Supporting Information for "Dense water production in Storfjorden, Svalbard, from a one-year time series of observations and a simple model: Are polynyas in a warming Arctic exporting heat to the deep ocean?"

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Introduction The supporting information presents the details of the box model with linear stratification developed to estimate the dense water production in the polynya, taking advantage of the continuous salinity observation from the mooring. The model is explained in Text S1 whereas additional details regarding the derivation of the model's

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equations are provided in Text S2. The supporting information also includes in Text S3 and Figure S1 an analysis of the bottom boundary layer at the mooring's location based on the classical Ekman boundary layer theory.

Text S1. A linearly stratified box model for dense water production.

This simple model to estimate the volume of BSW produced in the polynya takes advantage of the fact that salinity profiles are continuously measured over the freezing period. We allow both the surface salinity $S_o(t)$ and bottom salinity $S_b(t)$ to vary in time according to observations. An inflow v_i of surface water with constant salinity $S_o^\star$, set to the initial salinity at the onset of the freezing period compensates for the bottom outflow with volume flux v_e such that $\rho_b v_e = \rho_o^\star v_i$ for mass conservation. Rather than taking a box model with constant salinity layers, we use a linear stratification entirely defined by the surface and bottom values (Figure 3c),

$$S(z) = S_o - \frac{\tilde{S}}{h} z \tag{1}$$

$$\rho(z) = \rho_o - \frac{\rho_o \beta \tilde{S}}{h} z, \tag{2}$$

where $\tilde{S}(t) = S_b - S_o$, $\beta = -1/\rho \left(\partial\rho/\partial S\right)_{T,p}$ is the haline contraction coefficient ($\beta \approx 7.5 \times 10^{-4}$) and h is the depth of the domain. The domain has an area $\mathcal{A}$ and a liquid volume $\mathcal{V} = \mathcal{A} \times h$. The BSW reservoir is defined as the layer with salinity exceeding the threshold value $S_c = 34.8$, which in the case of a linear salinity profile lies below the depth $z = -h_c$ defined, when $S_o(t) \leq S_c \leq S_b(t)$, by

$$\frac{h_c(t)}{h} = \frac{S_c - S_o}{S_b - S_o}. \tag{3}$$

The volume of the BSW reservoir is therefore

$$V_b(t) = \begin{cases} 0 & \text{if} \quad S_b(t) \leq S_c \\ \mathcal{V}\frac{S_b - S_c}{S_b - S_o} & \text{if} \quad S_o(t) \leq S_c \leq S_b(t) \\ \mathcal{V} & \text{if} \quad S_o(t) \geq S_c. \end{cases} \tag{4}$$

Using mass and salt conservation (see Text S2), the exported BSW volume between t and $t + \Delta t$ can be expressed as the difference between the flux of salt into the ocean due to freezing, F_S, and the salt content variation of the liquid ocean, D_S:

$$v_e(t) = \begin{cases} 0 & \text{if} \quad S_b(t) < S_c \\ \frac{1}{\rho_b(S_b(t) - S_o^\star)} \left(F_S(t) - D_S(t)\right) & \text{otherwise.} \end{cases} \tag{5}$$

We used the simplified expression (8) of the main manuscript for F_S whereas the term D_S, obtained from the mooring's record, reads (Text S2):

$$D_S = \mathcal{V}\rho_o \left(\Delta S_m + \epsilon \left(\frac{3\Delta S_o - \Delta S_b}{2} \right) \right), \tag{6}$$

where $S_m(t) = (S_o + S_b)/2$ and $\epsilon = \beta\tilde{S}/2$. Since $\epsilon \ll 1$, the last term can be ignored.

The total production of BSW over the freezing season $[t_0, t_f]$ is therefore

$$V_{bsw}(t_f) = \sum_{t=t_0}^{t_f} v_e(t) + V_b(t_f), \tag{7}$$

where the last term is the remnant volume of BSW in the fjord interior at the end of the freezing period as defined by Equation (4).

The model is of course imperfect. Many processes are neglected, such as exchanges of waters with salinity different than $S_o^\star$ or S_b. We assume that the water coming in the surface has constant salinity, hence that no water mass transformation is taking place before this water reaches the Storfjorden polynya region. The main issue is perhaps errors in the two forcing terms: $F_S(t)$, which is obtained from a polynya model, includes errors and $D_S(t)$, which represents changes in the salt content of the entire basin, assumed to be spatially homogeneous, is estimated from measurements taken at a single location. These errors are difficult to quantify. They become obvious, however, when $F_S(t) - D_S(t) < 0$ since this situation yields an unphysical re-entering BSW flux ($v_e < 0$). We therefore split the flux into two parts. The corrected V_{bsw} reads

$$V_{bsw}(t_f) = \sum_{t=t_0}^{t_f} v_e^+(t) + V_b(t_f), \tag{8}$$

with

$$v_e^+(t) = \begin{cases} v_e(t) & \text{if} \quad v_e \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

The complementary part provides a rough metrics for the error

$$V_{err}(t_f) = \sum_{t=t_0}^{t_f} v_e^-(t) \qquad \text{with} \quad v_e^-(t) = v_e^+(t) - v_e(t). \tag{10}$$

The tuning of the model consists in minimizing V_{err} by low-pass filtering the input variables S_o and S_b. Low-pass filtering is justified as the model does not include transients and assumes instantaneous adjustment throughout the entire basin. The cut-off period T of the low-pass filter was gradually increased up to 80 days. The error metrics V_{err} sharply drops with increasing T from 1250 km^3 for no filtering to 270 km^3 for T=14 days, plateauing afterwards (not shown). We thus retained this cut-off period of 14 days to filter S_o and S_b.

Additionally we define V_{light} as the cumulative exported volume of dense water with salinity smaller than S_c, which therefore does not enter the definition of BSW.

$$V_{light}(t_f) = \sum_{t=t_0}^{t_f} \frac{F_S(t) - D_S(t)}{\rho_b(S_b(t) - S_o^\star)} \left(1 - \mathcal{H}(S_b(t) - S_c)\right), \tag{11}$$

where $\mathcal{H}$ is the Heaviside step function.

Text S2. Derivation of the terms of the box model for BSW production.

This section details the derivation of Equation (5) that gives the exported volume of BSW. The mass and salt budgets at time t are obtained after vertical integration of Equations (2) and (1):

$$M(t) = \rho_i V_i + \mathcal{A}\rho_o \int_{-h}^{0} (1 - \frac{\beta \tilde{S}}{h} z)\, dz \tag{12}$$

$$M_s(t) = \rho_i S_i V_i + \mathcal{A}\rho_o \int_{-h}^{0} \left[S_o - \frac{\tilde{S}}{h}(1 + \beta S_o) z \right] dz, \tag{13}$$

which gives, neglecting the quadratic term in z:

$$M(t) = \rho_i V_i + \mathcal{V}\rho_o \left(1 + \frac{\beta \tilde{S}}{2} \right) \tag{14}$$

$$M_s(t) = \rho_i S_i V_i + \mathcal{V}\rho_o \left(S_o + \frac{\tilde{S}}{2}(1 + \beta S_o) \right) \tag{15}$$

Writing $S_m = (S_o + Sb)/2$ and $\epsilon = \beta \tilde{S}/2 \ll 1$, the previous equations become

$$M(t) = \rho_i V_i + \mathcal{V}\rho_o (1 + \epsilon) \tag{16}$$

$$M_s(t) = \rho_i S_i V_i + \mathcal{V}\rho_o \left(S_m + \epsilon S_o \right) \tag{17}$$

Conservation of mass and salt between times t and $t + \Delta t$ yields

$$0 = \rho_i \Delta V_i + \mathcal{V}\rho_o \beta \left(\Delta S_m + \epsilon \Delta S_o \right) + \Delta \mathcal{V} \rho_o \left(1 + \epsilon \right) \tag{18}$$

$$-v_e \rho_b (S_b - S_o^\star) = \rho_i S_i \Delta V_i + \mathcal{V}\rho_o \left(\Delta S_m + \epsilon \Delta S_o + S_o \Delta \epsilon \right) + \Delta \mathcal{V} \rho_o \left(S_m + \epsilon S_o \right)$$
$$+ \mathcal{V}\rho_o \beta \left(S_m + \epsilon S_o \right) \Delta S_o, \tag{19}$$

where we have assumed $\rho_o^\star v_i = \rho_b v_e$ for mass conservation in (18). In the derivation we have further assumed that ρ_i and S_i are both constant ($S_i = \alpha S_o^\star$), and we have used the relation $\Delta \rho_o = \rho_o \beta \Delta S_o$. Equation (18) leads to

$$\Delta \mathcal{V} \rho_o \approx (1 - \epsilon) \left[-\rho_i \Delta V_i - \mathcal{V}\rho_o \beta \left(\Delta S_m + \epsilon \Delta S_o \right) \right]. \tag{20}$$

Substituting into Equation (19) and neglecting terms $\mathcal{O}(\epsilon^2)$ and higher yields

$$v_e \rho_b (S_b - S_o^\star) = \rho_i \Delta V_i \left[\left(S_m - \epsilon \frac{\tilde{S}}{2} \right) - S_i \right] - \mathcal{V}\rho_o \left(\Delta S_m + \epsilon \Delta S_o + S_o \Delta \epsilon \right)$$
$$+ \mathcal{V}\rho_o \beta S_m \frac{\Delta \tilde{S}}{2}, \tag{21}$$

which simplifies to

$$v_e\rho_b(S_b - S_o^\star) = \underbrace{\rho_i\Delta V_i\left[\left(S_m - \epsilon\frac{\tilde{S}}{2}\right) - S_i\right]}_{F_S} - \underbrace{\mathcal{V}\rho_o\left(\Delta S_m + \epsilon\left(\frac{3\Delta S_o - \Delta S_b}{2}\right)\right)}_{D_S}. \quad (22)$$

This is Equation (5) of the model, in which, however, a simplified expression of F_S is actually used.

At the end of the freezing period, F_S goes to zero and the flux is then, neglecting terms $\mathcal{O}(\epsilon)$, $v_e = -\mathcal{V}\Delta S_m/\tilde{S}^\star$ from then on until the BSW reservoir has drained out Quite logically, we can conveniently stop the integration at any time after the freezing period and add the remaining BSW volume $V_b(t_f)$ as is done in Equation (7) to get the total BSW, as we expect V_{bsw} to plateau after the freezing period if we assume that $v_e = -\Delta V_b$. This is not the case. Derivation of Equation (4) yields

$$\Delta V_b = \mathcal{V}\frac{1}{\tilde{S}}\left(\Delta S_m - \frac{S_m - S_c}{\tilde{S}}\Delta\tilde{S}\right) \quad (23)$$

Neglecting terms $\mathcal{O}(\epsilon)$

$$v_e + \Delta V_b = \mathcal{V}\left[\left(\frac{1}{\tilde{S}} - \frac{1}{\tilde{S}^\star}\right)\Delta S_m - \frac{S_m - S_c}{\tilde{S}^2}\Delta\tilde{S}\right] \quad (24)$$

which is not identically zero as one would intuitively expect, first because the salinity of the inflow water $S_o^\star$ is different from the surface salinity in Storfjorden $S_o(t)$.

Text S3. Bottom Boundary Layer. The progressive vector diagrams in Figure 6 indicate substantial veering with depth for the currents recorded at 90 m, that is 10 m above the bottom (mab), compared to the currents recorded by the ADCP at 82 m depth and above. We examine the consistency of this near bottom deflection of the current direction with the classical Ekman boundary layer theory. Assuming a steady flow in a homogeneous fluid of constant eddy viscosity ν, the complex horizontal velocity $\mathcal{U} = u+iv$ reads

$$\mathcal{U}(\zeta') = \mathcal{U}_\infty \left(1 - e^{-i\zeta'} e^{-\zeta'}\right), \tag{25}$$

where $\zeta' = \zeta/\delta_E$ is the height above the bottom scaled by the Ekman layer thickness $\delta_E = \sqrt{2\nu/f}$, and $\mathcal{U}_\infty$ is the interior geostrophic velocity, away from the influence of bottom friction ($\zeta \gg \delta_E$). Equation (25) is that of the classical Ekman spiral which shows a cyclonic deflection of the currents with depth (counterclockwise rotation with decreasing height). It is worth recalling that the deflection of the currents is not always cyclonic and depends on the frequency of the current for a periodic flow. While it is cyclonic for a steady flow, as considered here, it may be anticyclonic for a periodic current of period $\omega > f$ (Kundu et al., 1981).

The average horizontal current fits relatively well with an Ekman spiral for the period ranging from February to April 2017, when the water column is almost perfectly homogeneous, filled with BSW (Figure S1, left). A reasonable fit to (25) is obtained for a viscosity $\nu = 5 \times 10^{-3}\mathrm{m}^2\mathrm{s}^{-1}$, yielding a bottom layer thickness of 8.4 m. A fit to an Ekman spiral is less convincing for other periods of the year when the water column is more stratified and average currents at depth smaller, suggesting more complex bottom boundary layer dynamics (Figure S1, right).

References

Kundu, P. K., Blanton, J. O., & Janopaul, M. M. (1981). Analysis of current observations on the Georgia shelf. *Journal of Physical Oceanography*, *11*(8), 1139 - 1149. doi: 10.1175/1520-0485(1981)011⟨1139:AOCOOT⟩2.0.CO;2

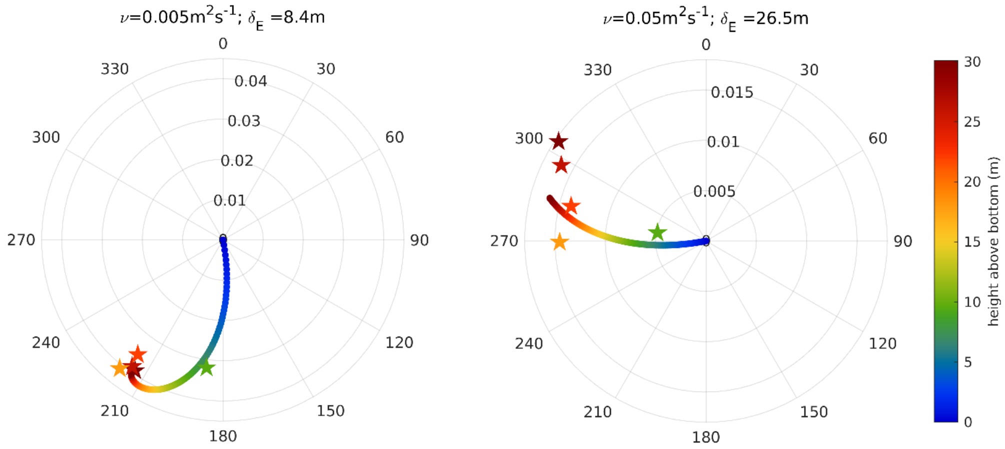

Figure S1. Mean horizontal currents at different heights above the bottom for the period February to April 2017 (left) and September to November 2016 (right) with a fitted Ekman spiral superimposed.