Supporting Information

1 Details on parameters used to describe species-environmental relationships

Here, we present how we derived the three ecological parameters (Fig. S1) describing species-environment relationships (i.e. the environmental optimum θ, the maximum probability of presence ψ_{max}, the SER width ω) from the regression coefficients.

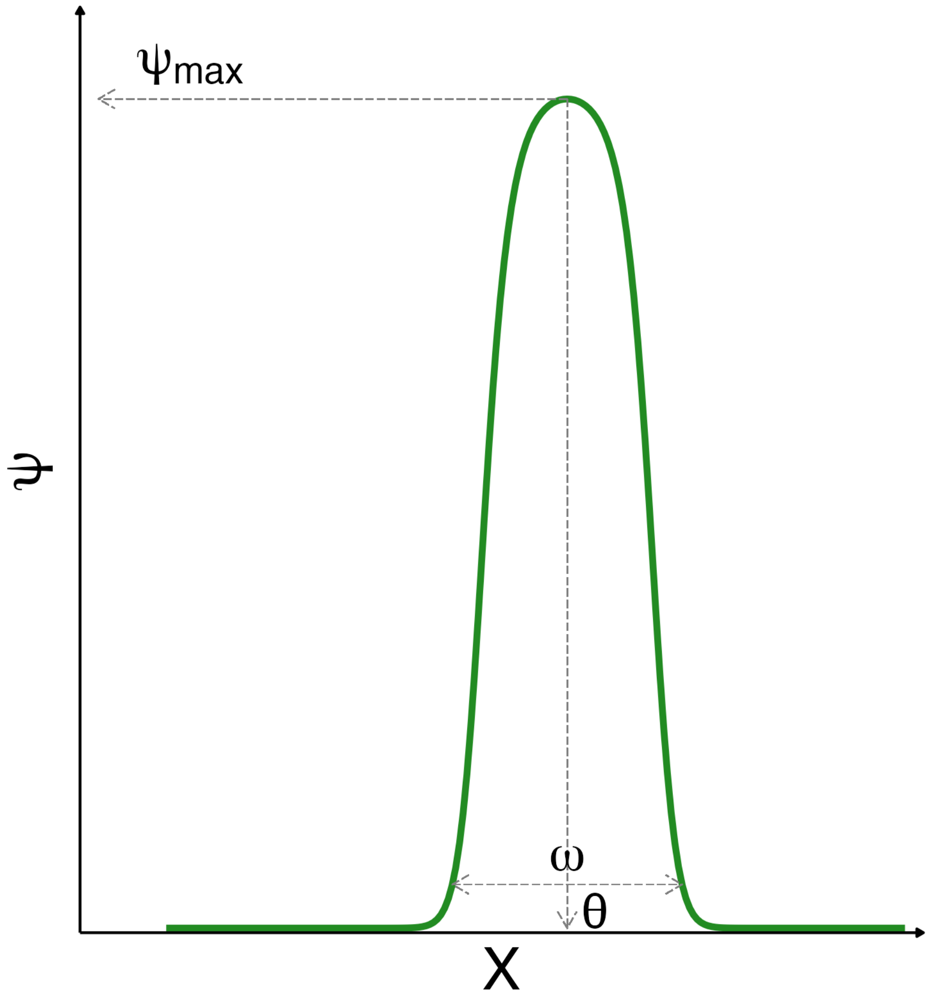

Figure S1: Illustration of a bell-shaped species-environment relationship with three associated ecological parameters: the maximum probability of presence ψ_{max}, the environmental optimum θ and the ecological width ω.

The regression modelling SER was specified as:

$$logit(\psi) = f(X) = \beta_0 + \beta_1 X + \beta_2 X^2$$

with ψ the probability of presence, X the environmental value and βs the regression coefficients.

1.1 Environmental optimum (θ)

The environmental optimum is the environmental value associated with the maximum probability of presence and is reached when the derivative of the regression equals zero:

$$f'(\theta) = 0 \tag{1}$$

$$\beta_1 + 2\beta_2\theta = 0 \tag{2}$$

$$\theta = \frac{-\beta_1}{2\beta_2}. \tag{3}$$

1.2 Maximum probability of presence (ψ_{max})

ψ_{max} the maximum probability of presence is reached at environmental optimum. Thus we have:

$$logit(\psi_{max}) = \beta_0 + \beta_1\theta + \beta_2\theta^2 \tag{4}$$

$$\psi_{max} = \frac{exp(\beta_0 + \beta_1\theta + \beta_2\theta^2)}{1 + exp(\beta_0 + \beta_1\theta + \beta_2\theta^2)}, \text{ with: } \theta = \frac{-\beta_1}{2\beta_2}. \tag{5}$$

1.3 SER width (ω)

ω the ecological width is a measure of the species ecological breadth (i.e. ecological tolerance) and is defined, for a bell-shaped SER, as the environmental distance between two points of the curve having a probability of presence of 0.05 (Michaelis & Diekmann, 2017). We denote this probability of presence as $\psi_\omega = 0.05$. Let's set x_1 and x_2 the solutions of the equation $\psi_\omega = 0.05 \Leftrightarrow \beta_0 + \beta_1 X + \beta_2 X^2 = logit(0.05)$, then we have:

$$\omega = |x_1 - x_2| \tag{6}$$

$$\omega = |\frac{-\beta_1 + \sqrt{\Delta}}{2\beta_2} - \frac{-\beta_1 - \sqrt{\Delta}}{2\beta_2}| \tag{7}$$

$$\omega = |\frac{\sqrt{\Delta}}{\beta_2}| \tag{8}$$

with $\Delta = \beta_1^2 - 4\beta_0'\beta_2 = \beta_1^2 - 4(\beta_0 - log(\frac{0.05}{1-0.05}))\beta_2$.

1.4 References

Michaelis J. & Diekmann M.R. (2017) Biased niches – Species response curves and niche attributes from Huisman-Olff-Fresco models change with differing species prevalence and frequency. *PLOS ONE*, **12**, e0183152.

2 Extended results

2.1 Estimates of species-environment relationships

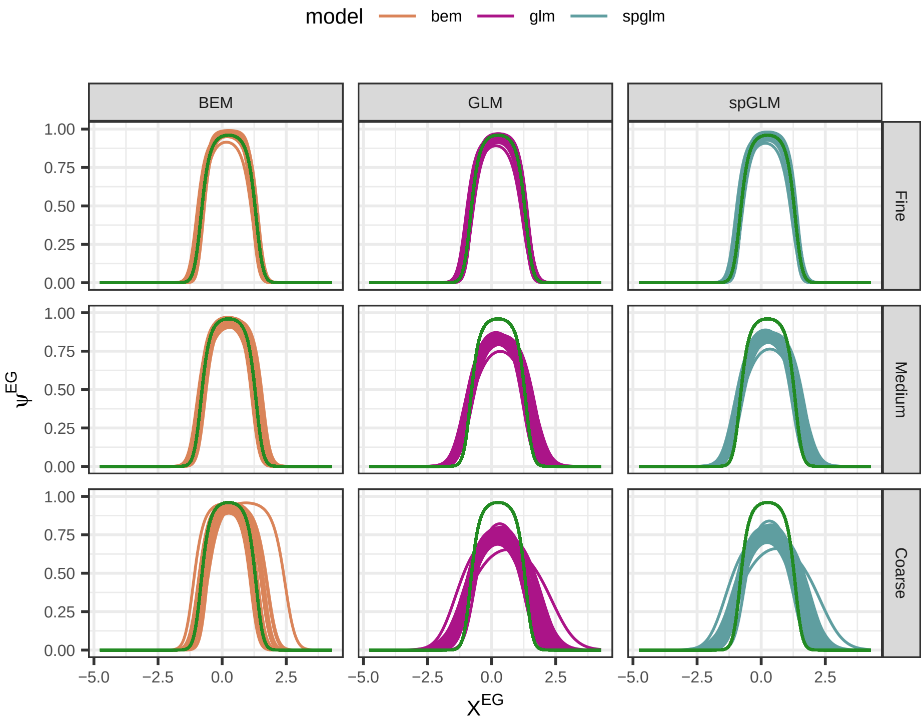

Figure S2: Species-environment relationships estimated by the three models for the 30 replications compared to the simulated relationship (green curve) for three covariate grain sizes.

2.2 Explanatory performance

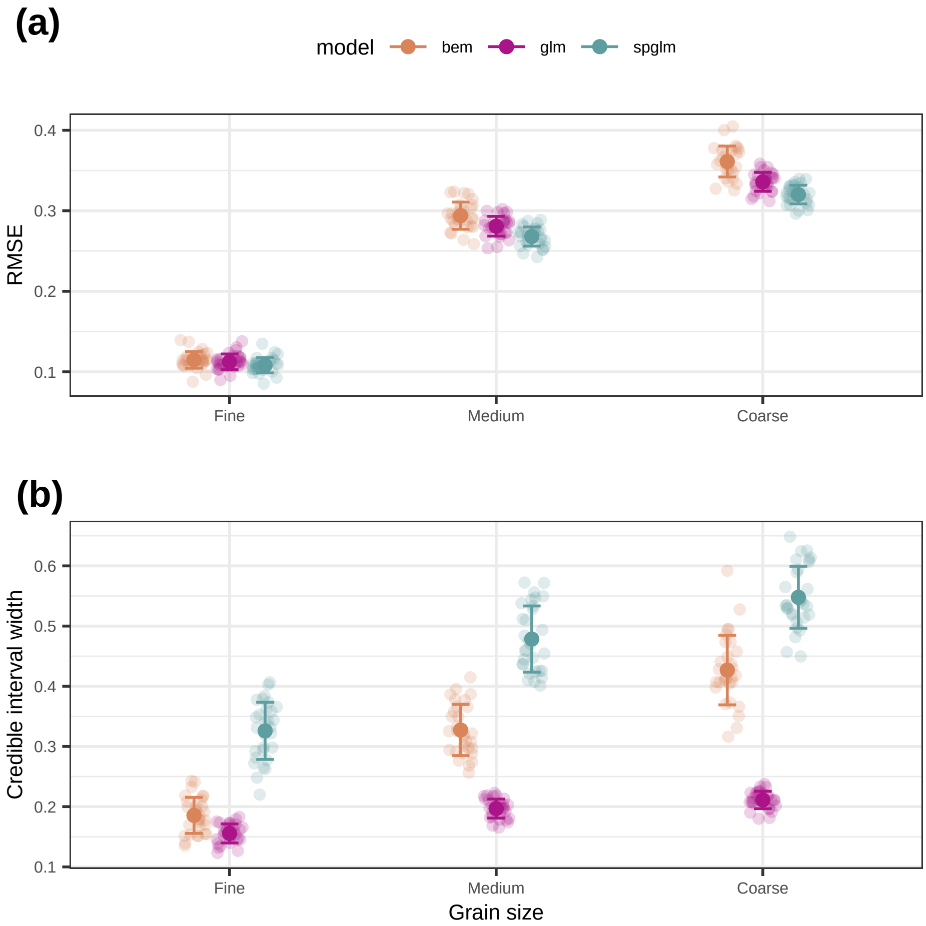

Figure S3: Explanatory performances of the three alternative models (BEM: Berkson-Error Model, GLM: Generalized Linear Model, spGLM: spatial GLM) fitted with environmental values at three grain sizes coarser than the ecological grain. Filled points represent mean performance metrics over the 30 simulated train datasets (shaded points) while vertical bars represent the associated standard deviations.

2.3 Predictive performance from environmental values at covariate grain.

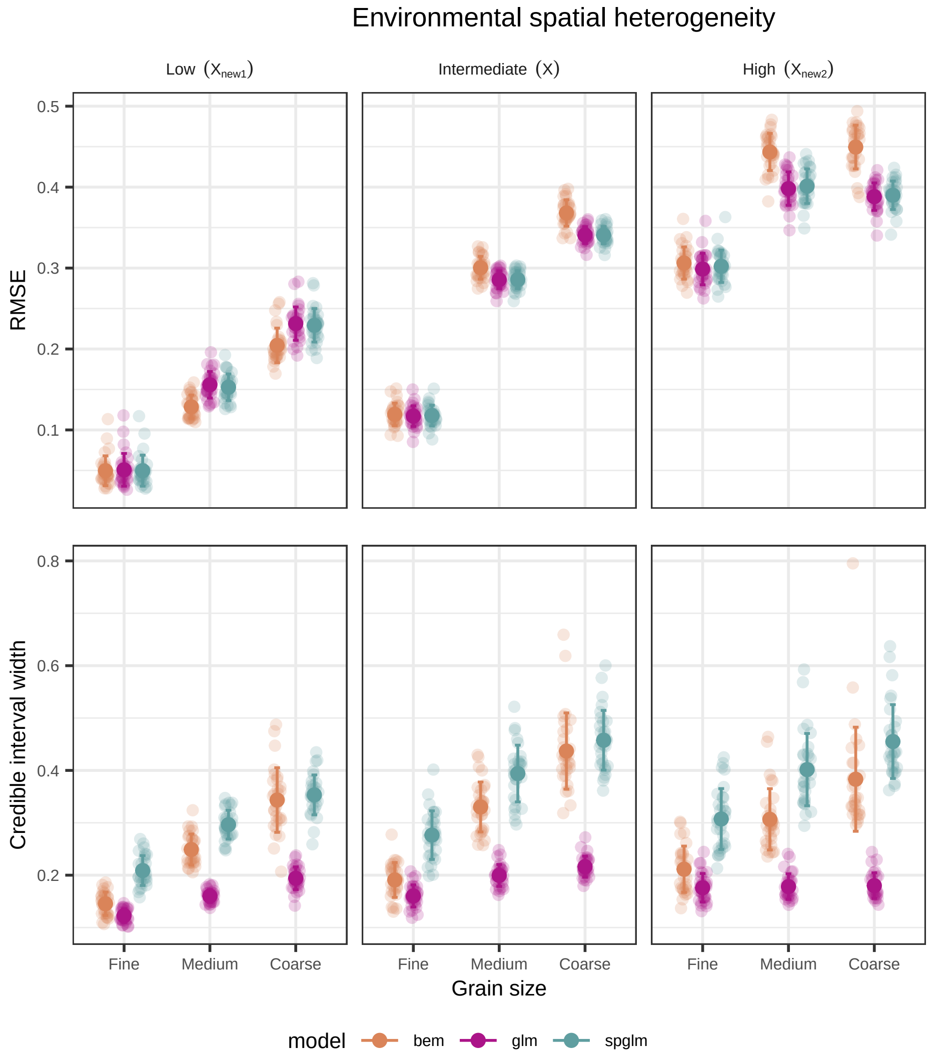

Figure S4: Evaluation of predictive performance of the three models fitted with area-to-point misaligned data with regards to their ability to predict species distribution at the ecological grain from environmental values at the covariate grain across three levels of environmental spatial heterogeneity (the three columns). Filled points represent mean performance metrics over the 30 simulated train datasets (shaded points) while vertical bars represent the associated standard deviations.

2.4 Predictive performance from environmental values at ecological grain.

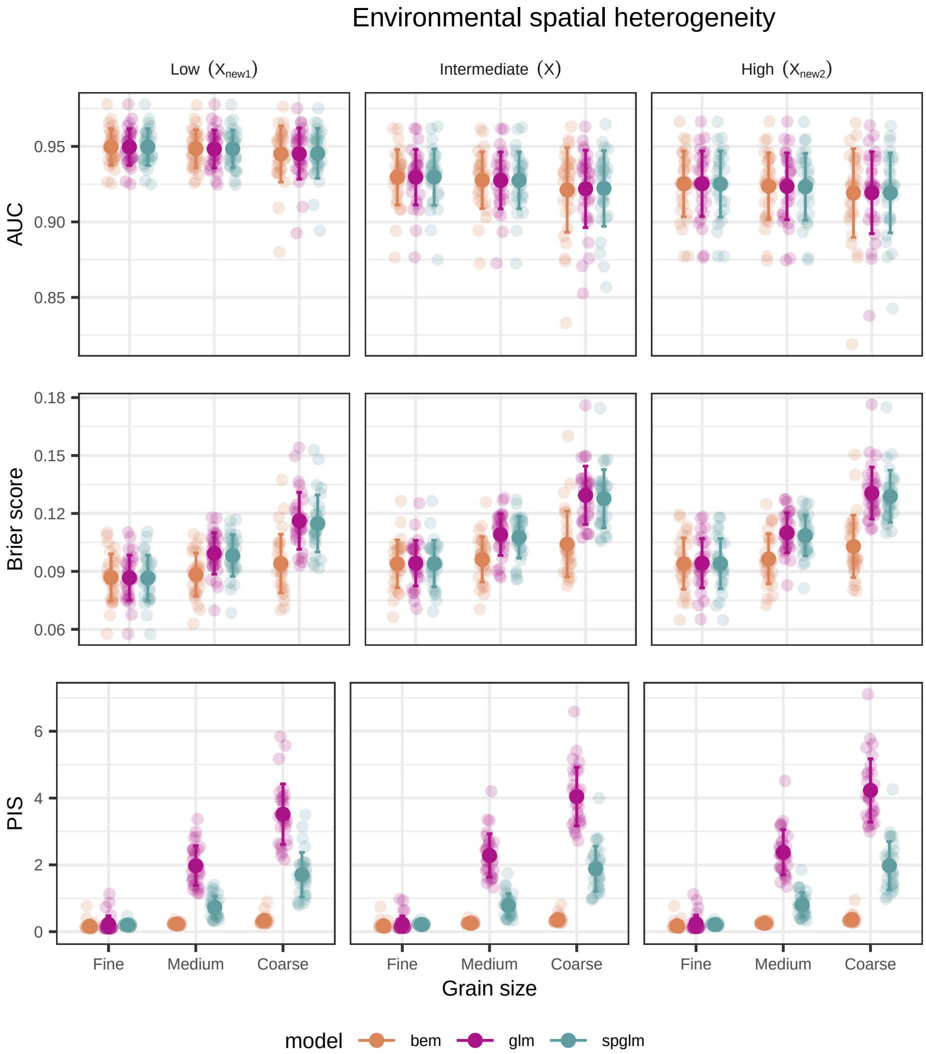

Figure S5: Evaluation of predictive performance of the three models fitted with area-to-point misaligned data with regards to their ability to predict species distribution at the ecological grain from environmental values at the **ecological grain** across three levels of environmental spatial heterogeneity (the three columns). Filled points represent mean performance metrics over the 30 simulated train datasets (shaded points) while vertical bars represent the associated standard deviations.

3 Loss of fine-grain heterogneity when coarsening environmental data.

For a given cell i we defined fine-grain heterogeneity (i.e. intra cell variability V_i^{intra}) as the variance in point-level environmental values $X_{j(i)}$ within the cell:

$$V_i^{intra} = \frac{1}{J_i - 1} \sum_{j=1}^{J_i} (X_{j(i)} - X_i)^2$$

with J_i the number of points within cell i and X_i the environmental value of cell i. To summarise it over the study area, we compute the mean:

$$\overline{V_i^{intra}} = \frac{1}{I} \sum_{i}^{I} V_i^{intra}$$

with I the number of cell in the study area. Finally to visualise the loss of fine-grain heterogeneity we plot the evolution of $1 - \overline{V_i^{intra}}$ with the increase of the grain size.

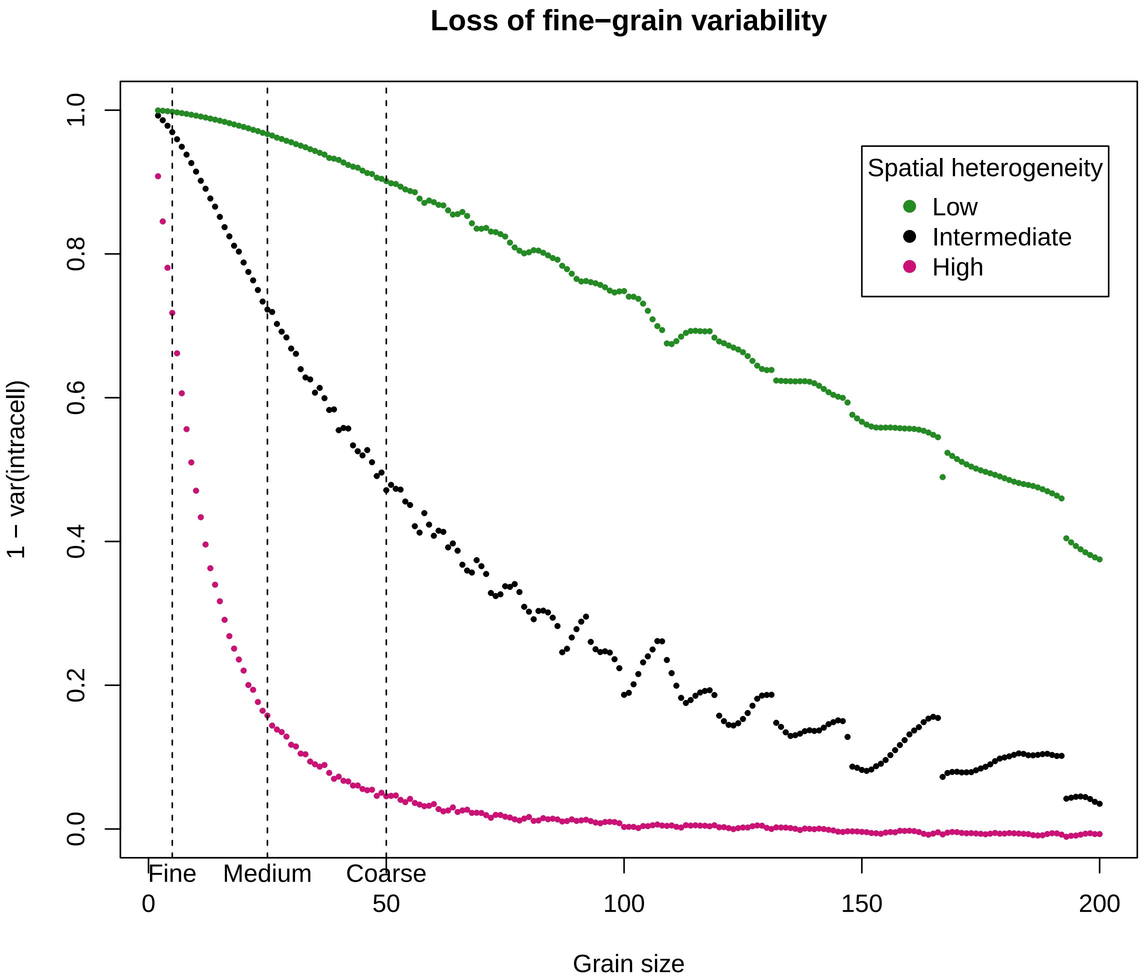

Figure S6: Loss of fine-grain variability in environmental values with increasing of covariate grain size for three environmental spatial heterogeneities. Vertical dashed bars indicate covariate grain sizes used in simulation.

4 Results of predictive performance assessment

Explanatory power

Table S1: Explanatory performance of the three models (BEM: Berkson-Error Model, GLM: Generalized Linear Model, spGLM: spatial GLM) fitted with simulated point-level presence-absence data (i.e. at ecological grain) and grid-level covariate resolved at different grain sizes coarser than the ecological grain (EG). Model performance was evaluated with regard to A) discrimination and B) calibration power by comparing the point-level presence-absence with the predicted point-level probabilities of presence from grid-level environmental values. Values represent mean performance metrics over the 30 simulated train datasets and values in brackets represent the associated standard deviations.

Covariate grain	AUC			Brier score		
	BEM	GLM	spGLM	BEM	GLM	spGLM
Fine (5 x EG)	0.92 (0.02)	0.92 (0.02)	0.94 (0.01)	0.1 (0.01)	0.1 (0.01)	0.09 (0.01)
Medium (25 x EG)	0.81 (0.02)	0.81 (0.02)	0.87 (0.02)	0.18 (0.02)	0.17 (0.01)	0.15 (0.01)
Coarse (50 x EG)	0.72 (0.03)	0.72 (0.03)	0.84 (0.02)	0.22 (0.02)	0.2 (0.01)	0.18 (0.01)

Predictive power

Table S2: Evaluation of predictive performance of the three models (BEM: Berkson-Error Model, GLM: Generalized Linear Model, spGLM: spatial GLM) fitted with a area-to-point misaligned data at three covariate grains. Predictive ability was assessed using three types of evaluation datasets presenting different levels of spatial heterogeneity in environmental conditions, which were either (i) less heterogeneous (X_{new1}), (ii) equally heterogeneous (X), or (iii) more heterogeneous (X_{new2}) than the fitting environment. Performance metrics were computed by comparing predicted probabilities of presence from covariate-grain environmental data with observed presence-absence. Different colours correspond to different models with respect to color-code in the previous figures (orange for the BEM, purple for the GLM and green for the spGLM). The first numbers represent mean performance metrics over the 30 simulated train datasets (shaded points). The numbers in brackets represent the associated standard deviations.

Covariate grain	Evaluation data	AUC			Brier Score			Predictive Interval Score (PIS)		
		BEM	GLM	spGLM	BEM	GLM	spGLM	BEM	GLM	spGLM
Fine (5 x EG)	Xnew1	0.95 (0.01)	0.95 (0.01)	0.95 (0.01)	0.09 (0.01)	0.09 (0.01)	0.09 (0.01)	0.21 (0.15)	0.28 (0.27)	0.21 (0.08)
Medium (25 x EG)	X	0.92 (0.02)	0.92 (0.02)	0.92 (0.02)	0.11 (0.01)	0.11 (0.01)	0.11 (0.01)	1 (0.22)	1.18 (0.24)	0.68 (0.14)
Coarse (50 x EG)	Xnew2	0.79 (0.03)	0.79 (0.03)	0.79 (0.03)	0.19 (0.02)	0.18 (0.01)	0.18 (0.02)	5.29 (0.63)	5.5 (0.57)	4.39 (0.6)
Fine (5 x EG)	Xnew1	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	0.1 (0.01)	0.11 (0.01)	0.11 (0.01)	0.78 (0.19)	2.67 (0.53)	1.22 (0.33)
Medium (25 x EG)	X	0.79 (0.03)	0.79 (0.03)	0.79 (0.03)	0.19 (0.01)	0.18 (0.01)	0.18 (0.01)	4.25 (0.41)	6.14 (0.53)	3.98 (0.36)
Coarse (50 x EG)	Xnew2	0.56 (0.04)	0.56 (0.04)	0.56 (0.04)	0.29 (0.02)	0.25 (0.02)	0.26 (0.02)	8.89 (0.87)	9.86 (0.62)	7.41 (0.75)
Fine (5 x EG)	Xnew1	0.9 (0.02)	0.9 (0.02)	0.9 (0.02)	0.13 (0.02)	0.14 (0.01)	0.14 (0.01)	1.61 (0.38)	4.86 (0.72)	2.8 (0.54)
Medium (25 x EG)	X	0.7 (0.03)	0.7 (0.03)	0.7 (0.03)	0.23 (0.02)	0.21 (0.01)	0.21 (0.01)	5.5 (0.63)	8.52 (0.44)	5.64 (0.35)
Coarse (50 x EG)	Xnew2	0.55 (0.03)	0.55 (0.03)	0.55 (0.03)	0.3 (0.03)	0.25 (0.01)	0.25 (0.01)	8.42 (1.12)	10.38 (0.67)	7.23 (0.6)