R - \varphi) c B d\varphi d\ln k \\
 351 \quad &\cong -\cot\theta M_{br}^t \cos\varphi_R B c_L^{np} \quad (21)
 \end{aligned}$$

352 where $c_L^{np} = c(K_L^{np})$ and M_{br}^t is tilt MTF for the Bragg scattering defined by Eq. (16).
 353 At $c_L^{np} \approx 2$ m/s, $|M_{br}^t| \approx 5$, and $P_{br}^{pp} = 0.5$, the residual velocity (δc_B^T) is approximately
 354 0.05 m/s and may therefore be omitted compared with the other factors.

355 C. Hydrodynamic modulations

356 Analysis of the effect of hydrodynamic modulations of the scattering facets on DV
 357 requires proper expression of spectral MTF. In radar applications, hydrodynamic MTF
 358 is usually defined in a relaxation approximation expressed as follows [44]:

$$359 \quad M^h(\mathbf{k}, \mathbf{K}) = -m_k \cos^2(\varphi - \varphi_K) \left(\frac{1-i\mu}{1+\mu^2} \right) \quad (22)$$

360 where \mathbf{k} and \mathbf{K} are the wavenumbers of the modulated short waves and modulating
 361 long waves, respectively, along their corresponding φ and φ_K directions. $m_k \equiv$
 362 $\partial \ln N / \partial \ln k$ is the ‘‘wavenumber exponent’’ of the short-wave action spectrum, which

363 is approximately $m_k \approx -9/2$ (for the Phillips's spectrum). $\mu = \omega/(\tau_r \Omega)$ is the
 364 relaxation parameter, τ_r is the dimensionless relaxation time, and ω and Ω are the
 365 frequencies of the short modulated wave and long modulated wave, respectively. Eq.
 366 (22) is expressed in a truncated form where we retained only the term that provides a
 367 nonzero contribution of the straining mechanism to the modulations of the integral
 368 wave parameters (such as modulations of the wave breaking) and omitted the
 369 mechanism of surface stress modulations on short wind-wave modulations. As argued
 370 in ([45], their Eqs. (17) and (18)), this stress mechanism is inefficient.

371 The determination of the relaxation scale is usually tuned to best compare the model
 372 with the MTF measurements. As proposed in [17] and [18], parameter τ_r can be
 373 determined using empirical data on dependence of the wave spectrum on wind speed
 374 (see Eqs. (36)–(39) in [17] for more details).

$$375 \quad 1/\tau_r = 2\beta(k)/m_* \quad (23)$$

376 where $m_* = \partial(\ln N)/\partial(\ln u_*)$ is the wind exponent of the wave action spectrum (N),
 377 and β is the dimensionless wind-wave growth rate defined as $\beta = c_\beta(u_*/c)^2$ with
 378 $c_\beta = 4 \times 10^{-2}$.

379 *Bragg wave spectrum*

380 Within the framework of the dual co-pol approach presented in this study, the Bragg
 381 wave wind exponent can be defined using Bragg scattering NRCS, i.e., Eq. (7), or PD
 382 ($\Delta\sigma_0 = \sigma_0^{vv} - \sigma_0^{hh}$), i.e., $m_* = \partial(\ln \Delta\sigma_0)/\partial(\ln u_{10})$. Each method is a good alternative
 383 to the Bragg wave spectrum. For C-band SAR, m_* varies from 1.0 to 1.5 (see Fig. 8 in

384 [38]). Moreover, the magnitude of the Bragg wave spectrum modulations depends on
385 relaxation parameter μ in Eq. (22) and can be expressed as follows:

$$386 \quad \mu_{br} = 2c_{\beta}C_D m_*^{-1} \alpha (u_{10}/c_{br})^3 \quad (24)$$

387 where $\alpha = C/u_{10}$ is the wave age of long modulating wave and C_D is the drag
388 coefficient. For spectral peak waves (which mostly contribute to DV) with α at
389 approximately one, the value of μ_{br} in the wind speed range from 10 to 20 m/s is 10.
390 In this case, Bragg-wave MTF is $M_{br}^h \propto m_k/\mu_{br} \propto 0.5$. This MTF is considerably
391 smaller than wave-breaking MTF (where $M_{wb}^h \approx 20$ according to [46]; see also the
392 section below). Given the comparability of the partial contributions of Bragg scattering
393 and NP returns, we can thus ignore the Bragg wave modulations compared with the
394 wave-breaking modulations.

395 *Wave breaking*

396 In contrast to the Bragg waves, a wave-breaking event is more “inertial” and can be
397 effectively modulated by long surface waves. Field measurements by [46] revealed that
398 the amplitude of white cap modulations (in terms of MTF) could be very high, which
399 reached approximately 20, and enhancement of wave breaking occurred at the crests of
400 long surface waves. NP scattering is proportional to the fraction of the sea surface
401 covered by breaking waves (consistent with the definition of σ_{np} in Eq. (4)). Hence,
402 NP NRCS, which is strongly modulated by long surface waves, significantly
403 contributes to DV.

404 This wave-breaking rate is a strongly nonlinear function of the wave spectrum and
 405 can be expressed as $\propto B^{n_g+1}$ [17]. Hence, the amplitude of the wave-breaking
 406 modulations is amplified by a factor of $(n_g + 1)$ compared with the spectrum
 407 modulations, i.e., Eq. (22). Dulov et al. (2021) demonstrated that a simple relaxation
 408 model of the white caps coverage modulations could be represented as follows [46]:

$$409 \quad M_{wb}^h(\mathbf{K}) = (n_g + 1) \int_{K/d}^{k_{wc}} M^h(\mathbf{K}, \mathbf{k}) \beta B d \ln k d\varphi / \int_{k < k_{wc}} \beta B d \ln k d\varphi \quad (25)$$

410 where k_{wc} is the upper limit of the breaking waves that generate white caps (k_{wc} is on
 411 the order of 10 rad/m). Here, saturation spectrum B is a constant, and M^h can be
 412 expressed by Eq. (22) with a relaxation parameter.

$$413 \quad \mu_{wc} = n_g \beta \omega / \Omega \quad (26)$$

414 where exponent n_g , which is set to five, reasonably agrees with the observations of
 415 white cap modulations (see Fig. 6 in [46]).

416 Furthermore, assuming that NP is proportional to the white cap coverage, we use Eq.
 417 (25) as MTF for NP scattering $M_{np}^h(\mathbf{K})$. To this end, we set the upper limit of the
 418 integration in Eq. (25) as $k_{wc} = k_{np} \equiv k_R/10$, which is linked to the upper limit of the
 419 shortest breaking waves that yield radar returns. After reorganization, Eq. (25) can be
 420 rewritten as follows:

$$421 \quad M_{np}^h(K, \varphi_L) = -\frac{n_k(n_g+1)}{2} A_{wb}(\varphi_L) k_{np}^{-1} \int_{K/d}^{k_{np}} \frac{1-i\mu_{wc}}{1+i\mu_{wc}^2} dk \quad (27)$$

422 where K/d refers to the lower limit of the breaking waves modulated by long waves
 423 with wavenumber K , the range of the modulating waves is $K < k_{np}d$, and $A_{wb}(\varphi_L)$ is
 424 the angular dependence of wave breaking MTF:

425
$$A_{wb}(\varphi_L) = \frac{\int_{-\pi/2}^{\pi/2} \cos^2(\varphi - \varphi_L) \cos^2 \varphi d\varphi}{\int_{-\pi/2}^{\pi/2} \cos^2 \varphi d\varphi} = 1 + 0.5 \cos(2\varphi_L) \quad (28)$$

426 To derive Eq. (27), we assume that $k_{np} \gg g/u_{10}^2$. Finally, we determine the
 427 contribution of the hydrodynamic modulations of the wave breaking due to the long
 428 surface waves (with spectrum $B(K, \varphi_L)$) to DV as follows:

429
$$c_{np}^H = \int_{K < k_{np}d} [M_{1np}^h \cos(\varphi_R - \varphi) + \cot\theta \cdot M_{2np}^h] \cdot cB(K, \varphi_L) d\varphi_L d\ln K \quad (29)$$

430 where $k_{np}d$ is the upper limit of the long waves that modulate the breakers: ($k_{np}d =$
 431 $(d/10)k_R = k_R/40$ at $d = 1/4$).

432 *D. Summary of the DV model*

433 To summarize, the DV model, namely the DP Dop model, can be expressed to obey
 434 the following:

435
$$V_D = u_s + (1 - P_{np})c_B + P_{np}c_{np} + c^T + P_{np}c_{np}^H \quad (30)$$

436 where the second and third terms at the right-hand side describe the contributions of the
 437 velocity of scattering facets, particularly the resonant Bragg waves (Eq. (9)) and the
 438 breakers (Eq. (12)) to DV. These contributions are weighted with partial contributions
 439 of the different scattering mechanisms to total NRCS, i.e., the radar returns from the
 440 breaking waves, which are defined as $P_{np} = \sigma_{np}/\sigma_0^{pp}$, and the resonant Bragg
 441 scattering, which is defined as $P_B = \sigma_B^{pp}/\sigma_0^{pp} = 1 - P_{np}$. These partial contributions
 442 are determined through NP expressed in Eq. (6), which is either derived from dual co-
 443 pol NRCS empirical GMF or from measurements. The fourth term (c^T) is given by Eq.
 444 (20) and describes the contribution of the scattering facets (Bragg waves and breakers)
 445 tilted by the long surface waves to DV. The last term represents the contribution of the
 446 hydrodynamic modulations to DV and is limited to the wave-breaking modulations

447 because the Bragg wave modulations are relatively small. The tilt and hydrodynamic
448 modulations provide dominant contributions to DV, as demonstrated hereunder. Both
449 terms are then crucially dependent on the spectrum of the surface waves, which serve
450 as the input parameter for the DPDop calculations.

451 The following factors are important. At the smallest considered incident angles of
452 approximately $\theta = 20^\circ$, the specular reflection from regular (nonbreaking) surface can
453 significantly contribute to (if not fully provide) the NP scattering. Therefore, we need
454 to be very careful when applying the proposed DV model to θ of less than 20° . At such
455 a small incident angle, NP scattering (and its relative contribution to total NRCS) must
456 additionally be subdivided into the contribution of specular points (regular surface) and
457 breaking waves as well as hydrodynamic modulations for specular points for
458 introduction and incorporation in the DV model, i.e., Eq. (30). This issue is outside the
459 scope of this study. Fortunately, the contribution of the specular reflections to NRCS
460 very rapidly decreases as the incident angle increases. As shown in [35] (see their Fig.
461 10, where the difference between the black solid and dotted lines gives specular
462 reflection NRCS) the specular reflections at 20° are comparable to the observed NP
463 values, but by 25° , they rapidly decrease and become an order of magnitude smaller
464 than NP. Thus, the DPDop model calculations in Eq. (30) at incident angles of
465 approximately 20° should be treated with great care but may be considered valid at
466 larger incident angles, e.g., more than 24° .

467

IV. Results

468 DPDop model simulations are performed using the JONSWAP-type spectrum for
469 wind waves as the input parameter.

470

$$S(\omega, \varphi) = b g^2 \omega^{-5} F(\omega/\omega_p) A_s(\varphi) \quad (31)$$

471 where b is the spectral level defined as $b = 7 \times 10^{-3} \alpha$, $\alpha = u_{10}/c_p$ is the inverse
472 wave age, subscript “ p ” denotes the spectral peak values, $F(\omega/\omega_p)$ is the spectral
473 shape function defined in its original form [47], and $A_s(\varphi)$ is the angular energy
474 distribution that obeys the condition $\int A_s(\varphi)d\varphi = 1$ [48]. The spectral level in Eq. (31)
475 slightly differs from the original level proposed in [47] but is consistent with the field
476 observations reported in [49], which reveals that developing waves obey the Toba law
477 in the form $eg^2/u_{10}^4 = 2 \times 10^{-3} \alpha^{-3}$, where e is the wave energy; $e =$
478 $\iint S(\omega, \varphi)d\omega d\varphi$.

479 A. Role of different mechanisms

480 The contributions of different mechanisms, inherent velocities of Bragg and breaker
481 facets, tilting, and hydrodynamic modulations to DPDop for VV and HH polarizations
482 are shown in Figs. 5 and 6. In general, all mechanisms affect DV. The effect of facet
483 tilting is important at any incident angle and wind speed, either in the upwind or
484 downwind direction.

485 In contrast to the tilt contribution, the effect of hydrodynamic modulations is well
486 expressed in the upwind direction and relatively weak in the downwind direction due
487 to the influence of the following: (i) shift in the wave-breaking modulations on the
488 forward slope of the modulating waves and (ii) azimuthal anisotropy of the NP
489 scattering (Fig. 3). The former factor causes the upwind–downwind asymmetry of
490 DPDop and its corresponding signal structure at the crosswind direction where the
491 contribution of the other mechanisms disappears. The magnitudes of DPDop for HH
492 polarization are larger than those for VV polarization due to the stronger contribution
493 of the NP scattering to total NRCS at HH compared with that at VV (Fig. 4).

494 B. Comparison with empirical GMF

495 The empirical GMF for DV in the C-band, CDOP model [21], and CDOP3SiX model
496 [24] are considered to investigate DP Dop. DV predicted by CDOP is a function of wind
497 speed, incident angle, and radar look direction and is not sensitive to specific features
498 of the surface wave field. In contrast to CDOP, CDOP3SiX DV not only depends on
499 the wind speed and geometry of the radar observations but also explicitly depends on
500 the wind wave and swell parameters (SWH, period, and direction of each wave system).

501 A comparison of the dependence of DV according to the DP Dop model on the
502 incident angle in the upwind, crosswind, and downwind radar look directions at various
503 wind speeds using the CDOP and CDOP3SiX (the inverse wave age is one) calculations
504 is shown in Fig. 7, which exhibits that the DP Dop model is not sensitive to the wave
505 age of the wind waves. The reason is that the effect of the downshift spectral peak on
506 the DP Dop simulations (increase in DV due to an increase in the spectral peak phase
507 velocity with decreasing $\alpha = u_{10}/c_p$) is compensated by a decrease in the spectral level
508 of the JONSWAP spectrum defined in Eq. (31). In general, a fairly good agreement
509 between the DP Dop model and empirical CDOP and CDOP3SiX models can be
510 observed. Moreover, the DP Dop and both CDOP and CDOP3SiX models exhibit a
511 good consistency in terms of the azimuthal distribution of DV (Fig. 8) with a
512 pronounced upwind-to-downwind asymmetry for incident angles of 24° and 37° at a
513 wind speed of 10 m/s.

514 Furthermore, the wind dependence of the DP Dop model versus the CDOP and
515 CDOP3SiX models (Fig. 9) reveals good consistency in the upwind direction among
516 the models. Overall, the DP Dop model is consistent with the empirical CDOP and

517 CDOP3SiX models in terms of producing similar trends with respect to the incident
518 angle, wind speed, and azimuth direction. However, some discrepancies are also
519 noticed, and they are most likely due to the empirical CDOP model assumption in which
520 the sea state (normally represented in the real ocean by mixed seas) after global
521 averaging is entirely related to the wind speed. In contrast, the DP Dop simulations are
522 highly dependent on the 2D wave spectrum, which can be represented as a
523 superposition of the wind sea and swell. Therefore, the DP Dop model simulations of
524 DV for a given wind speed can vary depending on the input wave spectrum parameter
525 representation.

526 A comparison of the DP Dop and CDOP3SiX models for mixed sea consisting of
527 wind seas with inverse wave age $\alpha = u_{10}/c_p = 1$ and swell with SWH $H_s=1.9$ m and
528 period $T_s=9.1$ s, which travel along, across, and opposite the wind, is shown in Fig. 10.
529 These swell parameters are the mean parameters used for training the CDOP3SiX
530 model in the Norwegian coastal zone [24]. Except for the upwind observations at a
531 wind speed of 5 m/s, when the DP Dop model overestimates the CDOP3SiX values, the
532 agreement between the semiempirical and empirical models is quite good. Both models
533 exhibit a remarkable increase in DV when the swell travels downwind and a decrease
534 in DV if the swell travels opposite the wind direction. Simultaneously, the effect of the
535 swell on DV is more pronounced in the upwind radar look directions compared with
536 that in the downwind direction.

537 **V. Conclusion**

538 A new DP Dop model based on the decomposition of NRCS on the ocean surface is
539 proposed. This model can qualify as a semiempirical model, i.e., it is between the pure
540 empirical (e.g., CDOP [22], CDOP3SX [24], and KaDOP [19]) and physical models of
541 the DopRIM type [11], which are based on scattering and statistical properties of the
542 sea surface described under certain physical approximations.

543 The key element of the DP Dop model is related to the decomposition of VV- and
544 HH-polarized radar data (either from direct measurements or GMF) on the polarized
545 and NP scattering components [26], [35]. The former is associated with resonant Bragg
546 scattering, and the latter accounts for the radar returns from breaking waves. Such
547 decomposition provides a quantitative description of the Bragg and NP scattering, their
548 relative contributions to total NRCS, and their dependence on the incident angle,
549 azimuth, and wind speed.

550 By acquiring this quantitative information, we can determine the contributions of the
551 scattering facet velocities (Bragg wave and breaker velocities) and the long waves that
552 provide tilt and hydrodynamic modulations to these facets, which lead to the
553 corresponding Doppler shifts. Measured/empirical tilt MTF is used to estimate the
554 contribution of the tilt modulations to DV. The hydrodynamic modulations of NRCS
555 are mainly dominated by the wave-breaking modulations (modulations of the NP
556 component), which are estimated using a semiempirical model based on the
557 measurements reported in [46].

558 In addition to the VV- and HH-polarized radar data, the main input parameter of the
559 DP Dop model is the wave spectra that predetermine DV for a given geometry of radar

560 observations. The DP Dop model simulations have demonstrated qualitative and
561 quantitative consistencies with the empirical CDOP and CDOP3SiX models. However,
562 the simulations need to be further tested and assessed by comparison with satellite DV
563 measurements and collocated in situ measurements of the surface wave spectra and
564 wind field. The results of such a comparison are presented in a companion paper [29].

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575 **APPENDIX A: Reconstruction of the directional distribution of scattering facets**

576 We consider $PD = VV - HH$ or the Bragg scattering component of NRCS as defined
 577 by Eq. (7) using dual co-pol decomposition as a proxy of the Bragg wave spectrum.
 578 Either of these quantities provides important information on the angular spreading of
 579 the Bragg wave spectrum and its dependence on wind speed.

580 The input data, namely, VV- and HH-polarized NRCS, are usually represented in the
 581 form of a truncated Fourier series.

582
$$\sigma_0^{pp} = A_0^{pp} + A_1^{pp} \cos\varphi + A_2^{pp} \cos 2\varphi \quad (\text{A1})$$

583 where $A_j^{pp} = A_j^{pp}(u_{10}, \theta)$ is the empirical coefficient, and φ is the angle between the
 584 radar look direction and wind velocity (usually, $\varphi = 0$ corresponds to the upwind radar
 585 look direction).

586 *Bragg waves*

587 PD, namely, $\Delta\sigma_0 = \sigma_0^{vv} - \sigma_0^{hh}$, or the Bragg component of NRCS, which is defined
 588 by Eq. (7), can also be represented in a form similar to Eq. (A1) in which the
 589 corresponding Fourier coefficient is expressed through A_j^{pp} . The Fourier coefficients of
 590 PD expressed in form of Eq. (A1) are calculated as follows: $\Delta A_j = A_j^{vv} - A_j^{hh}$.

591 According to the angular distribution of the Bragg wave spectrum, we only need the
 592 second harmonic. We assume that the saturation (folded) spectrum of the Bragg waves
 593 is proportional to PD, and $B_{br}(k_{br}, \varphi) \propto \Delta\sigma$. Hence, its angular distribution A_{br}^f is
 594 calculated as follows:

595
$$A_{br}^f = 1 + \delta \cos(2\varphi) \quad (\text{A2})$$

596 where $\delta = \Delta A_2 / \Delta A_0$ is a parameter for the angular distribution of the folded spectrum
 597 of the Bragg waves expressed using the radar scattering coefficients.

598 The angular distribution of the folded spectrum is related to the angular distribution
 599 of the directional spectrum $A_{br}(\varphi)$:

$$600 \quad A_{br}^f(\varphi) = 1/2 [A_{br}(\varphi) + A_{br}(\varphi + \pi)] \quad (\text{A3})$$

601 We search the angular distribution in the form $A_{br}(\varphi) = m \exp[-2n\varphi^2]$, where m
 602 and n are unknown parameters and φ by definition is in the range of $-\pi < \varphi \leq \pi$.
 603 These parameters can be found from the two equations (Eq. (A3)) for the upwind and
 604 crosswind directions, where the values of $A_{br}^f(\varphi)$ are known and equal to $A_{br}^f(0) =$
 605 $1 + \delta$ and $A_{br}^f(\pi/2) = 1 - \delta$, respectively. Then, we obtain the following:

$$606 \quad A_{br}(\varphi) = 2(1 + \delta) \exp \left[-\ln \left(\frac{2(1+\delta)}{1-\delta} \right) \left(\frac{2\varphi}{\pi} \right)^2 \right]. \quad (\text{A4})$$

607 Fig. A1 shows examples of the reconstruction of the directional spectrum from the
 608 folded spectrum at different parameters of azimuthal anisotropy δ .

609 *Breakers*

610 A similar procedure is implemented to determine the angular distribution of the
 611 breaking waves that provide NP radar returns. To determine the angular distribution of
 612 breaker facets $A_{wb}(\varphi)$ in Eq. (12), we need to expand NP defined in Eq. (6) into a
 613 truncated Fourier series in the form of Eq. (A1) and find the Fourier coefficient A_j^{np} .
 614 On the other hand, these coefficients can be directly obtained from the NP values in the
 615 upwind, downwind, and crosswind directions, namely, σ_{npU} , σ_{npD} , and σ_{npC} ,

616 respectively. In particular, coefficient of azimuthal anisotropy $\delta_{np} = A_2^{np} / A_0^{np}$ for NP
617 is calculated as follows:

$$618 \quad \delta_{np} = \frac{\sigma_{npU} + \sigma_{npD} - 2\sigma_{npC}}{\sigma_{npU} + \sigma_{npD} + 2\sigma_{npC}}. \quad (\text{A5})$$

619 Then, the directional distribution of the breaker facets is defined by a relationship that
620 is similar to Eq. (A4).

$$621 \quad A_{np}(\varphi) = 1(1 + \delta_{np}) \exp \left[-\ln \left(\frac{2(1 + \delta_{np})}{1 - \delta_{np}} \right) \left(\frac{2\varphi}{\pi} \right)^2 \right] \quad (\text{A6})$$

622 which, together with Eq. (13), defines the DV of the breaker facets, i.e., Eq. (12).

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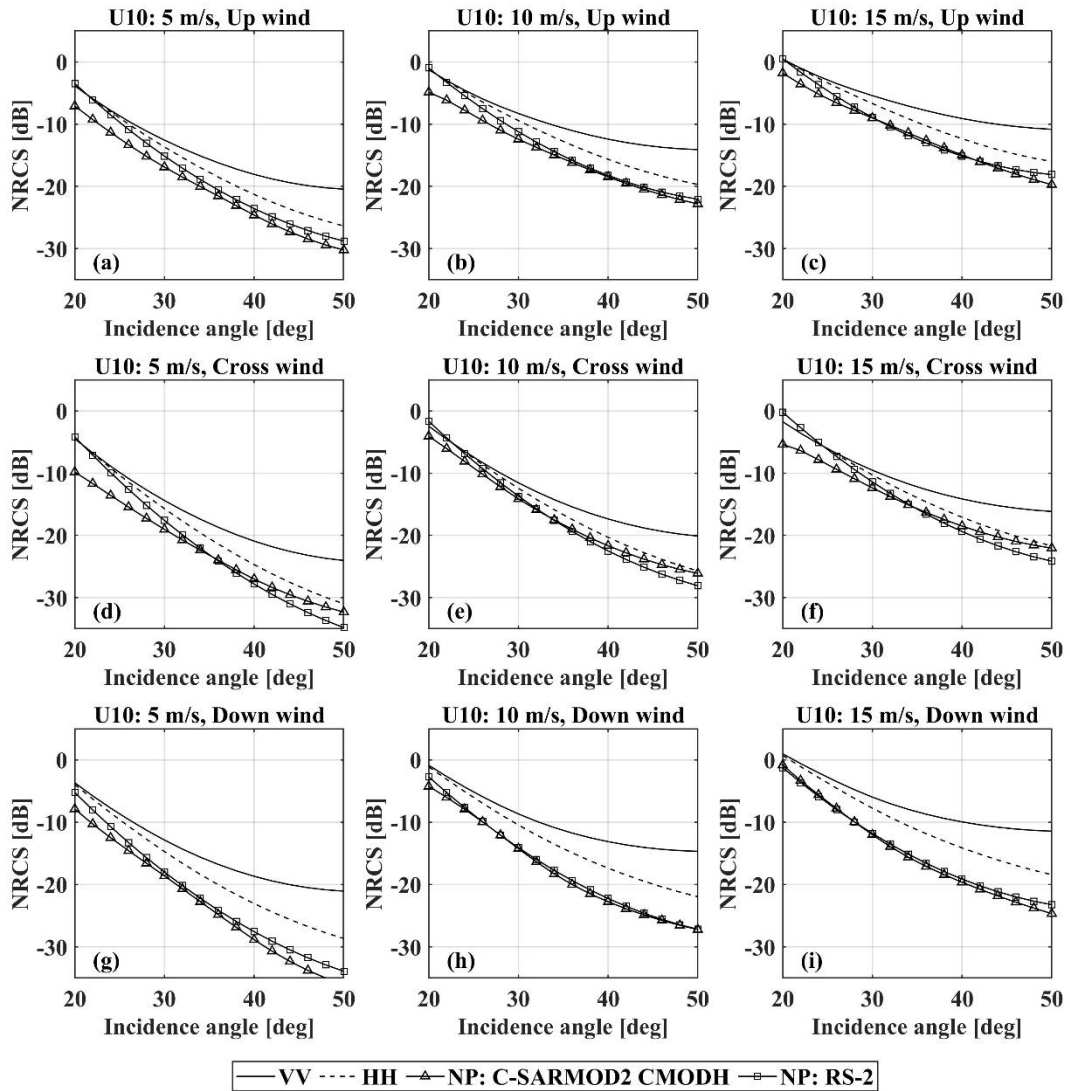
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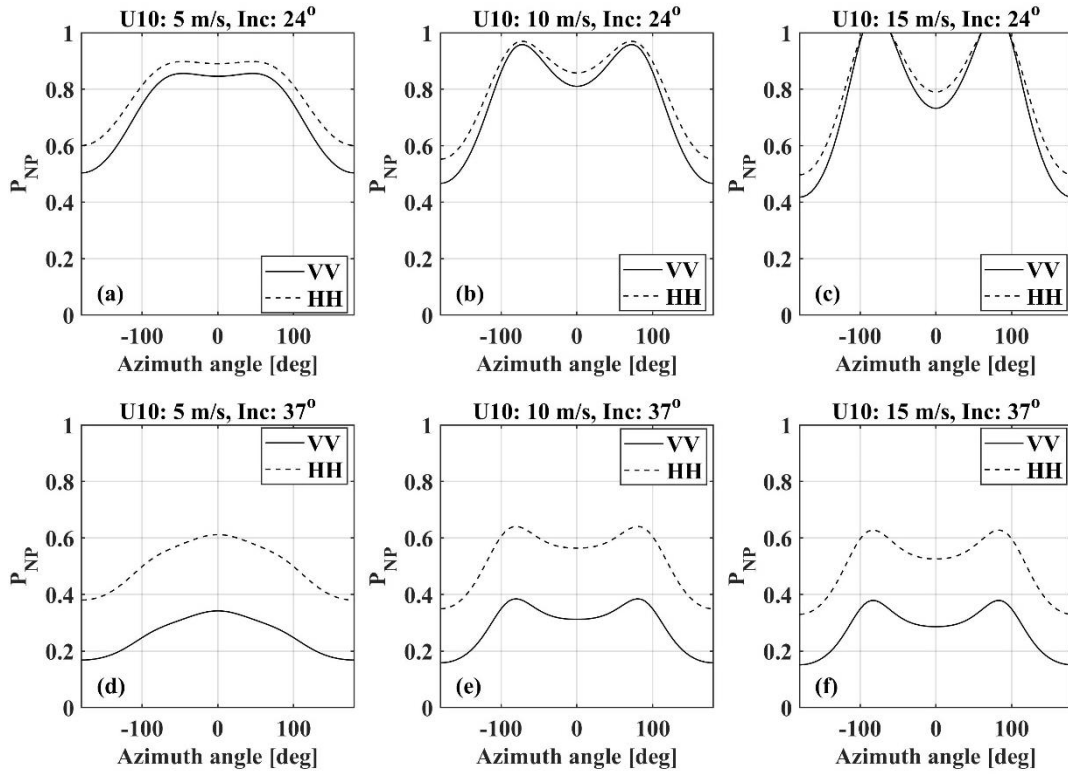
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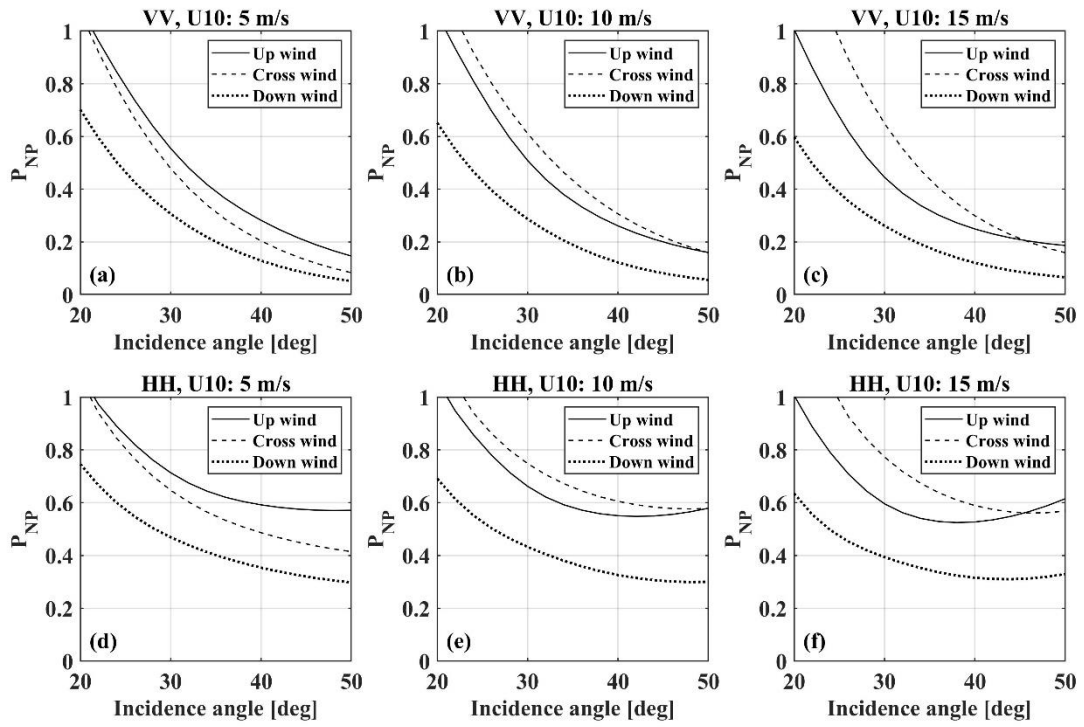
791 **Fig. 1.** Relationships between NRCS and incident angles at wind speeds of 5, 10, and
 792 15 m/s (columns from left to right) along the upwind, crosswind, and downwind
 793 directions (from upper to lower rows).

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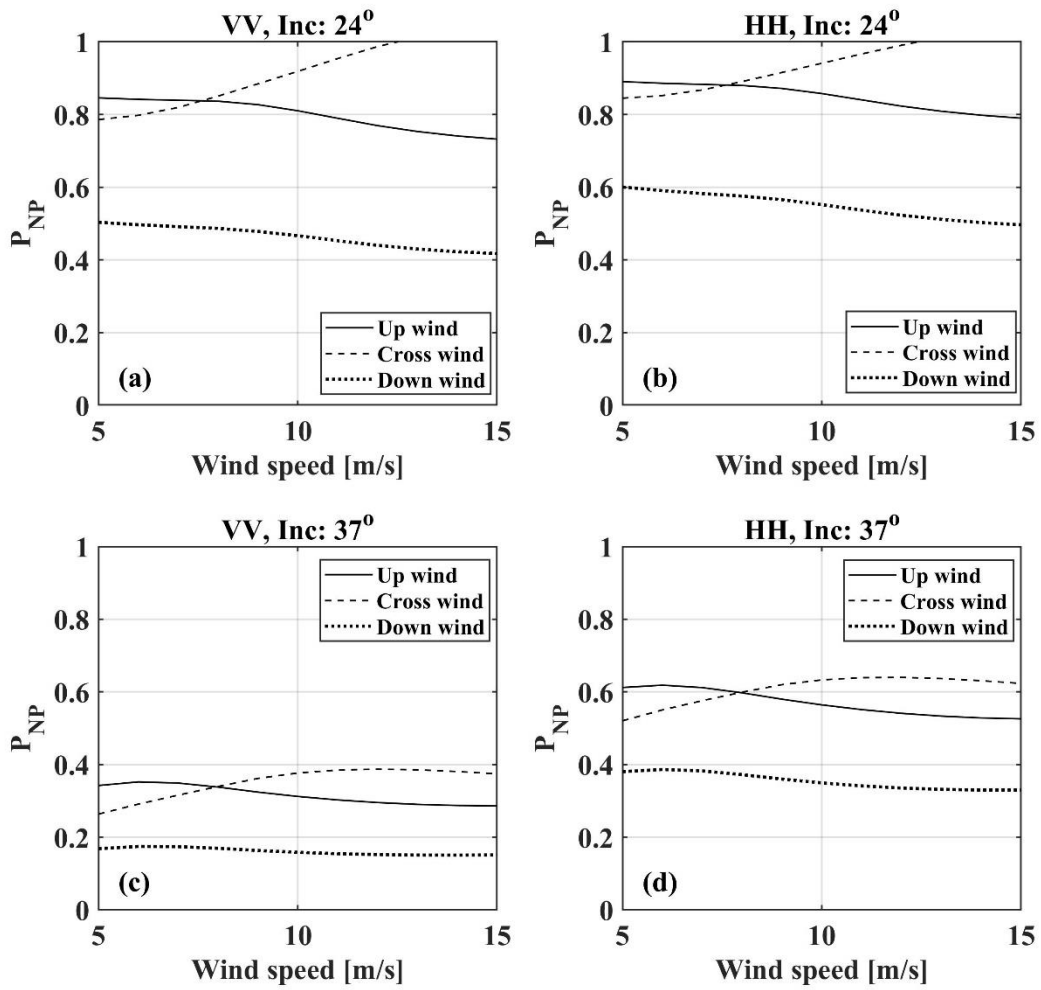


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796 **Fig. 2.** Azimuthal dependence of the relative contribution of NP to total NRCS $P_{np}^{pp} =$
 797 $\sigma_{np}/\sigma_0^{pp}$ at wind speeds of 5, 10, and 15 m/s (columns from left to right) and incident
 798 angles of 24° (upper) and 37° (lower) .

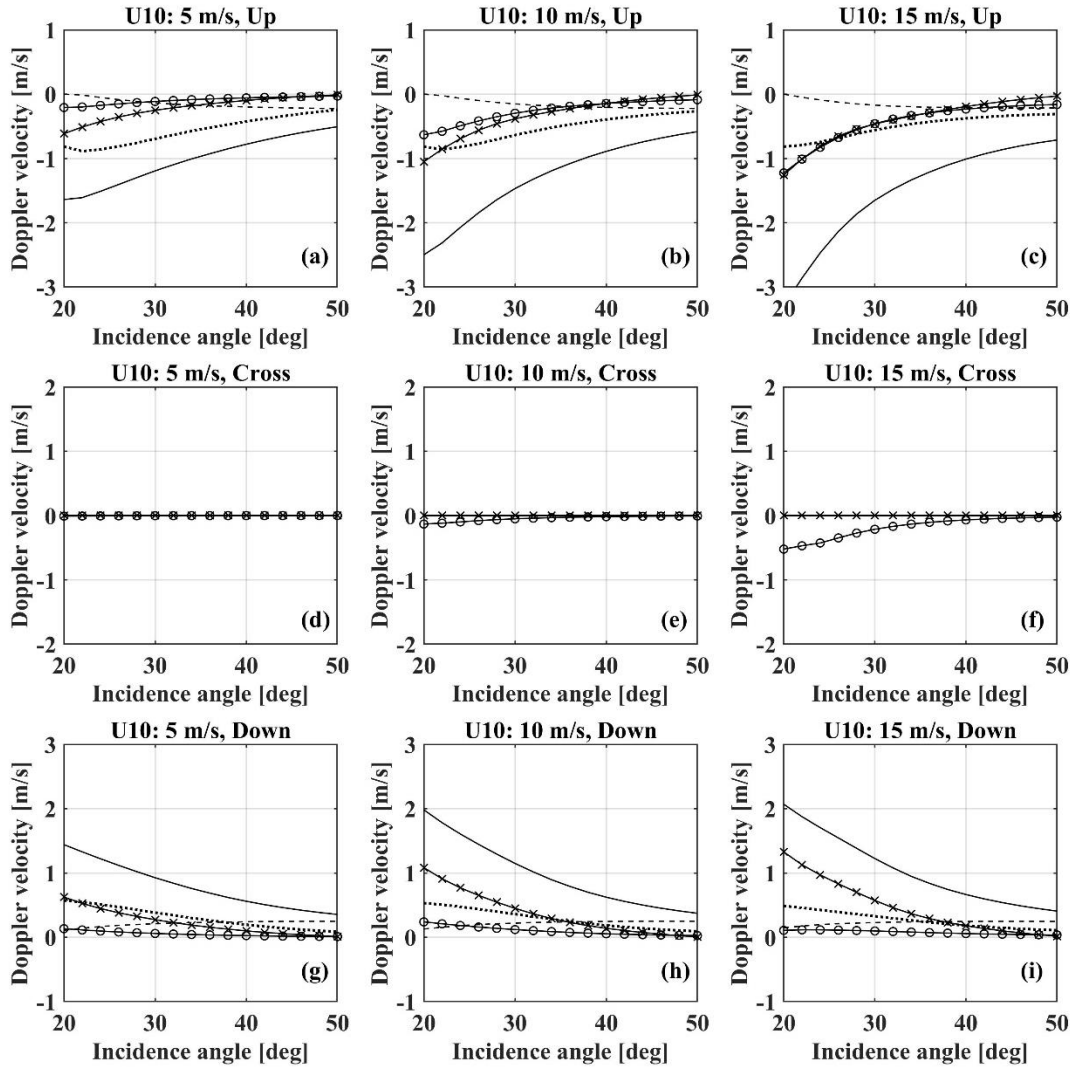


799 **Fig. 3.** Relative contribution of NP to VV (upper row) and to HH (lower row) NRCS
800 as a function of the incident angle at wind speeds of 5, 10, and 15 m/s (columns from
801 left to right) at the upwind, crosswind, and downwind directions.

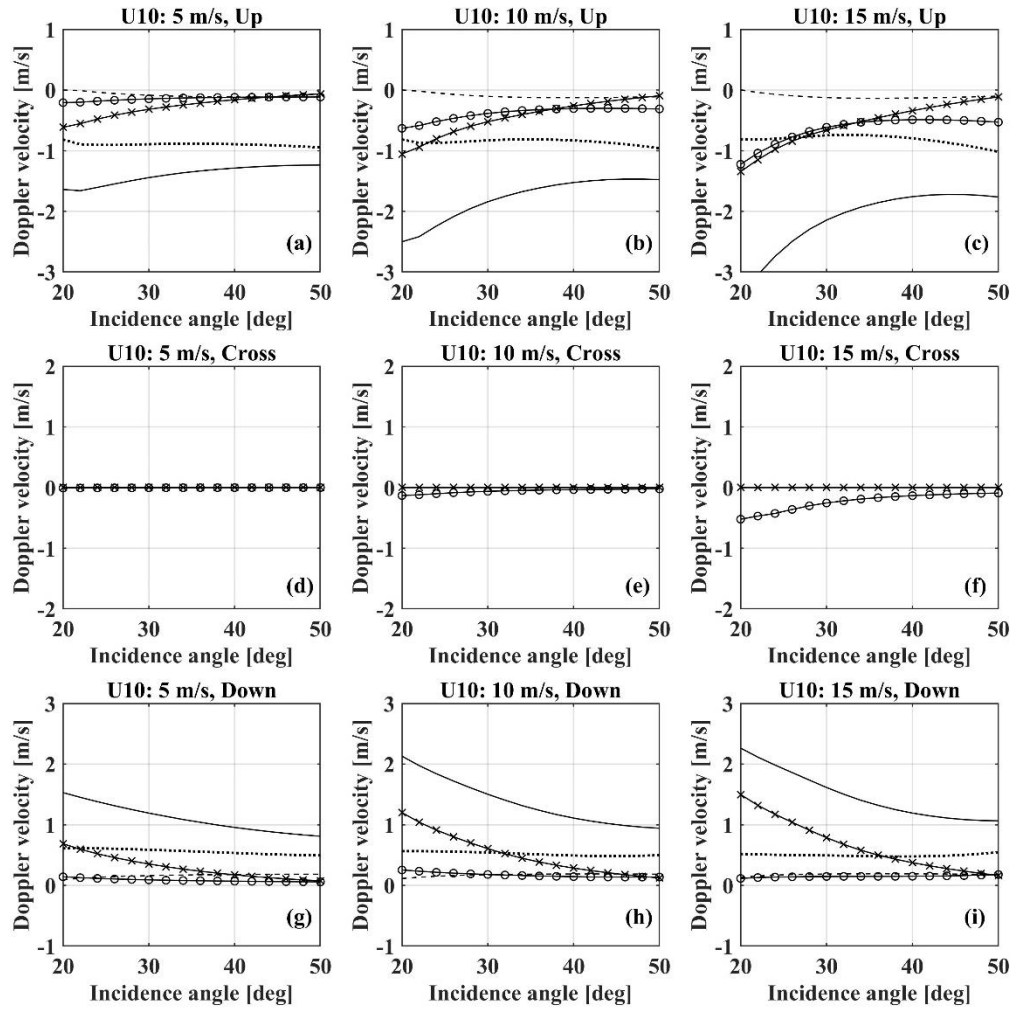


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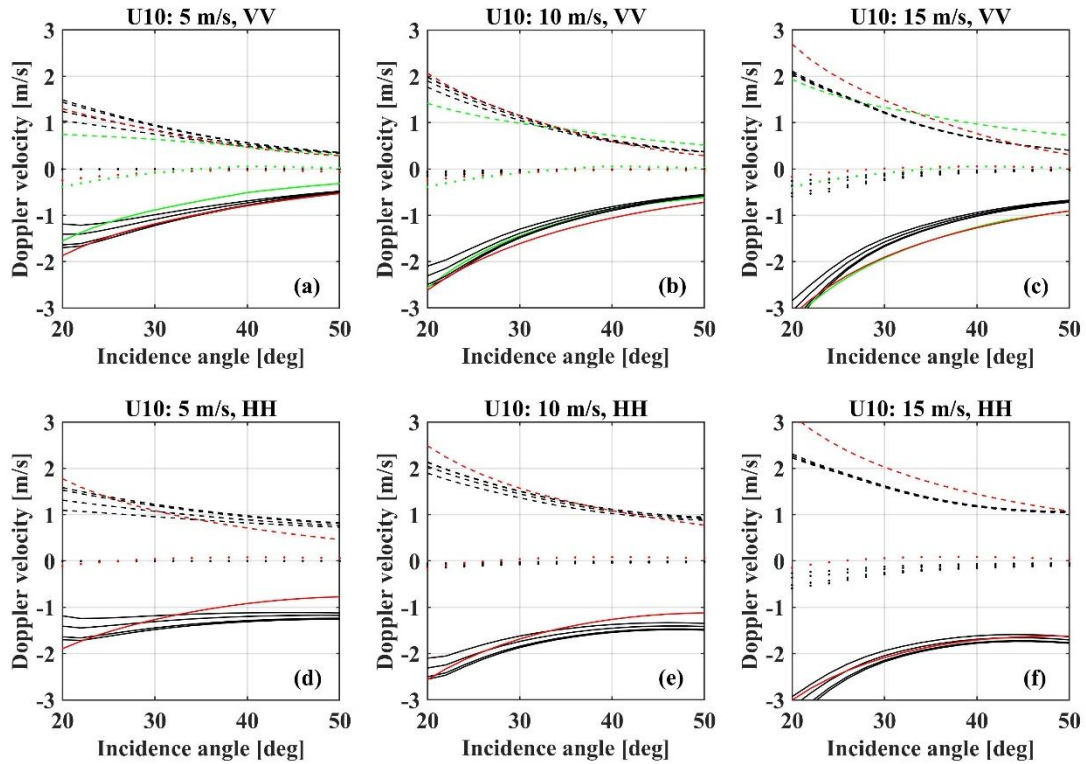
803 **Fig. 4.** Wind speed dependence of the relative NP contribution to VV (left column) and
 804 HH (right column) NRCS for the upwind, crosswind, and downwind azimuth directions
 805 at incident angles of 24° (upper row) and 37° (lower row).



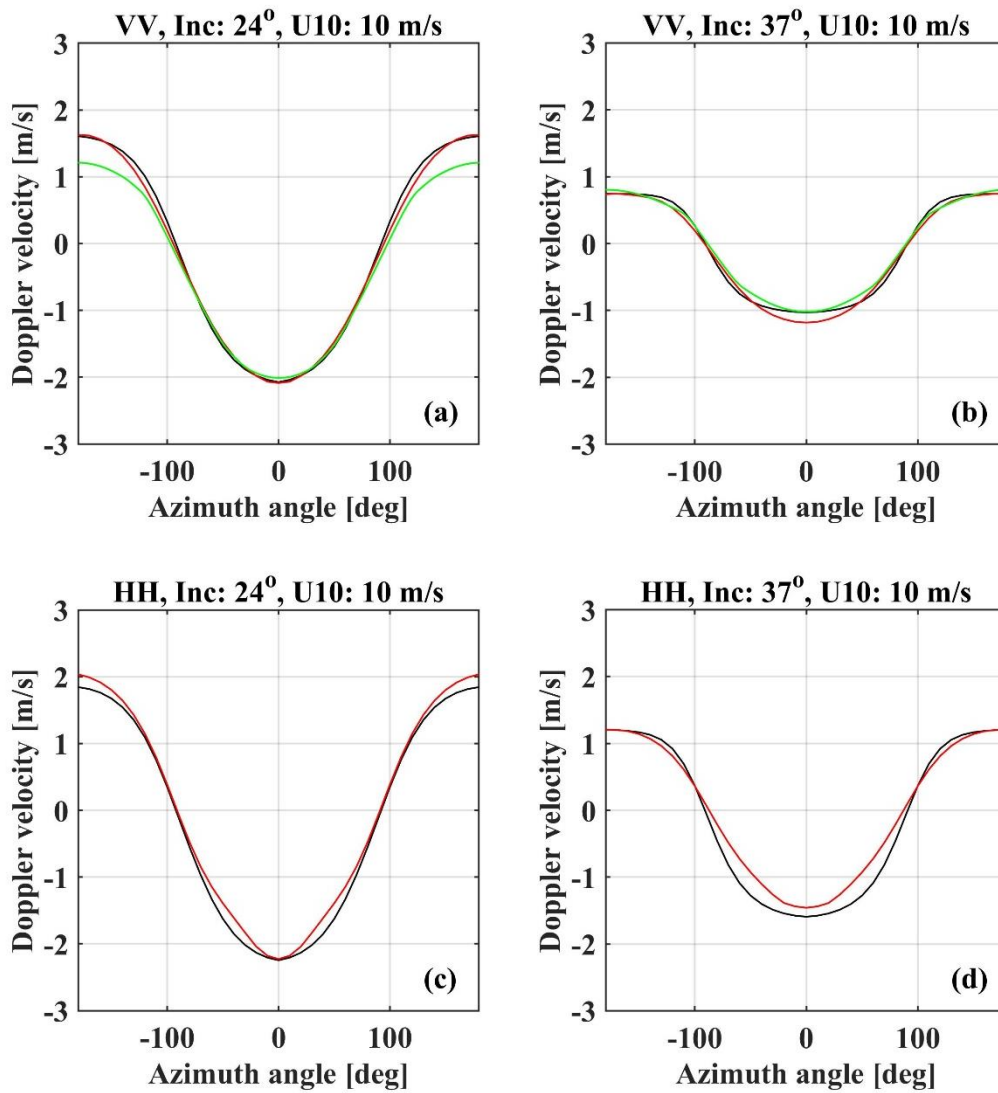
806 **Fig. 5.** Contributions of the different mechanisms to DPDop in the VV polarization in
807 the C-band at upwind, crosswind, and downwind directions (upper, middle, and lower
808 rows, respectively, and at wind speeds of 5, 10, and 15 m/s (left to right columns).
809 Dashed line: velocity of the Bragg facets; dotted line: velocity of the breaker facets;
810 line with crosses: tilt modulations; line with circles: hydrodynamic modulations of
811 wave breaking; solid line: total.



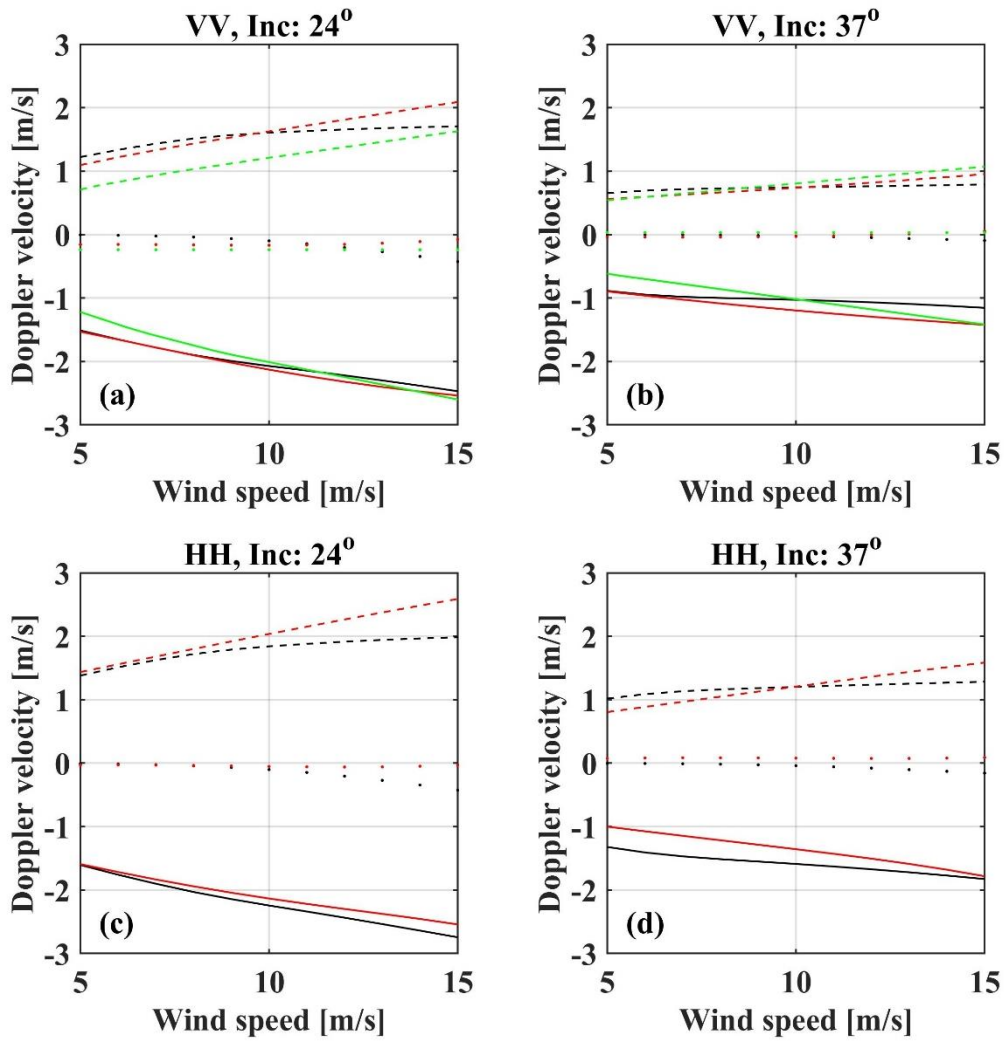
812 **Fig. 6.** Similar to that shown in Fig. 5, but for HH polarization.



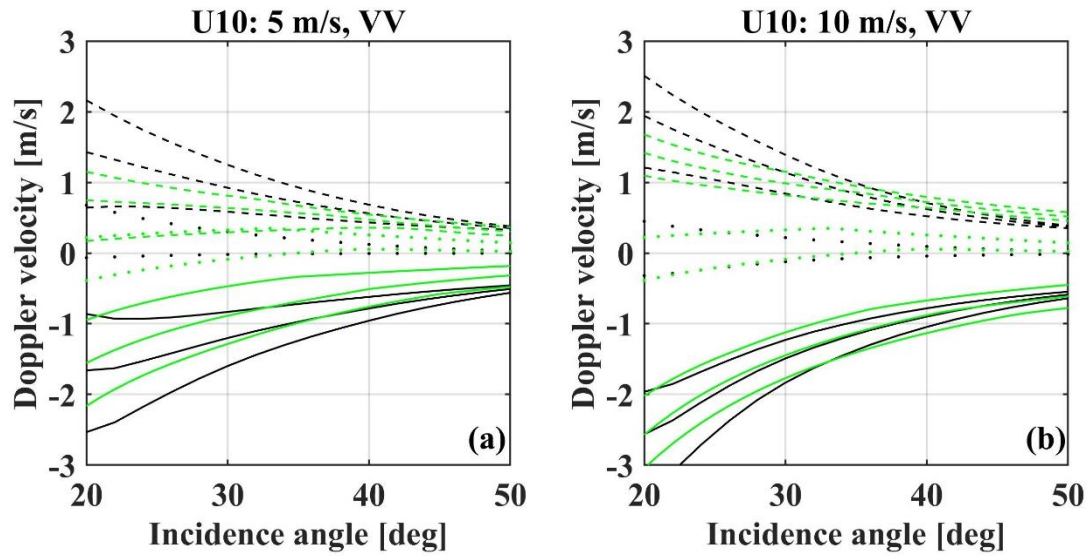
813 **Fig. 7.** Comparison of the DPDop model (black) with the CDOP (red) and CDOP3SiX
814 (green) models for VV (upper column) and HH (lower column) polarizations at wind
815 speeds of 5, 10, and 15 m/s (left to right). The black lines indicate the model simulation
816 results at different wind speeds with inverse wave ages of 0.85, 1, 1.5, and 2 at upwind
817 (solid lines from bottom to top), downwind (dashed lines from top to bottom) and
818 crosswind (dotted lines from top to bottom) directions. CDOP3SiX calculations are
819 performed for the inverse wave age of one.



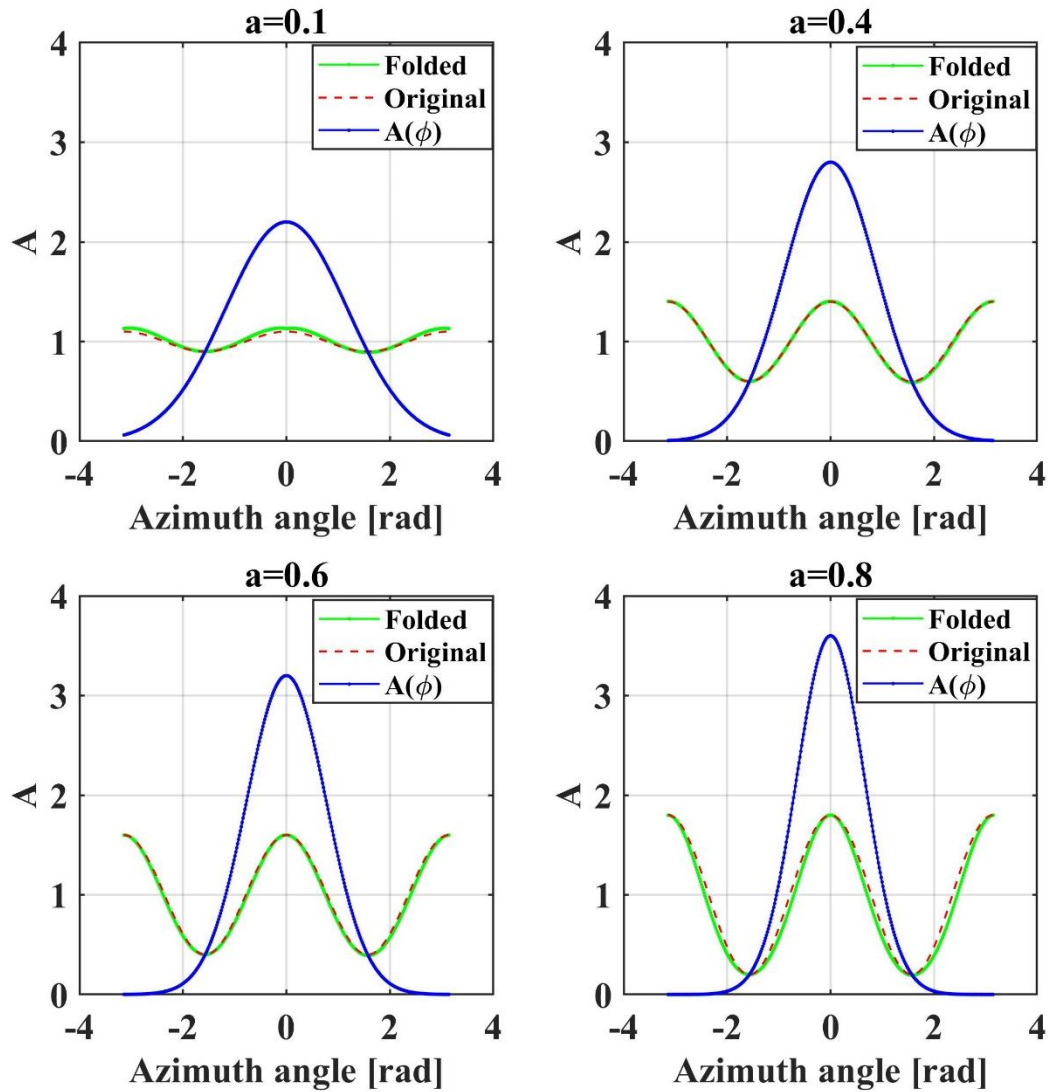
820 **Fig. 8.** Azimuthal distribution of DV at two incident angles at 10 m/s wind speed for
 821 HH (upper row) and (lower row) VV polarizations for DPDop (black), CDOP (red),
 822 and CDOP3SiX (green) models.



823 **Fig. 9.** Wind dependence of DV under incident angles of 24° (left column) and 37°
824 (right column) on VV (upper row) and HH (lower row) polarizations at upwind (solid
825 lines), crosswind (dotted lines), and downwind (dashed lines) directions calculated
826 using DP Dop (black), CDOP (red) and CDOP3SiX (green) models.



827 **Fig. 10.** DP Dop (black lines) and CDOP3SiX (green lines) simulations of DV of mixed
 828 sea consisting of wind waves with inverse wave age of one and swell with $H_s = 1.9$ m
 829 and period $T_s = 9.1$ s that travel opposite, along, and across the wind direction (maximal,
 830 minimal, and intermediate magnitudes of DV at a given radar look direction) at (a) wind
 831 speeds of 5 m/s and (b) 10 m/s for upwind (solid lines), downwind (dashed lines), and
 832 crosswind (dotted lines) radar look directions.



833

834 **Fig. A1.** Examples of the reconstruction of the angular distribution (blue lines) of the
 835 directional spectrum (Eq. (A4)) of the Bragg waves from the angular distribution
 836 (dashed red lines) of the Bragg wave folded spectrum (Eq. (A2)) for different
 837 parameters of azimuthal anisotropy δ . The green lines show the inverse calculation of
 838 the folded distribution from the directional distribution using Eq. (A3).

839