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Supporting Information for ‘Diagnostic of Ocean Near-Surface Horizontal Momentum Balance from pre-SWOT altimetric data, drifter trajectories, and wind reanalysis’

Margot Demol[1], Aurélien Ponte[1], Pierre Garreau[1], Jean-François Piollé[1],

Clément Ubelmann[2], Nicolas Rascle [1]

[1]Ifremer, LOPS, Plouzané, France

[2]Datlas, Grenoble, France

Contents of this file

1. Texts S1 to S2
2. Figures S1 to S4

Introduction This supporting information document contains demonstration of equations (8) of the article in Text S1 as well as a theoretical development for the predictions of the impact of colocation and scaling errors on the momentum balance reconstructions in Text S2. Temporal mismatch sensitivity is shown on Figure S1 and geographical statistical errors on the residual MS on Figure S2. Figures S3 and S4 illustrate Text S2.

Corresponding author: M. Demol, margot.demol@ifremer.fr

Text S1. Statistics : balanced and residual contribution decomposition's demonstration This section aims at demonstrating the following equations :

$$\begin{cases} \beta_i = \dfrac{1}{2}(A_i - \mathcal{E} + \mathcal{E}_{-i}) \\ \mathcal{E}_i = \dfrac{1}{2}(A_i + \mathcal{E} - \mathcal{E}_{-i}). \end{cases} \tag{1}$$

From the definitions of the residual MS and balanced signal contributions we get:

$$\begin{cases} A_i = \langle a_i(\epsilon - \epsilon_{-i})\rangle = \mathcal{E}_i + \beta_i \\ \\ \mathcal{E} = \langle (a_i + \epsilon_{-i})^2\rangle = A_i + \mathcal{E}_{-i} + 2\langle a_i \epsilon_{-i}\rangle = A_i + \mathcal{E}_{-i} - 2\beta_i. \end{cases} \tag{2}$$

Combining the two equations of the system (2) finally leads to (1).

Text S2. Errors impact on momentum balance reconstruction

This section aims at deriving the impact of different type of errors on the momentum balance reconstruction diagnostic variables (e.g. $\mathcal{E}$, β, β_i, $\mathcal{E}_i$, $X_{i,j}$). The case of colocation errors and scaling errors is considered in deeper details (section 2 and 3 respectively).

1. General case

Consider two different reconstructions that distinguish themselves by the estimation of the k-term:

$$\sum_i a_i = \epsilon \tag{3}$$

$$\sum_{i \neq k} a_i + a_k^* = \epsilon^* \tag{4}$$

where the a_k estimate of the k-term of (4) is replaced by the a_k^* estimate. Denoting all of the metrics related with reconstruction (4) with an * and the difference in between these two estimates $d_k = a_k^* - a_k = \epsilon^* - \epsilon$ (i.e. the additional error), we can derive the effect induced by this difference on the different relevant metrics :

$$\beta^* = \beta - 2\langle d_k \epsilon_{-k} \rangle \tag{5}$$

$$\mathcal{E}^* = \langle (\epsilon + d_k)^2 \rangle \tag{6}$$

$$= \mathcal{E} + D_k + 2\langle d_k \epsilon \rangle. \tag{7}$$

For $i \neq k$:

$$\begin{cases} \beta_i^* = -\sum_{j \neq i} \langle a_i a_j \rangle - \langle d_k a_i \rangle = \beta_i - \langle d_k a_i \rangle \\ \mathcal{E}_i^* = \langle a_i \epsilon^* \rangle = \mathcal{E}_i + \langle d_k a_i \rangle. \end{cases} \tag{8}$$

For $i = k$:

$$\begin{cases} \beta_k^* & = -\sum_{j \neq k} \langle a_k^* a_j \rangle = \beta_k - \sum_{j \neq k} \langle d_k a_j \rangle = \beta_k - \langle d_k \epsilon_{-k} \rangle \\ \mathcal{E}_k^* & = \langle a_k^* \epsilon^* \rangle = \langle (a_k + d_k)(\epsilon + d_k + a_k - a_k) \rangle = \langle a_k(s - a_k) + d_k(s - a_k) + (a_k + d_k)^2 \rangle \\ & = \mathcal{E}_k + \langle d_k \epsilon_{-k} \rangle + A_k^* - A_k \\ & = \mathcal{E}_k + D_k + \langle d_k(\epsilon + a_k) \rangle. \end{cases} \tag{9}$$

For i,j $\neq$ k :

$$X_{i,j}^* = X_{i,j}. \tag{10}$$

For $i = k$ and $j \neq k$:

$$X_{k,j}^* = X_{k,j} - 2\langle d_k a_j \rangle. \tag{11}$$

Eq.(5) to (11) points toward several expected and desired properties:

- As d_k vanishes, all diagnostic variables associated with (4) converge towards those associated with (3).

- When the additional error d_k is uncorrelated with all terms from (4) and, consequently, its residual, the residuals $\mathcal{E}$ and $\mathcal{E}_k$ are the sole diagnostics affected and the modification consists in the addition of the positive definite D_k term, e.g. $\mathcal{E}^* = \mathcal{E} + D_k$

and $\mathcal{E}_k^* = \mathcal{E}_k + D_k$. Importantly, this points toward the fact that correlated errors are necessary in order to alter balanced signal components (e.g. β^*, β_i^*).

- When the alternative formulation of the k-term a_k^* is uncorrelated with all terms from (4), i.e. a terrible estimate, the paired contributions concerned by the k-term are null ($X_{ki}^* = 0$), so the balanced signal contribution of the k-term is null too ($\beta_k^* = 0$) whereas its residual contribution is equal to its MS ($\mathcal{E}_k^* = A_k^*$). In consequence, the balanced component decreases ($\beta^* = \beta - 2\beta_k$) and the residual MS is $\mathcal{E}^* = A_k^* + \mathcal{E}_{-k}$. Otherwise, we also have for $i \neq k$ the following relationships $\beta_i^* = \beta_i - 1/2X_{ki}$ and $\mathcal{E}_i^* = \mathcal{E}_i + 1/2X_{ki}$.

- As far as paired contributions are concerned, the impact of the modification of term a_k is only felt on paired contributions involving a_k.

2. Colocation error case

We first consider that the reconstruction (3) is a reconstruction where the estimation of a_k is free of colocation error i.e. it is estimated at the same position and time than other terms. Then, in reconstruction (4), we were only able to approach the k-term of (3) by its estimate at a different position and/or time a_k^*, introducing some colocation error in the reconstruction. This is typically what happens when reconstructing the momentum conservation with along-track altimetry and drifter trajectories: in this case, the k-term is the pressure gradient term, that we were able to estimate at the altimeter matchup but not at the drifter-matchup.

As shown by Figures 3c and 3d, both the residual and the balanced signal contributions $\mathcal{E}_i$ and β_i are sensitive to colocation errors. These sensitivities are also mirrored, as predicted by the equations (8) and (9) : the residual contribution of a term increases as much as its balanced signal contribution decreases. The colocation error d_k is thus necessarily correlated to the other terms. The residual MS consequently grows with the spatial mismatch (Figure 3a), and according to Eq.(7), this increase can be explained by two terms only: D_k, which is here the second order spatial structure function of the pressure gradient, and a correlation term $2\langle d_k \epsilon \rangle$. To investigate the composition of colocation errors in details we compare AVISO-altimeter-matchup reconstruction (i.e. with colocation error) and AVISO-drifter-matchup reconstruction (i.e. without colocation error). This analysis shows that in Eq.(7) it is the second order spatial structure function of the pressure gradient D_k that dominates the residual MS over the correlation term $2\langle d_k \epsilon \rangle$ and controls its increase for spatial mismatches larger than about 10 km (Figure S3). Regarding residual contributions, colocation errors mainly affect those associated with the pressure gradient and the Coriolis acceleration (no shown) as for reconstructions with along-track data (Figure 3).

3. Scaling error $d_k = (\alpha - 1)a_k$ case

We know consider that we introduced a scaling error that misestimates a_k with a factor α, i.e. $a_k^* = \alpha a_k$. This may have happens taking global parameters in the Rio, Mulet, and Picot (2014)'s model for the wind term. Taking $d_k = (\alpha - 1)a_k$ in section 1, we get for the residual MS :

$$\mathcal{E}^* - \mathcal{E} = (\alpha^2 - 1)A_k + 2(\alpha - 1)\sum_{j \neq k} \langle a_k a_j \rangle. \tag{12}$$

Scaling error also affect the balanced signal signal contributions and residual contributions :

For $i \neq k$:

$$\begin{cases} \beta_i^* = \beta_i + (1-\alpha)\langle a_i a_k \rangle \\ \mathcal{E}_i^* = \mathcal{E}_i - (1-\alpha)\langle a_i a_k \rangle . \end{cases} \tag{13}$$

For $i = k$:

$$\begin{cases} \beta_k^* = \beta_k - (1-\alpha)\beta_k \\ \mathcal{E}_k^* = \mathcal{E}_k + (\alpha^2 - 1)A_k + (\alpha - 1)\sum_{j \neq k} \langle a_j d_k \rangle . \end{cases} \tag{14}$$

And pairs contributions related to the k-term : For $i, j \neq k$:

$$X_{i,j}^* = X_{i,j}. \tag{15}$$

For $i = k, j \neq k$:

$$X_{k,j}^* = X_{k,j} - 2(1-\alpha)\langle a_j d_k \rangle . \tag{16}$$

The effect of applying a factor 0.5 and 1.5 on the wind term has been tested and give the reconstruction described by Figure S4.

References

Rio, M.-H., Mulet, S., & Picot, N. (2014, December). Beyond GOCE for the ocean circulation estimate: Synergetic use of altimetry, gravimetry, and in situ data provides new insight into geostrophic and Ekman currents: Ocean circulation beyond GOCE. *Geophysical Research Letters*, *41*(24), 8918–8925. Retrieved 2022-12-22, from `http://doi.wiley.com/10.1002/2014GL061773` doi: 10.1002/2014GL061773

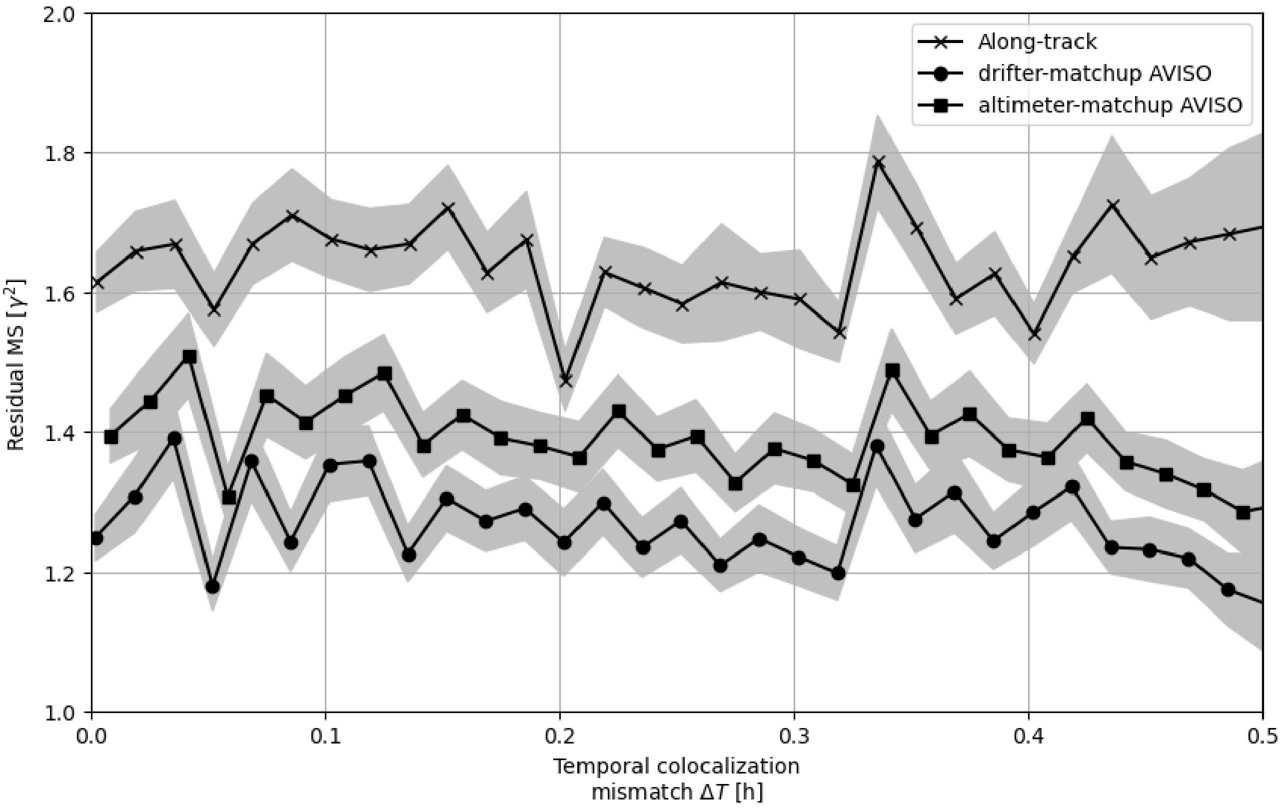

Figure S1. Dependency of the mean square value (MS) of the residual on temporal colocation mismatch ΔT. Residual MS are averaged over colocations in one minute temporal mismatch bins ($|\Delta T - dt| < 1$ min for given dt). Residual MS shows no clear tendency while the temporal mismatch increases and its variations of order 0.1 γ^2 can be related to statistical noise.

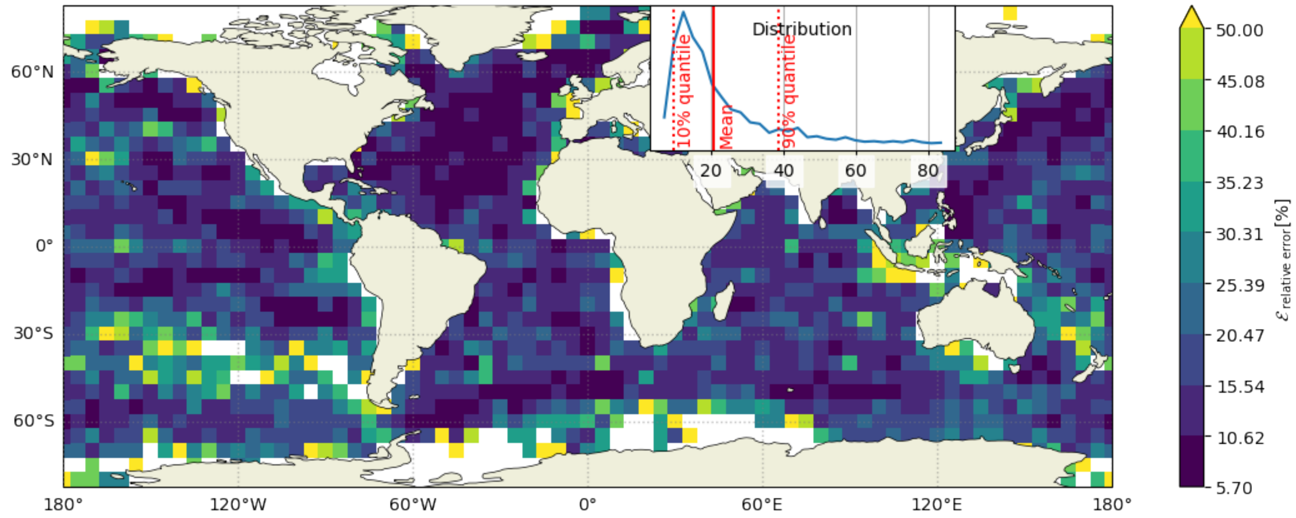

Figure S2. Relative statistical error on residual MS mapped in 5°-geographical bins. Statistical errors on the residual are computed with the bootstrap method and normalized by the binned residual value. Only bins below 50% are represented

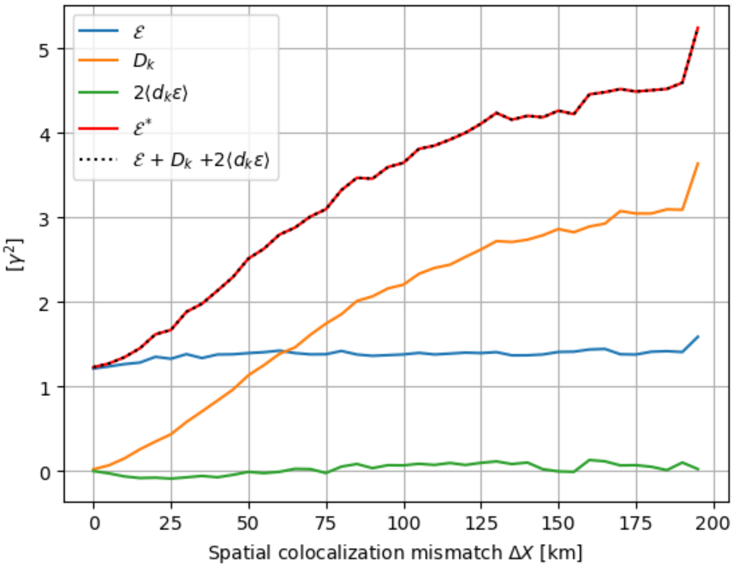

Figure S3. Illustration of the impact of colocation errors on the residual MS. $\mathcal{E}$ is the residual MS for the drifter-matchup AVISO reconstruction and $\mathcal{E}^*$ the residual MS for the altimeter-matchup AVISO reconstruction (with colocation errors). The difference in between these two residual is explained by the terms D_k and $2\langle d_k \epsilon$, also plotted.

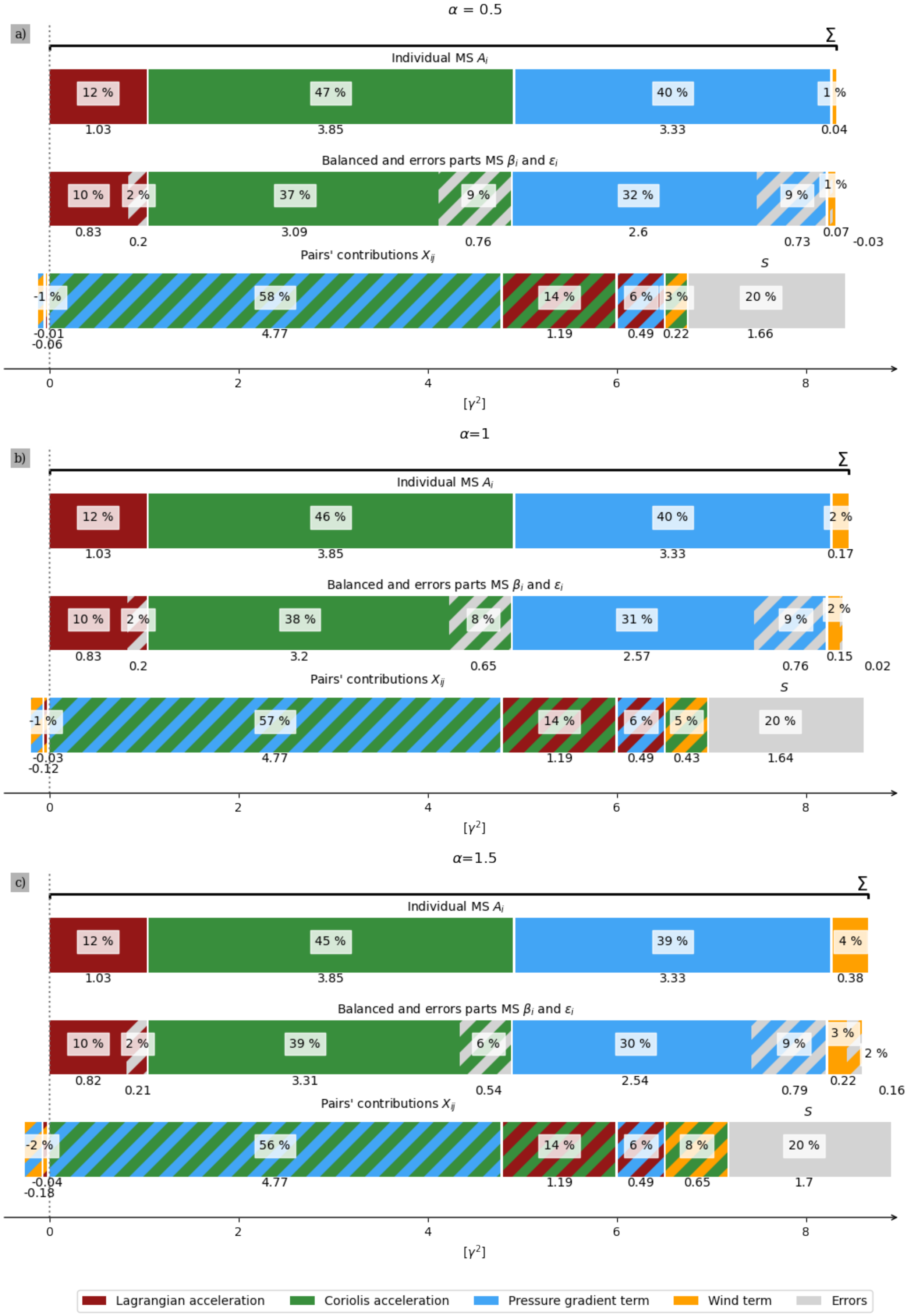

Figure S4. Impact of scaling errors on the wind term on the along-track reconstruction : a) applying a factor 0.5, b) no factor, reference case, c) applying a factor 1.5