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WAVES
Session 3

A STATISTICAL RELATIONSHIP BETWEEN INDIVIDUAL HEIGHTS AND PERIODS OF STORM WAVES

CONTRIBUTION

By A. Cavanié M. Arhan and Ezraty, Centre Océanologique de Bretagne, France

Introduction

– The goal of this short communication is to present a theoretical joint probability distribution for wave heights and periods developped in 1975 at the Centre Océanologique de Bretagne. This research program was proposed and financially supported by the Association de Recherche Action des Elements (group formed of 9 french companies or government agencies) which also carried out analysis of 200 twenty minute storm wave recordings in the North Sea selected from Waverider data kindly placed at our disposal by the United Kingdom Offshore Operators Association. –

To stay within the limits of alloted space, mathematical developments will be cut short and physical guidelines, as well as highlights of the results stressed. Further information may be obtained from the A.R.A.E. internal report actually in print [1] .

I - The "zero up-crossing wave" concept in view of storm wave spectra

Engineers for a number of reasons, are attached to the individual wave concept while oceanographers have, to a large extent, abandonned it in favor of spectral models. Cutting off that 10 % of the total energy in the high frequency tail of Pierson-Moskowitz spectra [3] leads to spectra with a constant small value of the width parameter ϵ ; experimentally 152 storm wave spectra in the North Sea submitted to such a cut-off gave for ϵ a value of 0.463 ± 0.32 quite close to that of 0.425 for the P.M. spectrum. This indicates that storm spectra are in practice "narrow" so that zero up-crossing wave analysis can be applicable.

(*) C.N.E.X.O. Centre Océanologique de Bretagne B.P. 337 29273 BREST cédex (France)
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II - The theoretical joint probability distribution

A sinusoidal wave may be defined, either by its amplitude and period, or by its amplitude and second derivative with respect to time at the crest, from which its period is readily found anew. This second definition led us to seek the statistical distribution of individual wave "heights" and "periods" defined at the positive maxima of second order gaussian noise, starting from the basis of Cartwright and Longuet-Higgins'fundamental paper [2] . This leads after some algebra to the following joint probability distribution

$$1)\quad p(h,\tau) = \frac{\alpha^3 h^2}{4(2\pi)^{1/2}\,\epsilon(1-\epsilon^2)\,\bar{\tau}^4\,\tau^5}\,\exp\left\{-\frac{h^2(\bar{\tau}\tau)^{-4}}{8\epsilon^2}\left[(\bar{\tau}^2\tau^2-\alpha^2)^2+a^2\alpha^4\right]\right\}$$

where $\epsilon^2 = (m_o\, m_4 - m_2^2)/m_o m_4$ $\qquad m_n = \int_o^\infty \omega^n\,\Phi(\omega)\,d\omega$

$$\alpha = \left[1 + (1-\epsilon^2)^{1/2}\right]/2 \qquad a^2 = \epsilon^2/(1-\epsilon^2)$$

The variable h is the non dimentional variable, ratio of twice the positive maximum's vertical displacement to $\sqrt{m_o}$; τ is the ratio of the wave period T (defined by the acceleration of the free surface at the maximum) to $\bar{\tau}\ T_c$ where T_c is the mean time internal between successive positive maxima which for gaussian noise is given by

$$T_c = 2\pi\,(m_2/m_4)^{1/2}\,\alpha^{-1}$$

and $\bar{\tau}$, function of ϵ , is the mean value of T/T_c which must be determined by numerical integration of equation 1 (the values of $\bar{\tau}$ remain close to 1.0 for values of ϵ up to 0.95). Figure 1 shows the evolution of $p(h,\tau)$ as a function of ϵ .

III - Comparison with storm condition recordings

Figure 2 shows both isodensity curves determined experimentally from 28 240 zero up-crossing waves ($\bar{T}$ being the mean zero up-crossing period of each recording) and theoretically from equation 1). The fit is quite good in view of the large value of ϵ, 0.865, mean value of the 182 recordings used (standard deviation 0.031). Comparison of present theory which takes into account only positive maxima, that obtained by considering the double amplitude of both positive and negative maxima (deduced from reference [2]) where applicable, and observations in the North Sea is made in figure 3 ; here as in figure 4, ten recordings have been analyzed and their spectra artificially narrowed by elimination of high frequency components , retaining 100, 95, 90 and 80 per cent of the energy at the low frequencies.

Conclusion

From the complete set of figures it must be concluded that theoretical fit with measurements is rather good, under the only restriction that the experimental non-dimensionnal "period" used be the ratio $T_{z.u.c.} / \overline{T}$ in place of τ. In view of the basic hypothesis, the best fit between theory and observations is obtained for low values of ϵ , but at least for the severe sea state conditions examined, agreement is still satisfactory values of ϵ up to 0.9 ; practically it is important to notice that the highest waves will have a maximum probability of occuring with values of T only slightly superior to $\overline{T}$ as indicated both by figures 2 and 3.

In conclusion, it must be stressed that the theory developped applies to all fairly narrow band gaussian noise so that its use may be extended to quite a number of other applications.

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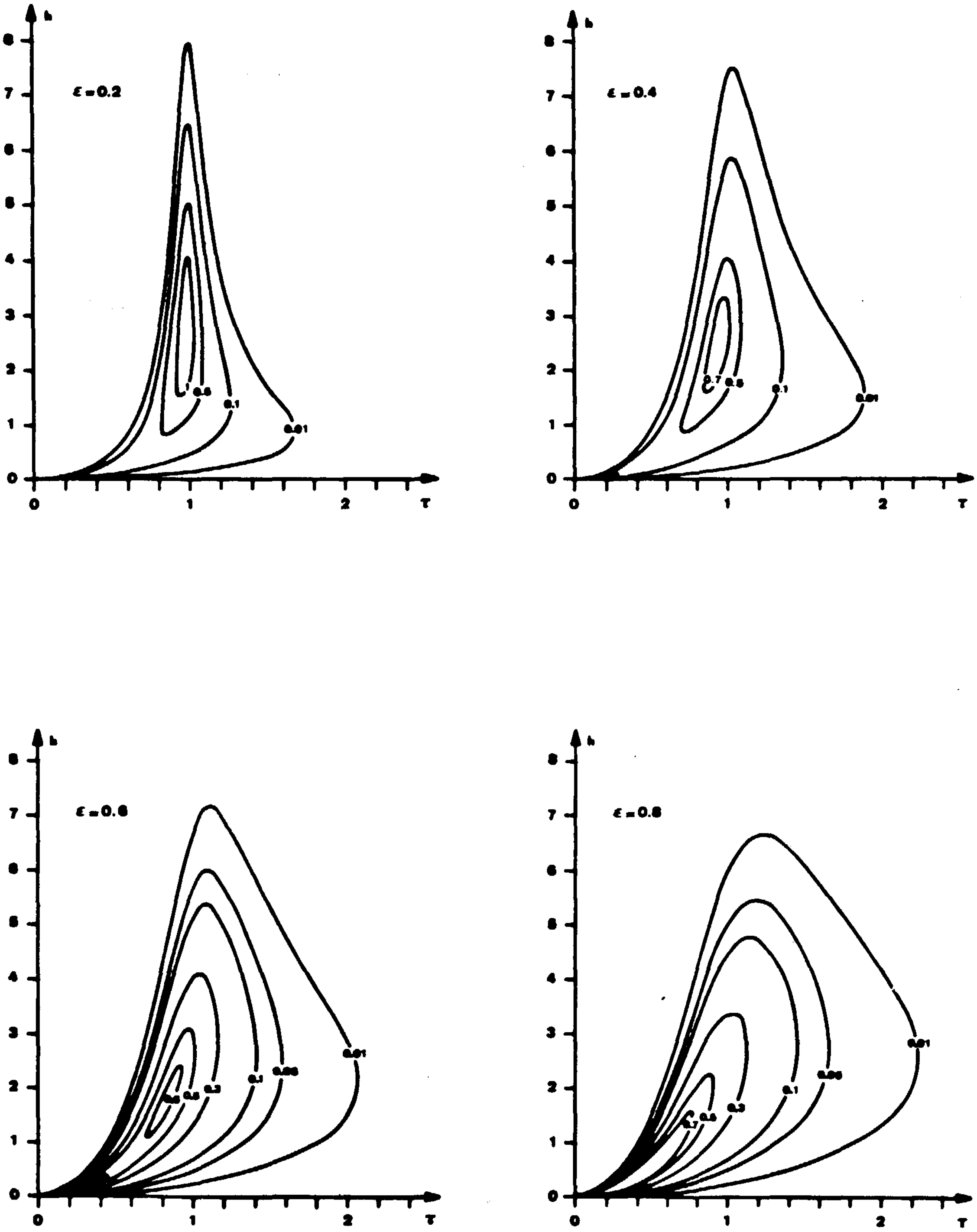
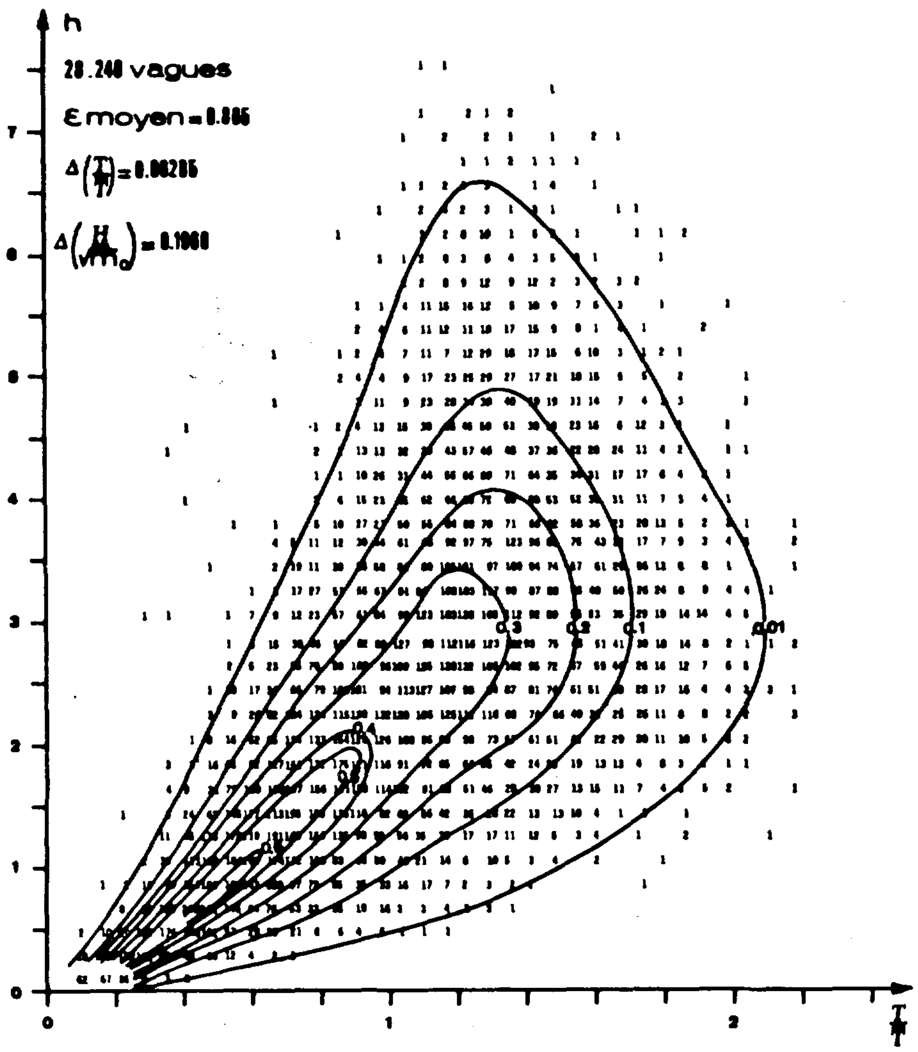
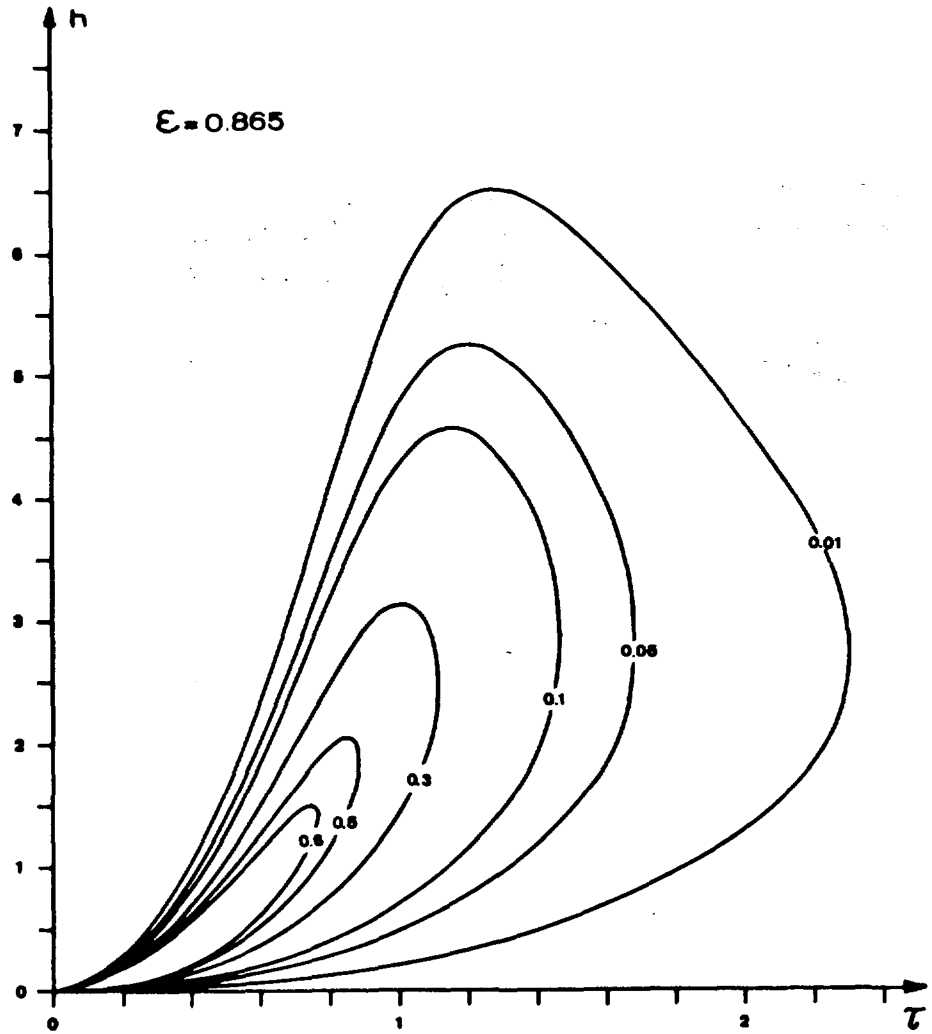


Evolution of p(h,τ) with ε

Figure n°1



North Sea Storms Probability Density



Theoretical Probability Density

Figure n°2

Evolution of different parameters with ε

- - - - - Cartwright & Longuet Higgins (1956)

——— Prevent Theory

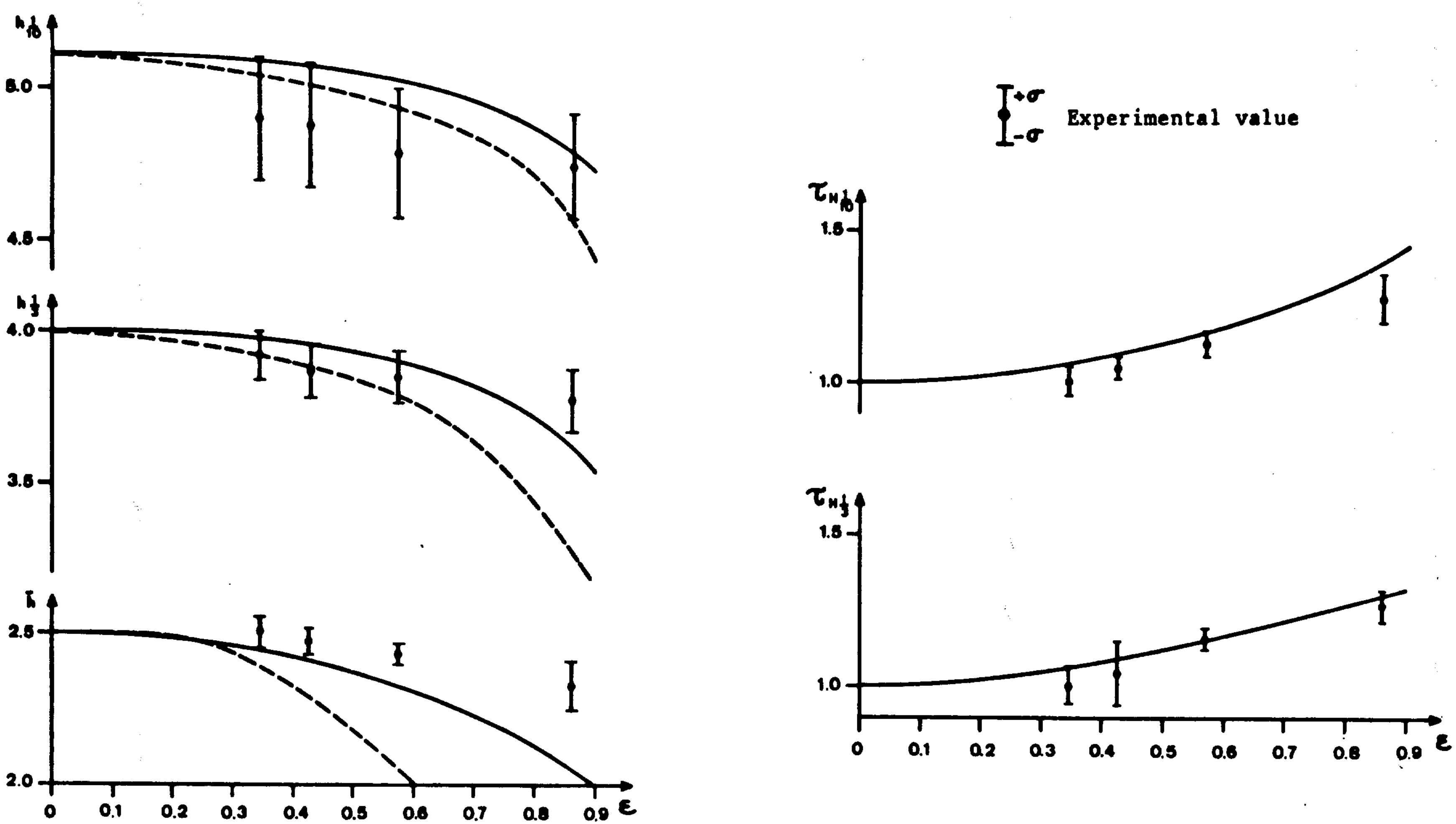
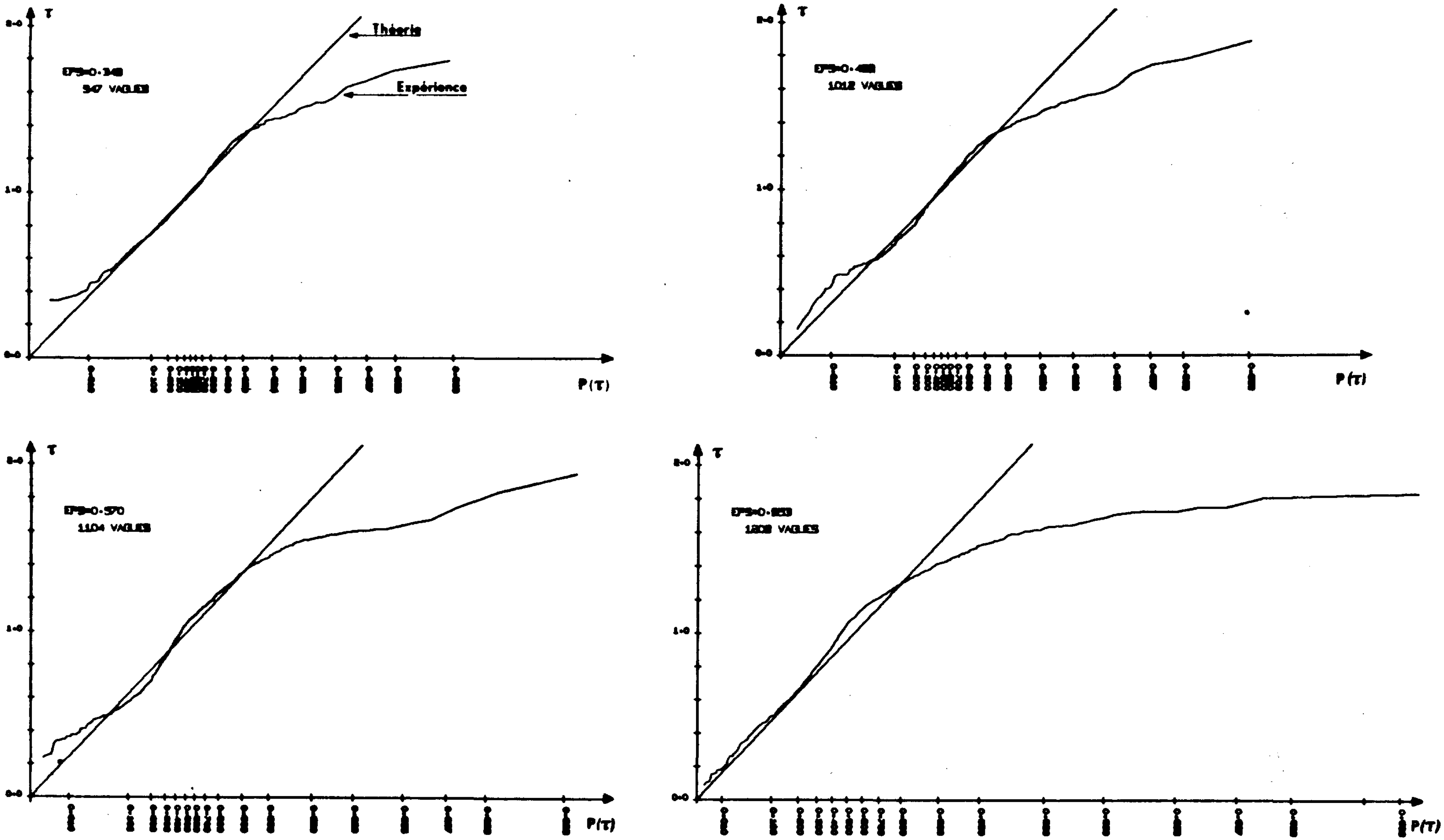


Figure n°3



Evolution with ε of experimental cumulative probability compared with theory (straight line)

Figure n°4

"A STATISTICAL RELATIONSHIP..." BY CAVANIÉ(ET AL.)

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DISCUSSION

By Mr. H. Rye, The Norwegian Institute of Technology, Norway

I guess that my comment is not only directed towards the paper presented by Cavanié et.al. in particular, but is relevant to many other papers presented at this conference as well. My comment deals with the various spectral shape parameters applied at this conference and the conflicting results that these parameters produce.

The spectral parameter ε was mentioned by Dr. Milgram to give numbers generally lower than .7, while Cavanié et.al. report on a mean value of .865. The spectral narrowness parameter ν applied by Houmb and Overvik at this conference, was found to be located basically between .5 and .6 for JONSWAP-type spectra, while Liu [1] and Longuet - Higgins [2] have applied the numbers .425 and .234, respectively.

Both these parameters have been shown to be unstable; i.e. they are dependent on the high-frequency cut-off choice when the moments of the spectrum are computed [3]. This is unfortunate because the energy content located in the high-frequency tail in the spectrum is generally of little or no significance for ocean engineers. Personally, I am in favour of the peakedness parameter Q_p applied by Goda at this conference:

$$Q_p = \frac{2}{m_0^2} \int_0^\infty [S(f)]^2 f df$$

where m_o is the zeroth moment of the spectrum S(f). This parameter is shown to be stable [3]. If ε or ν are to be used, the location of the spectral peak as well as the high-frequency cut-off have to be specified for each separate case.

Generally, it is unfortunate that the spectral moments are applied to specify the wave field. These concepts were originally applied for the analysis of radio signals. Wave spectra, however, usually contain a high-frequency tail proportional to f^{-5} which causes the moments to depend on the high-frequency cut-off choice. Fig. 1 (below) shows the dependency of the second moment to the high-frequency choice. The figure shows a variation of a factor of two for m_2 for a Pierson-Moskowitz type spectrum ($\gamma=1$), within the range $1.5\ f_p - 10\ f_p$ where f_p is the peak frequency. These limits are chosen because both have been applied by various investigators [4,5].

It is therefore obvious that a variety of results may be obtained by varying the choice of the cut-off frequency!

For parameters related to the third and the fourth moment, the cut-off dependency is even larger. I therefore believe that moments larger than the first one should be avoided in favour of, say, Q_p and the peak frequency.

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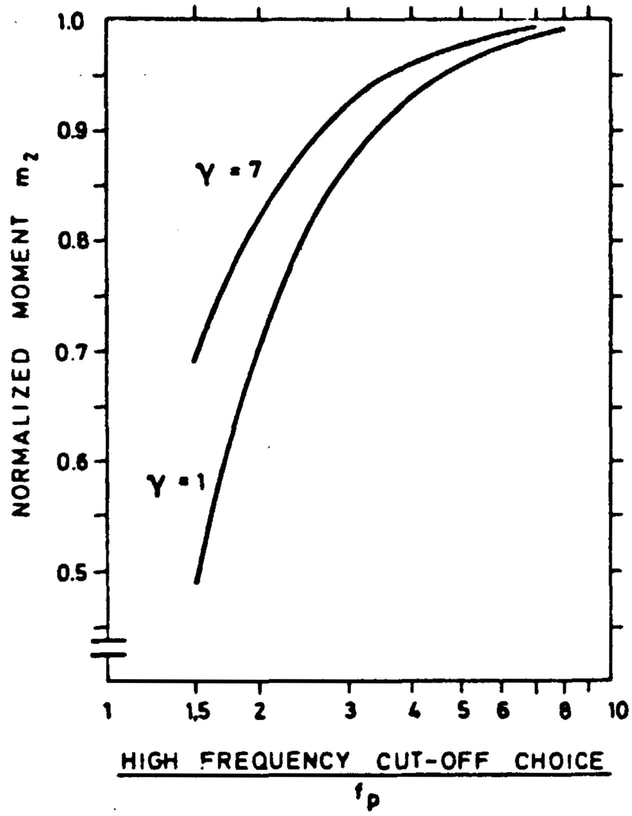

FIGURE 1. The second moment m_2 as a function of the high-frequency cut-off choice. m_2 is computed by means of numerically integrating the JONSWAP spectrum equation [5]. The results are normalized to a cut-off choice equal to 10 f_p. $f_p = f_m$ = peak frequency = arbitrary. α = arbitrary. γ = peakedness factor in the JONSWAP spectrum equation [5]. $\sigma_a = 0.07$. $\sigma_b = 0.09$.