Supporting Information for "Distribution, Mixing, and Transformation of a Loop Current Ring Waters: The Case of Gulf of Mexico"

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• Text S1: The dissipation thermal variance rates (χ) (Osborn & Cox, 1972), were derived from spectra of temperature gradients (Ψ) and the Batchelor spectrum (Ψ_B) (Batchelor, 1959), through an iterative calculation as outlined by Scheifele et al. (2018):

$$\chi = \chi_{lW} + \chi_{obs} + \chi_{hW} = 6D_T \left(\int_0^{k_l} \Psi_B \, dk + \int_{k_l}^{k_u} \Psi \, dk + \int_{k_u}^{\infty} \Psi_B \, dk \right), \quad (1)$$

The iterative calculation process involves the following steps: (i) Fit the Batchelor spectrum, Ψ_B , to the observed spectrum Ψ for each iteration; (ii) Calculate χ_{obs} by integrating the observed spectrum, between the wavenumbers k_l and k_u (Fig. S2); (iii) For wavenumbers outside this range, where the observed spectrum is unreliable, we integrated Ψ_B to obtain the correction terms χ_{lW} and χ_{hW} ; (iv) The factor of 6 is introduced based on the assumption of isotropic flow, and D_T is the molecular diffusivity coefficient of temperature.

• Text S2: We use the parameterization developed by Middleton et al. (2021) to es-29 timate the dissipation rate associated with double-diffusive convection. This pa-30 rameterization works by estimating the turbulent buoyancy flux $\langle w'b' \rangle$, and as-31 suming it is in balance with the dissipation rate ε . They estimate the turbulent 32 buoyancy flux by using an assumption of balance in the variance equation for buoy-33 ancy, following Osborn and Cox (1972). In other words, they assume that the avail-34 able potential energy within the small scale turbulence is in a quasi-steady state, 35 so the primary balance is between the diapycnal buoyancy flux: 36

$$\Phi_d = \left\langle \frac{(\kappa_T + \kappa_S)}{2b_z^*} |\nabla b|^2 + \frac{(\kappa_T - \kappa_S)}{2b_z^*} \nabla b \cdot \nabla s_p \right\rangle,\tag{2}$$

and the turbulent buoyancy flux $\langle w'b' \rangle$, averaged over the space between observations. Here b_z^* is the adiabatically resorted buoyancy profile, and s_p denotes the 'spice', which is defined using a linear equation of state as $s_p = g\alpha T + g\beta S$ for the purposes of the parameterization.

The diapycnal buoyancy flux Φ_d is estimated from observations by assuming that 41 spice has a steeper spectral slope for its power spectrum than does buoyancy. So 42 the buoyancy gradient is estimated using observations of N^2 , and we assume a spec-43 tral slope of k^{-1} for the power spectrum of spice on sub-observational scales. The magnitude of the spice gradient at the overturning scale $|\nabla s_p|$ is estimated by fit-45 ting a power spectrum between each pair of observations using a two-point cor-46 relation along an isopycnal. The assumed slope of the spectrum can be altered to 47 account for lesser degrees of stirring of spice. Details on the iterative method used 48 to calculate Φ_d can be found in Middleton et al. (2021). This method assumes double-49 diffusive convection is present, as it relies on the second term of Φ_d which is purely 50 double diffusive (Notice for equal molecular diffusivities κ_T and κ_S , this term dis-51 sapears). The parameterization also assumes an anti-correlation between ∇b and 52 ∇s_p on overturning scales, which amounts to an assumption that double-diffusive 53 convection is present. 54

• Figure S1 shows the parameters used for the optimal multiparameter analysis: con-55 servative temperature (θ) , absolute salinity (S_A) , dissolved oxygen (O_2) , and po-56 tential vorticity (PV). The water mass types are defined as (quasi) continuous lines 57 in the parameter space considering the most characteristic values of the water masses 58 involved. We used the CTD cast data to find the characteristic parameter values 59 in the entire profile from the Loop Current Ring (LCR) center (red line) and those 60 taken outside of the LCR (blue line), based on the range defined in Portela et al. 61 (2018). We focused on complete profiles because we aim to examine the transi-62 tional waters between the pure LCR or Caribbean waters and the mature forms 63 of the Gulf waters, such as Gulf Common Water, as glider samples are collected 64 near the LCR boundary. 65

• Figure S2 shows temperature gradient spectra (Ψ) of randomly selected vertical profile sections. Integration limits for χ estimation are clearly marked by red circles (k_{min}) and red squares (k_{max}). Data points falling below the noise spectrum (red dotted line) have been excluded to prevent contamination by instrumental noise. Additionally, high-wavenumber data that deviate from the theoretical Batchelor spectrum (dashed lines) have been removed to avoid fine-scale contamination.

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- Figure S3 shows the histogram of the Ozmidov length scale, $(L_O = (\varepsilon/N^3)^{1/2})$, which is a metric of the size of the turbulent overturns. The mean size of the turbulent overturns is 1.67 m for all data (grey) and 0.17 m for data below the mixed layer (orange).
- Figure S4 shows the glider's cross-section of conservative temperature, absolute salinity and density anomaly, revealing interleaving layers of warmer, saltier water with cooler, fresher layers. These layers are steeper than isopycnals suggesting that advection or stirring by the mesoscale eddy may be shaping these structures (Meunier et al., 2019), supported by evidence that submesoscale processes primarily drive spice distribution (Fig. S6).
- Figure S5 describes the distribution of temperature staircases in the water column. 82 Double-diffusive convection (DDC) may be characterized by a Turner Angle of -83 $45/-90 \text{ rad}^{-1}$ and $45/90 \text{ rad}^{-1}$ for the diffusive convection (DC) and salt finger-84 ing (SF) conditions, respectively. The Turner angle shows two areas susceptible 85 to DDC conditions, (i) the thermohaline intrusions (blue square), and (ii) salt-fingers 86 favourable conditions (red square). In the blue square, spice anomalies are greater 87 than density anomalies (panel c), in average by a factor 2, which is a typical pat-88 tern of thermohaline intrusions or layering (Meunier et al., 2019). These structures 89 present some thermohaline staircases reaching up to 5 m of vertical length (panel 90 d), which are smaller than the spice anomalies (up to 20 m). A second area, be-91

low 200 m depth, shows SF conductive conditions (red square in panel b). As shown
in panel e, spice anomalies are smaller or compensated by density anomalies. High resolution temperature profiles from the thermistor reveal indistinct thermoha line staircases (panel f).

• Figure S6 shows the averaged power spectral density of spice anomalies variance, 96 calculated within the isopycnal range from 24.7 kg m^{-3} to 26.1 kg m^{-3} . The slope 97 of $k^{-2.2}$ at high wavenumbers range $10^{-4.5} < k_h < 10^{-3.5} \text{ m}^{-1}$ (wavelengths 3-30 km) is slightly steeper than the expected k^{-2} for quasigeostrophic turbulence. 98 99 This suggests that spice anomalies are stirred around a coherent vortex, as observed 100 in a similar structure by Meunier et al. (2019). Furthermore, the flatter slope $k^{-2.2}$ 101 observed at high wavenumbers, compared to the typical enstrophy cascading in 102 two-dimensional turbulence (k^{-3}) , indicates that the spice distribution in the LCR 103 is primarily driven by submesoscale processes. 104

• Figure S7 shows shipboard-averaged measurements from the R/V Pelican across the eddy. The stratification is stronger than the vertical shear (by an order of magnitude), resulting in a high Richardson number (Ri > 1), which indicates a dynamically stable water column. Additionally, the potential vorticity (PV) remains positive, suggesting that conditions are not conducive to symmetric instabilities (PV * f < 0).

• Figure S8 shows the buoyancy Reynolds number (R_{eb}) estimated from the verti-111 cal microstructure profiler, the glider-microstructure and the DDC parameteri-112 zation from Middleton et al. (2021). R_{eb} is calculated as the ratio of the dissipa-113 tion rates, which promotes vertical overturns, to the potential energy of stratifi-114 cation, which suppresses these overturns. A threshold for the buoyancy Reynolds 115 number is ~ 10 ; values below this threshold generally indicate that diapycnal tur-116 bulent mixing is suppressed (Stillinger et al., 1983; Shih et al., 2005; Ivey et al., 117 2008; Bouffard & Boegman, 2013). A large number of estimates, ranging from $\sim 63\%$ 118 to 77%, occurred under conditions where $Re_b < 10$, regardless of the measurement 119 platform. This suggests that stratification suppresses shear-productions in most 120 cases, indicating that turbulent fluxes are predominantly driven by DDC. How-121 ever, a bimodal distribution is observed for vertical profiler and glider estimates, 122 with a peak in the turbulent regime $Re_b > 10$, mainly induced by intense mixing 123 in the surface mixed layer. The bimodal distribution is not captured by double-124 diffusive convection parameterization $(Re_{b_{Pred}})$, because it fails to represent shear-125 driven mixing or internal wave breaking. 126



Figure S1. Water mass types definition in the parameter space (thick lines) used for the optimal multiparameter analysis. The blue line represents the Gulf waters outside the LCR and the red line shows the LCR's core waters. Black dots are the CTD data used to separate the profiles inside from outside the eddy. (a) θ - S_A , (b) θ - O_2 , and (c) θ -PV diagrams.



Figure S2. Temperature gradient spectra (Ψ) of randomly selected vertical profile sections from the thermistor. The spectra of temperature gradients were integrated between k_{min} (red circles) and k_{max} (red squares). The dotted lines represent the corresponding empirical spectra obtained through fitting to the Batchelor spectrum. The red dotted line indicates the thermistor's theoretical noise curve.



Figure S3. Histogram of the Ozmidov length scale $(L_O; m)$, with all data in grey and only data below the mixed-layer in orange. For both datasets, mean values are indicated as $\overline{L_O}$.



Figure S4. Glider cross-section of (a) conservative temperature, (b) absolute salinity, and (c) density anomaly in the layering region (eddy periphery). Isopycnal layers are represented by black contours.



Figure S5. Glider section of (a) squared Brunt-Väisälä frequency and (b) Turner angle, where regions that are susceptible to double-diffusive convection are indicated by values of -45/-90 rad⁻¹ (double-convection: light blue), and 45/90 rad⁻¹ (salt-finger: yellow). Regions of thermohaline intrusions (blue square) and salt finger conditions (red square) were highlighted. (c) and (e) are one of the spice (blue) and density (red) anomaly profiles from the blue and red squares in (b) respectively. (d) and (f) are selected temperature profiles (presented as relative temperature, shifted by and offset of 0.5° C) recorded by the FP07 fast thermistor in the blue and red squares in (b), respectively. The blue and red profiles are those represented in (c) and (e), respectively.



Figure S6. Power spectral density in isopycnal coordinates of spice variance anomalies. Spectra are averaged over the isopycnal range from 24.7 kg m⁻³ to 26.1 kg m⁻³. The red line represents the linear fit of the spectra $10^{-4.5} < k_h < 10^{-3.5}$ m⁻¹ in the wavenumber range (wavelengths of 3-30 km), and the k^{-2} slope is marked by the dashed black line.



Figure S7. Shipboard averaged measurements from the R/V Pelican across the eddy: (a) squared Brunt-Väisälä frequency, (b) vertical shear from L-ADCP, (c) Richardson number, and (d) potential vorticity. The blue, green, and orange lines and colored dots represent measurements taken at different locations: outside the eddy, at its periphery, and at its core, respectively.



Figure S8. (a, b, c) Log-histograms comparing predicted buoyancy Reynolds number $(Re_{b_{Pred}})$ from double-diffusive convection parameterization (Middleton et al., 2021), with estimates from microstructure $(Re_{b_{MicT}})$ and VMP $(Re_{b_{VMP}})$, covering areas outside the eddy (a), its periphery (b), and center (c), respectively.

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