## Supplementary Information to

## The open ocean kinetic energy cascade is strongest in late winter and spring

René Schubert<sup>1,\*</sup>, Oscar Vergara<sup>2</sup>, and Jonathan Gula<sup>1,3</sup>

<sup>1</sup>Univ Brest, CNRS, IRD, Ifremer, Laboratoire d'Océanographie Physique et Spatiale (LOPS), IUEM, Plouzané, France

<sup>2</sup> Collecte Localisation Satellites (CLS), Ramonville Saint-Agne, France

<sup>3</sup> Institut Universitaire de France (IUF), Paris, France

\*corresponding author, rene.schubert@univ-brest.fr

## Supplementary Note 1: Effects of tides on the kinetic energy cascade and its estimation from along-track altimetry

To investigate the effects of tides on the cascade and its estimation, a parallel simulation experiment with explicitly simulated tides has been performed. Including tides in ocean general circulation models is a rather new development<sup>1,2</sup>. Tides enhance the forward cascade, in particular at smaller scales, in summer, in the tropics, and in regions of strong internal tide generation including the Azores, Cape Verde Islands, and the Guyot Province (Fig. 1a). Furthermore, tides strengthen the unbalanced part of the flow and thus reduce the dominance of balanced flows over the unbalanced. This introduces further errors in the computation of the geostrophic flow from the SSH gradients at the respective scales and in the respective regions and seasons (Fig. 1b). In mid-latitude regions, at scales larger than the local *T*, the  $5^{\circ} \times 5^{\circ}$  averaged geostrophic flux shows similar amplitudes and patterns as in the non-tidal run (Fig. 1c). When the tropics between 25°S and 25°N are excluded from the computation of the estimation coefficient *C* for scales of [30,40,50,60,70,80,90,100,120,140,160,180,200] km, values of [0.32,0.26,0.23,0.22,0.19,0.17,0.16,0.16,0.14,0.13,0.12,0.11,0.10] are resulting for the tidal run. These are very close to the one of the non-tidal run which implies that one can use these values regardless of how much tidal effects are included in the observations for the estimation of the scale kinetic energy flux, as long as *L* is reasonably larger than *T*. The estimation works similarly well for the non-tidal run (Fig. 1d). Large differences are also only found in the core Gulf Stream extension, where the flux is overestimated.



**Supplementary Figure 1.** The scale kinetic energy flux from a simulation with tides *The scale kinetic energy flux*  $\Pi$  *computed from GIGATL1 with simulated tides in the period Apr 2008 - Mar 2009 from the total velocity (a), the geostrophic velocity (b), the geostrophic velocity and subsequently averaged over*  $5^{\circ} \times 5^{\circ}$  *domains (c), and estimated with equation (1), see main study, from SSH cut from the simulation along Jason-3 tracks (d). In a) to d), the time-mean flux at 60 km scales is shown.*  $25^{\circ}S$  and  $25^{\circ}N$  are shown with black horizontal lines. North respectively South of these latitudes, black lines mark regions, where the time-mean T is larger than 60 km. Hatches mark regions that have been excluded for the computation of the estimation coefficient C. In e) to f), Hovmöller plots of the area-averaged flux in the North Atlantic are shown (domain marked with a black box in a) to d)). The scale of 60 km is marked with a thick black line. Thin black lines show the area-mean T (solid) plus and minus one standard deviation (dashed) in the North Atlantic box.

## **Supplementary References**

- 1. Arbic, B. K., Wallcraft, A. J. & Metzger, E. J. Concurrent simulation of the eddying general circulation and tides in a global ocean model. *Ocean. Model.* 32, 175–187 (2010).
- 2. Müller, M., Cherniawsky, J., Foreman, M. & von Storch, J.-S. Global m2 internal tide and its seasonal variability from high resolution ocean circulation and tide modeling. *Geophys. Res. Lett.* **39** (2012).