Abstract—We examine how the rough sea surface scattering of L-band celestial sky radiation might affect the measurements of the future European Space Agency Soil Moisture and Ocean Salinity (SMOS) mission. For this purpose, we combined data from several surveys to build a comprehensive all-sky L-band celestial sky brightness temperature map for the SMOS mission that includes the continuum radiation and the hydrogen line emission rescaled for the SMOS bandwidth. We also constructed a separate map of strong and very localized sources that may exhibit L-band brightness temperatures exceeding 1000 K. Scattering by the roughened ocean surface of radiation from even the strongest localized sources is found to reduce the contributions from these localized strong sources to negligible levels, and rough surface scattering solutions may be obtained with a map much coarser than the original continuum maps. In rough ocean surface conditions, the contribution of the scattered celestial noise to the reconstructed brightness temperatures is not significantly modified by the synthetic antenna weighting function, which makes integration over the synthetic beam unnecessary. The contamination of the reconstructed brightness temperatures by celestial noise exhibits a strong annual cycle with the largest contamination occurring in the descending swaths in September and October, when the specular projection of the field of view is aligned with the galactic equator. Ocean surface roughness may alter the contamination by over 0.1 K in 30% of the SMOS measurements. Given this potentially large impact of surface roughness, an operational method is proposed to account for it in the SMOS level 2 sea surface salinity algorithm.

Index Terms—Microwave radiometry, sea surface electromagnetic scattering.

I. INTRODUCTION

CELESTIAL sky L-band radiation scattered by the ocean surface can contaminate spaceborne measurements of upwelling sea surface brightness temperature used to retrieve sea surface salinity (SSS). The sensitivity of the linearly polarized sea surface brightness temperature to salinity ranges from about 0.2 to 0.8 K/psu [1] (depending on ocean surface temperature, incidence angle, and polarization). Since the open ocean surface salinity generally ranges from 32 to 37 psu, the expected dynamical range of L-band brightness temperatures associated with variations in SSS alone is small relative to the total brightness temperature, which is less than approximately 45 K for open ocean conditions.

For the Aquarius/SAC-D mission, it was reported in [2] that, under the assumption of a flat perfectly conducting Earth surface (with a reflectivity of 1), the total celestial sky radiation contribution to the antenna temperature varies from a little less than 4 K to more than 9 K. For a perfectly flat dielectric 52 sea surface, the reflectivity may range from about 30% to 80% at 1.4 GHz for incidence angles below 50°, depending on the SSS, sea surface physical temperature, and observation polarization. In this case, the contamination ranges from about 1 to 7 K. As discussed in [3] (hereinafter referred to as Part I), ocean surface roughness both decreases the surface reflectivity and directionally spreads the impact of the source brightness. For specular points in the vicinity of the galactic equator, the spreading effect of the rough surface greatly reduces the impact along the equator and broadens the contamination far beyond the narrow bright source concentrated along the equator. Nevertheless, for specular points far from strong sources, sea surface roughness has a negligible impact on the reflected signal. Overall, the intensity of the scattered celestial noise ranges from 30% to 70% of the flat ocean surface reflected values, with most of the variation associated with the directional spreading of the radiation.

The nonuniform distribution of celestial radiation has an important systematic impact on the measurements. The future European Space Agency Soil Moisture and Ocean Salinity (SMOS) and National Aeronautics and Space Administration (NASA)/Comisión Nacional de Actividades Espaciales Aquarius/ SAC-D satellites that are dedicated to SSS remote sensing will be launched in the near future into sun-synchronous orbits, and considering these orbits along with the Earth’s orbit around the Sun, the celestial sky glitter contamination will exhibit strong geographic and seasonal dependence. As such, flagging and correction strategies for such contamination must be developed to reduce large-scale seasonal and geographical biases in the retrieved surface salinity fields.

To achieve the 0.1-psu accuracy goal for the retrieved salinity, the sky glitter contribution must be estimated with an uncertainty not exceeding 0.05 K. This is a stringent constraint that may be difficult to satisfy given the accuracies of both the future SMOS radiometric measurements and the sky brightness
temperature maps. This constraint also presents potential difficulties for the forward modeling of the scene brightness temperatures since this modeling is plagued by uncertainties and potential biases associated with rough sea surface scattering and emissivity models. Moreover, for the SMOS mission, the multidirectional nature of the measurements incorporated into the salinity retrieval on Earth results in a potentially wide range of celestial noise contamination for any given retrieval, so that failure to correct for the contamination prior to salinity retrieval may result in retrieved salinity errors and biases that are not easily correctable by further processing at a later stage.

The focus of this paper is given as follows: 1) to analyze the expected annual cycle of contamination of SMOS multiangular reconstructed brightness temperatures by scattered celestial radiation over the ocean and 2) to propose a method for the proper detection of and correct for this sky glitter within the SMOS ocean surface salinity retrieval algorithm.

For these two purposes, a sky brightness temperature map at L-band was generated for SMOS based on an existing all-sky continuum map using an approach similar to [2], and the method used to build this map is reviewed in Section II. Missing data in the vicinity of Cassiopeia A and other strong sources in the continuum map can potentially lead to underestimation of the reflected sky noise, particularly over smooth sea surfaces, and to address this issue, we derived an error map by using higher resolution surveys to identify the locations and brightness temperatures of sources that may introduce substantial errors associated with estimating the downwelling celestial radiation from the continuum map alone. Since these localized strong sources might require the use of very high-resolution grids when applying the modeling methodology developed in Part I, we evaluated the impact of resolution on the numerical scattering calculations and determined an acceptable discretization of the celestial noise map.

In Section III, we formulate expressions for the rough surface scattered celestial radiation incident at the SMOS antenna. The interferometric nature of the Microwave Imaging Radiometer (MIRAS) results in a formulation that is distinct from that for a real aperture radiometer. In developing these expressions, we consider, in turn, simplifications that are obtained by assuming that the sea surface is perfectly smooth and approximating the synthetic antenna weighting function by an isotropic function in director cosine (DC) coordinates.

Given the potential for scattered celestial noise to introduce seasonal and regional biases in retrieved surface salinity, we examined the seasonality and spatial distribution of the expected celestial sky glitter contamination for SMOS by performing a series of monthly orbit propagations in which we collected dwell lines, i.e., sets of multiangular scene brightness temperatures at a fixed location on Earth, over a fixed Earth grid that spans one complete orbit. The scattered celestial radiation was calculated for one orbit per month over a one-year period using idealized descriptions of the ocean surface state. Results of these calculations are presented in Section IV. The SMOS configuration, with its sun-synchronous orbit and large field of view (FOV), provides reconstructed brightness temperatures over a large range of incidence and azimuth angles (and therefore a large range of specular sky locations) at each point on Earth, so that a large portion of the sky will contribute to the contamination at any given time. Moreover, given the sun-synchronous nature of the SMOS orbit, the celestial sky glitter contamination is a function of time with a distinct annual cycle. This is distinct from the situation with the Aquarius/SAC-D mission, for which the set of all specular sky locations (or specular projection of the FOV) at any given time of year is a large closed loop on the celestial sphere. For SMOS, the error associated with assuming a perfectly smooth sea surface may exceed 0.1 K over large portions of the measurements in an orbit. Both the contamination and the potential error associated with assuming a perfectly flat surface are greatest for the descending passes from August to October.

Given the potential for significant impact of the rough ocean surface and the large computational burden associated with the scattering calculations, in Section V, we propose a practical correction and flagging strategy that may be used in a salinity retrieval algorithm. In Section VI, we summarize the results and briefly discuss potential sources of error as well as validation issues for the proposed celestial sky glitter correction.

II. GENERATION OF AN L-BAND SKY MAP TO BE USED FOR SMOS DATA PROCESSING

Three components are required to build a map of the sky emission at L-band [2].

1) The cosmic microwave background (approximately a constant value of 2.725 K).

2) The neutral hydrogen line (HI in astronomer’s shorthand): This strong emitting line is centered at 1420.4058 MHz and is spread over a finite band by an additional Doppler shift. In surveys of the continuum, this source is usually rejected by a stopband filter.

3) The continuum at ~1.4 GHz, which originates from a variety of emission mechanisms (other lines than HI, synchrotron, free–free, thermal, blended emission of discrete radio sources, . . . ).

The final merged map to be used with SMOS, which is termed here as the “nominal map,” is expressed in the Besselian Epoch B1950 [4], and in what follows, we display the results and perform the scattering calculations in an equatorial coordinate system (with coordinates given by right ascension and declination) in this reference frame.

A. Main Sources of Data

To provide coverage of the whole sky, measurements obtained by different instruments situated in both the Northern and Southern Hemispheres must be combined. This requires extensive data collection and calibration to ensure sufficient data quality. The merging of these data sets requires cross-calibration and consideration for differences in instrument angular resolutions. This work was conducted by experts in the field of radio astronomy, and the maps introduced here are based upon products produced by these experts.

1) Continuum: The data set identified here is a combination of the Northern Sky survey made with the Stockert radio
telescope [5]–[7] and the Southern Sky survey made with the 200 radio telescope of the Instituto Argentino de Radioastronomía 202 (IAR) [8]. When the bandwidth of the receiver was overlapping 203 the HI emission, a stopband filter centered over the HI line 204 and 2 MHz wide was applied to the measurement to reject it. 205 Data were sampled with a 0.25° resolution in both declination 206 and right ascension (equatorial coordinates, B1950 system). 207 The sensitivity (defined as three times the root-mean-square 208 brightness temperature noise) of the merged data set is 0.05 K. 209 In the following, this data set will be referred to as the Reich 210 and Testori map (Fig. 1).

It is assumed that the “continuum” radiation, with unpolar- 211 ized brightness temperature $T_{\text{cont}}$, is broadband and therefore 213 does not vary appreciably within the SMOS band. Thus, data 214 from surveys made at slightly different center frequencies and 215 with slightly different bandwidths may be directly combined. 216 The continuum data set includes the constant 2.725 K cosmic 217 background radiation.

2) Hydrogen Line: To account for the hydrogen line emission, we used the Leiden–Argentina–Bonn (LAB) survey [9]. 200 The LAB survey contains the final data release of observations 219 of 21-cm emission from galactic neutral hydrogen 220 over the entire sky and is a merged product based on the 221 Leiden–Dwingeloo survey of the sky north of $-30^\circ$ [10] and 222 the IAR survey of the sky south of $-25^\circ$ [11], [12]. The 223 source velocities away from the Earth range from $-450$ to 224 $+400$ km·s$^{-1}$ and are resolved in the data to 1.3 km·s$^{-1}$. 227 The root-mean-square error of the brightness temperatures in 228 the merged data set is 0.07–0.09 K (for each 1.3 km·s$^{-1}$ layer). 229 Data were sampled with a 0.5° resolution in both latitude and 230 longitude (in galactic coordinates). Hereinafter, this data set 231 will be referred to as the HI map.

3) Integration of HI Into the Continuum Map: As mentioned earlier, the continuum signal is broadband, with almost 233 constant brightness temperature $T_{\text{cont}}$ throughout the SMOS 235 bandwidth. By contrast, the hydrogen line emission exists only 236 over a very narrow band, but MIRAS measures radiation over 237 a bandwidth $B_{\text{SMOS}}$ of 19 MHz that includes the HI line 238 (1420.4058 MHz) so that this narrow source must be integrated 239 into the continuum map. The merged all-sky map provided by

Reich and Testori includes both $T_{\text{cont}}$ and the constant 2.725 K 240 cosmic microwave background $T_{\text{CMB}}$. The HI data [9] do not 241 include $T_{\text{CMB}}$ (Fig. 2).

To derive the HI line contribution over the SMOS bandwidth 243 from HI line velocity range data, we used a Doppler relation 244 between velocity range and frequency shift. The HI line fre- 245 quency is $f_0 = 1420.4058$ MHz, and the Doppler shift is given 246 by $f = f_0(c/(c + v))$, where $c$ is the speed of light and $v$ is the 247 relative speed of the source away from the Earth. The stopband 248 filter applied to the Reich and Reich measurements is centered 249 on $f_0$ and is $B_{\text{HI}} = 2$ MHz wide. This corresponds to outward 250 velocities ranging from $-211.2$ to $+211.4$ km·s$^{-1}$. Over this 251 bandwidth, the contribution of HI signal is

$$\tilde{T}_{\text{HI}} = \frac{1}{422.6 \text{ km} \cdot \text{s}^{-1}} \int_{-211.2 \text{ km} \cdot \text{s}^{-1}}^{211.4 \text{ km} \cdot \text{s}^{-1}} T_{\text{HI}}(\nu) d\nu.$$  \hspace{1cm} (1)

Finally, the resulting sky noise to be considered for SMOS is

$$T_{\text{sky}} = T_{\text{CMB}} + T_{\text{cont}} + \tilde{T}_{\text{HI}} B_{\text{HI}}.$$ \hspace{1cm} (2)

B. Gaps in the Continuum Survey: Use of Alternative Surveys 254 and Source Catalogs for Missing Data Integration

The Reich and Testori continuum survey is not complete and 256 contains regions with inadequate coverage. The most prominent 257 such region is Cassiopeia A, where the high flux prevented 258 accurate measurement using standard procedures. In addition, 259 highly localized strong sources are not properly taken into account in the continuum survey. Higher resolution surveys that 260 can alleviate this problem by providing auxiliary 1.4-GHz flux 261 measurements for these problematic areas are available. These 262 data sets usually come in two forms.

1) Higher resolution local sky maps where for a given area of the sky a radio flux is associated to each [right ascension, declination] cell. This enables an assessment 267 of the slow variations of the background flux when it

![Fig. 1. Reich and Testori continuum map. Dark blue is for 0 K, and red is for 20 K.](image)

![Fig. 2. HI map rescaled over SMOS bandwidth. Dark blue is for 0 K, and red is for 3 K.](image)
results from the combination of minor sources that cannot be individually identified. Once rescaled and converted to the proper geometry, these data sets can be used to patch the continuum map where data are missing.

2) Source catalogs that provide flux measurements for specific strong sources with small angular extents. These data sets can be useful to identify strong sources in otherwise quiet areas of the sky.

In the case of strong sources of small angular extent, it is difficult to determine whether they are properly taken into account in the full sky survey map. To evaluate the extent to which strong sources are properly accounted for in the continuum map, a map of strong sources was generated from L-band source catalogs [13], [14], and we computed the corresponding brightness temperatures that would be collected by the Stockert/Testori/IR radio telescopes (the ones that were used to generate the Reich and Testori map). These source data were obtained from both the [NRAO (National Radio Astronomy Observatory) VLA (Very Large Array) Sky Survey (NVSS)] [14] (Northern Hemisphere) and the Parkes [13] (Southern Hemisphere) catalogs. Only sources stronger than 0.3 Jy were considered, since smaller fluxes would introduce less than 0.015-K error in the Reich and Testori map.

The resulting brightness temperatures were compared with the combination of the continuum and HI maps. Fig. 3 shows that most sources exhibit a brightness temperature that does not exceed the corresponding continuum value (which is generally the case when the sources are embedded in regions of strong emission that dominate the total signal within the relatively large beam of the telescope). Nevertheless, some strong sources, such as Cygnus A and Taurus A, can be identified. Fig. 4 shows the locations of the sources for which the fluxes are underestimated in the nominal sky map. Most differences in the flux are quite small and are expected to be strongly reduced when integrated over the SMOS synthetic beam. The strongest discrepancies occur in the vicinity of Cygnus A and Cassiopeia A. The nominal sky map generated for SMOS was not corrected for these strong sources; instead, a separate sky map that contains only the strong source brightness temperatures averaged to the Stockert/Testori beamwidth was developed. As in [2], we do not account for possible polarization in either the nominal or the strong source maps.

C. Impact of Strong Sources

To quantify the maximum expected impact of strong point sources, we calculated the scattered signal along cross sections from the survey data of Reich and Testori, the rescaled HI line data from the survey in [9], and the Effelsberg survey data in the vicinity of Cassiopeia A. Only sources that exhibit a brightness temperature (for a 35-arcmin beam) larger than that in the nominal sky map are displayed.

Fig. 3. Comparison between brightness temperatures derived from the individual source catalogs (NVSS + Parkes) and brightness temperatures extracted from the survey maps (i.e., a combination of the merged data from Reich and Testori, the rescaled HI line data from the survey in [9], and the Effelsberg survey data in the vicinity of Cassiopeia A). The diagonal line shows a one-to-one correspondence.

Fig. 4. Strong sources superimposed on the nominal SMOS sky map derived from the survey data of Reich and Testori, the rescaled HI line data from the survey in [9], and the Effelsberg survey data in the vicinity of Cassiopeia A. Only sources that exhibit a brightness temperature (for a 35-arcmin beam) larger than that in the nominal sky map are displayed.

The scattered signal was calculated at a wind speed of 7 m·s⁻¹. The total scattered signal in the direction of the instrument is obtained by integrating the product of this weighting function and the sky signal in the direction of the instrument is obtained by integrating the product of this weighting function and the sky signal. The downwind dihedral reflection $\phi_r$ relative to the scattering azimuth $\phi_s$ (both defined to be positive counterclockwise from due east) is $\phi_r - \phi_s = 0^\circ$. This weighting function has been normalized to a maximum, and contours are shown at 0.1, 0.3, 0.5, 0.7, and 0.9. Clearly, the weighting function extends well beyond the 333 localized strong source, so it is expected that the impact of such 334 a source should be small. In Fig. 5(b), we show the scattered unpolarized signal $1/2(T_w + T_h)$ along a cross section at 336 constant declination in the celestial sphere, considering only the strong source map. In this cross section, the wind speed $338$ is fixed at 7 m·s⁻¹, the specular declination is $58.25^\circ$, the scattered field incidence angle is $0^\circ$, and the specular right ascension ranges from $-40^\circ$ to $+20^\circ$. The resulting glint is 341 shown for both the Kirchhoff approximation (KA) [15] and the 342 first-order small slope approximation (SSA-1) [16] scattering models described in [3] and for both the full-resolution and 344 reduced-resolution maps. The resulting glint never exceeds 345.
Fig. 5. (a) Incident unpolarized celestial noise. Overlaid is the bistatic scattering weighting function (discussed further in the text) for horizontal polarization for a specular point defined by \((\alpha_s = 350.25^\circ, \delta_s = 58.5^\circ)\) (Kirchhoff model; Kudryavtsev wave spectrum), which is normalized to unit amplitude. Thin contour lines correspond to the normalized levels \((0.5, 0.9)\), and thick contour lines correspond to the normalized levels \((0.1, 0.3, 0.7)\). (b) Cross section, at constant declination, of the unpolarized \((1/2(T_a + T_b))\) scattered radiation from strong sources as determined using the KA [15] and SSA-1 [16] electromagnetic models and the Kudryavtsev equilibrium ocean surface wave spectrum. Curves are defined in the inset of (c). The specularly reflected signal (solid black curve) is scaled to the right axis, whereas the scattered signals are scaled to the left axis. (c) Same as in (b) except for the nominal celestial noise map without strong sources. The scattered signal is evaluated at an incidence angle of \(\theta_s = 0^\circ\) and the incidence plane orientation angle \(\psi_{ab} = 0^\circ\). The surface wind speed is \(u_{10} = 7\ m\cdot s^{-1}\); the downwind direction relative to the scattering azimuth is \(\varphi_w - \phi_s = 0^\circ\). The maximum incident signal in the strong source map exceeds 1700 K, while in the nominal map the maximum signal in the vicinity of this strong source is approximately 150 K [with a maximum at a slightly displaced position of \((\alpha_s = 350.2^\circ, \delta_s = 58.2^\circ)\)]. Units are in kelvin.

III. CELESTIAL SKY GLITTER CONTRIBUTION

A. General Formulation

Considering the simple case of unpolarized celestial radiation with scalar brightness temperature \(T_{sky}\) and assuming a simple exponential model for attenuation on both downward and upward paths, it was shown in Part I that the total antenna temperature Stokes component \(p\) (where \(p\) corresponds to either horizontal or vertical polarization) associated with rough sea scattered celestial radiation is

\[
\mathbf{T}_p^a = \frac{1}{\Omega_{a}} \int \frac{\mathbf{G M}_a}{4\pi \cos \theta_s} e^{-a \sec \theta_s} \times \int_{\Omega_0(\Omega_a)} \left[ \sigma_{pp}(\Omega_0) + \sigma_{pq}(\Omega_0) \right] e^{-a \sec \theta_0 T_{sky}^a(\Omega_0) d\Omega_0 d\Omega_a} d\Omega
\]

where the factor in front of the outermost integral normalizes the antenna gain pattern \(\mathbf{G}\). As detailed in Part I, \(\mathbf{M}_a\) is a 381 composite transformation matrix accounting for polarization 382 basis and Faraday rotation. The scattered field incidence angle 383 is \(\theta_s\), and \(\sigma_{pq}\) are the normalized bistatic scattering cross 384 sections of the rough sea surface defined using the forward 385 scattering alignment polarization basis convention as discussed 386 in Part I and in [17]; \(\Omega_{a}\) refers to the solid angle domain 387 of integration over the antenna pattern and is associated with 388 antenna incident and azimuth angles \(\theta_a\) and \(\varphi_a\), respectively; 389 \(\Omega_0\) refers to the entire upper hemisphere of over which sky 390 radiation is incident at the target; \(a\) is the zenith atmospheric 391
attenuation. Note that the normalized bistatic scattering cross sections used in this paper and in Part I differ from the bistatic scattering coefficients $\gamma_{pq}$ used in [1] in that the normalized bistatic scattering cross sections relate scattered wave energy flux across the undisturbed scattering surface to the incident flux in the incident wave propagation direction, whereas the bistatic scattering coefficients relate the scattered energy flux across the surface to the incident energy flux across the surface, so that

$$\sigma_{pq} = \cos \theta_0 \gamma_{pq}. $$

This distinction is briefly discussed in [18].

In the case of an interferometric instrument such as MIRAS, we are concerned with modeling the reconstructed brightness temperature rather than the conventional antenna temperature as obtained from a real aperture radiometer. The reconstructed brightness temperature in direction $(\theta_a, \phi_a)$ is still given by an equation of the form (3), but the outermost integral over $\Omega_a$ is evaluated over a rather narrow synthetic antenna weighting function centered at synthetic boresight direction $(\theta_a, \phi_a)$, and the real aperture antenna gain matrix $G$ is replaced by a synthetic antenna weighting function that, in general, depends upon both the instrument and the image reconstruction method.

In what follows, we will use the terms “reconstructed brightness temperatures” and “brightness temperature measurements” interchangeably.

If we introduce instrument DC coordinates $(\xi, \eta)$

$$\xi = \sin \theta_a \cos \phi_a$$

$$\eta = \sin \theta_a \sin \phi_a$$

then the SMOS synthetic antenna weighting function, which is also called equivalent array factor (AF), may be written [19] as

$$\text{AF}_{\text{eq}}(\xi, \eta, \eta') = \frac{\sqrt{3}}{2} d^2 \sum_{m} \sum_{n} W(u_{mn}, v_{mn})$$

$$\times \cos \left(2\pi \frac{(u_{mn} \cdot \xi + v_{mn} \cdot \eta)}{f_0} \right)$$

$$\times e^{i2\pi (u_{mn}(\xi - \xi') + v_{mn}(\eta - \eta'))}$$

(6)

where $W$ is the apodization function; $\tilde{r}$ is the fringe-washing factor (FWF), which accounts for the spatial decorrelation between antennas; $u, v$ are the baseline coordinates in the frequency domain; $d$ is the dimensionless antenna element spacing (0.875); $f_0$ is the central frequency (1413 MHz); $\xi, \eta$ are the central (i.e., synthetic boresight) DC coordinates; and $\xi', \eta'$ are the running DC coordinates. Defining $D = (\xi', \eta': \xi'^2 + \eta'^2 < 1)$ as the domain of integration within the synthetic beam and noting that

$$d\Omega_a = \sin \theta_a d\theta_a d\phi_a = \frac{\sin \theta_a \cos \theta_a d\xi d\eta}{\sqrt{1 - \xi^2 - \eta^2}}$$

(7)

the expression for the contribution of the polarized celestial sky glitter to the reconstructed brightness temperature in the Ludwig-3 polarization basis is given by

$$\langle T_p(\xi, \eta) \rangle = \int_D \int \frac{\text{AF}_{\text{eq}}(\xi', \eta', \eta')}{\sqrt{1 - (\xi' - \xi)^2 - (\eta' - \eta)^2}}$$

$$\times [M_a(\xi', \eta') A_u(\xi', \eta') \langle \text{AF}(\xi', \eta', \eta') \rangle d\xi' d\eta'.$$

(8)

where $\text{AF}(\xi', \eta', \eta')$ is the Stokes vector of the surface scattered celestial noise in the target polarization basis, and $A_u$ is an upward atmospheric attenuation matrix defined in Part I. The variations in atmospheric attenuation and geometrical rotation are sufficiently small within the narrow (approximately 3') synthetic beam that these factors may be approximated by their values at the synthetic beam center, i.e., $\langle \text{AF}(\xi', \eta', \eta') \rangle$, so that

$$\langle T_p(\xi, \eta) \rangle \approx M_a(\xi, \eta) A_u(\xi, \eta) \int_D \int \frac{\text{AF}_{\text{eq}}(\xi', \eta', \eta')}{\sqrt{1 - (\xi' - \xi)^2 - (\eta' - \eta)^2}}$$

$$\times [\sigma_{pp}(\Omega_0) + \sigma_{pq}(\Omega_0)] e^{-a \sec \theta_o T_{\text{sky}}(\Omega_0)} d\phi_o d\rho_o \sqrt{1 - \rho^2}$$

(9)

As shown in [20], if one neglects the FWF, the AF may be approximated by a rather narrow centrosymmetric function that is independent of the location of the synthetic boresight $(\xi, \eta)$ within the FOV. The following explicit formula has been developed to approximate the actual AF with no FWF effect:

$$\text{AF}_{\text{eq}}(\xi, \eta, \eta') \approx 4\pi \cos \theta_s \text{AF}(\xi, \eta, \eta')$$

$$= \max \left\{0, \left[\sin k_f \rho \right]^{k} \frac{1}{1 + k_g \rho^{k}} \right\}$$

(10)

where $\rho = \sqrt{(\xi' - \xi)^2 + (\eta' - \eta)^2}$ is the distance in DC coordinates, $k_f = 73.30$, $k_g = 524.5$, $k_h = 2.1030$, and $k_k = 443.1.4936$. Throughout the rest of this paper, we refer to the FWF approximation AF expression (10) as the antenna weighting function, i.e., WEF. If we adopt this approximation formulation for the AF and assume that the downwelling sky radiation is unpolarized, then the total contamination of reconstructed brightness temperatures by scattered celestial sky glitter (3) becomes

$$\langle T_p(\xi, \eta) \rangle \approx M_a(\xi, \eta)$$

$$\times \int_D \int \frac{\sigma_{pp}(\Omega_0) + \sigma_{pq}(\Omega_0)}{\sqrt{1 - \rho^2}} e^{-a \sec \theta_o T_{\text{sky}}(\Omega_0)} d\phi_o d\rho_o \sqrt{1 - \rho^2}$$

(11)

In this equation, $D_p = \{ \rho, \phi : (\xi'(\rho, \phi))^2 + (\eta'(\rho, \phi))^2 < 1 \}$ is the polar coordinate domain corresponding to the Cartesian 452 domain $D$.

B. Perfectly Smooth (Flat) Sea Surface Case

When the sea surface is perfectly flat, the scattered celestial 455 sky glitter incident at the instrument in the surface polarization 456 basis reduces to

$$\langle T_p(\xi, \eta) \rangle = R_{pp}^{(0)}(S, T_s, \theta_s) T_{\text{sky}}(\theta_s, \phi_s - \pi) e^{-a \sec \theta_s}$$

(12)

where $R_{pp}^{(0)}(S, T_s, \theta_s)$ are the Fresnel reflection coefficients of the flat sea surface with salinity $S$, physical surface temperature $T_s$, incidence angle $\theta_s$, and linear polarization $p$. In this case, 460
the contribution to the reconstructed brightness temperature
from the sky glitter incident at the antenna from direction \((\xi, \eta)\),
which is expressed in the instrument polarization basis, is

\[
T_p^{fa}(\xi, \eta) \approx (M_\alpha(\xi, \eta)A_u(\xi, \eta)) \int_{D} \frac{AF_{veq}(\xi, \xi', \eta, \eta')}{\sqrt{1 - (\xi'-\xi)^2 - (\eta'-\eta)^2}}
\times \left| R^{(0)}_{pp}(S, T_s, \theta_s(\xi', \eta')) \right|^2 \times T_{sky}(\xi, \eta, \xi', \eta') e^{-a \sec \theta_s(\xi', \eta')} \, d\xi' \, d\eta'.
\]

(13)

The Fresnel power reflection coefficients vary weakly over the
significant portion of the synthetic beam, so that

\[
\left| R^{(0)}_{pp}(S, T_s, \theta_s(\xi', \eta')) \right|^2 \simeq \left| R^{(0)}_{pp}(S, T_s, \theta_s(\xi, \eta)) \right|^2.
\]

(14)

With this approximation, the celestial sky glitter contribution at
the SMOS antenna for a perfectly flat sea surface becomes

\[
T_p^{fa}(\xi, \eta) \approx M_\alpha(\xi, \eta) e^{-a \sec \theta_s(\xi, \eta)} \left| R^{(0)}_{pp}(S, T_s, \theta_s(\xi, \eta)) \right|^2 \times \int_{D} \frac{\rho F_{veq}(\rho)}{\sqrt{1 - \rho^2}} T_{sky}(\xi, \eta, \rho, \phi) e^{-a \sec \theta_0(\phi, \rho)} \, d\rho \, d\phi.
\]

(15)

If we ignore the downward atmospheric attenuation, then the
approximate formulation (15) for a perfectly smooth ocean
surface is particularly attractive from a processing point of
view because it allows the incorporation of the antenna pattern
effect by a presmoothing of the sky brightness temperature map
[i.e., the integral factor in (15)] with the idealized synthetic
antenna weighting function. The synthetic beam-weighted re-
flected celestial sky glitter contamination may then be obtained
for arbitrary viewing geometry with a simple interpolation from
the smoothed map followed by a matrix multiplication.

C. Antenna Pattern Smoothing Versus Roughness Spreading

Following (11), to properly account for the celestial glitter in
the presence of surface roughness, we must compute the scat-
erred noise throughout the synthetic beam and then integrate the
product of the weighting function \(F_{veq}\) and this scattered signal.
Such a computation is not practical, and hence, it is useful to
498 determine if we can avoid this averaging operation. To assess
the impact of the synthetic beam averaging, we selected a time
499 and satellite configuration such that a small but strong bright-
500 ness source exists inside the SMOS FOV. We then established
a fine mesh over a small portion of the FOV surrounding this
source (a 65 \times 65 regular grid covering a 0.2 \times 0.2 domain
in the antenna DC coordinates) and calculated the scattered
horizontally polarized signal (in the surface polarization basis)
at each grid point. The flat surface reflected signal with no
horizontal synthetic beam smoothing, which is shown in Fig. 6(a), exhibits
a maximum brightness temperature of approximately 50 K.
502 The corresponding signal as smoothed by the synthetic beam,
which is shown in Fig. 6(b), is significantly smoother, with a
maximum brightness temperature of approximately 16 K.

Fig. 6. (a) Flat sea surface specularly reflected signal at horizontal polariza-
tion. (b) Flat sea surface specularly reflected signal at horizontal polarization
weighted by centrosymmetric WEF. (c) Bistatically scattered signal at horizon-
tal polarization for a wind speed of 3 m/s. (d) Difference between scattered horizontally polarized celestial noise with and without weighting by the WEF.
Units are in kelvin.

However, based on the scattering solutions obtained in this paper, the impact of synthetic beam smoothing is far less than that owing to the directional spreading of the radiation by the roughened ocean surface, even at a wind speed of 3 m \cdot s^{-1}.
Fig. 6(c)]. This rough surface smoothing is sufficiently large that applying the WEF smoothing to the scattering solutions yields little change (generally less than about 0.05 K), as shown in Fig. 6(d). We conclude that the application of the WEF is not necessary in the presence of surface roughness, so long as this roughness is uniform within the synthetic antenna beam. Although the WEF impact might be nonnegligible for surface roughness at wind speeds lower than 3 m \cdot s^{-1} or for highly heterogeneous rough surfaces, in what follows, we neglect this impact except for perfectly smooth surface conditions.

Without the WEF smoothing, the contribution of rough sur-
face scattered celestial sky glitter to the reconstructed bright-
ness temperatures reduces to

\[
T_p^{fa}(\xi, \eta) = \frac{M_\alpha(\xi, \eta)}{4\pi \cos \theta_s(\xi, \eta)} e^{-a \sec \theta_s(\xi, \eta)} \times \int_{\Omega_0} \left[ (\sigma_{pp}(\Omega_0) + \sigma_{pq}(\Omega_0)) e^{-a \sec \theta_0 T_{sky}(\Omega_0)} \right] \, d\Omega_0
\]

(16)

and this equation is the basis for the results that follow.

IV. ANNUAL CYCLE OF CELESTIAL SKY GLITTER
CONTAMINATION FOR SMOS

A. Orbit Propagation and Dwell Line Generation

Having established a reasonable approximation for the im-
pact of scattered celestial noise on the measurements, we now quantify the impact of celestial sky glitter on SMOS
measurements throughout the year. Given the sun-synchronous nature of the SMOS orbit, the sky noise impact is expected to exhibit a distinct annual cycle. To examine this annual cycle, we performed a series of orbit propagations, with successive orbits spaced roughly one month apart. Although we considered both idealized and realistic geophysical conditions, here, we present only the results from the idealized simulations (with constant roughness conditions) to emphasize the impact of the viewing geometry on the results. Introducing spatial and temporal variabilities in the surface wind speed complicates the interpretation of statistics and obscures the results, and the expected behavior in variable wind conditions may be anticipated from the results presented in Part I. For the present simulations, bistatic scattering cross sections were evaluated at a constant SSS of 35 psu and temperature of 15 °C. As discussed in Part I, these two geophysical parameters will have a small impact on diffuse scattering of celestial sky radiation. For comparison purposes, these surface conditions were also used to estimate the contamination assuming a perfectly smooth ocean surface. Moreover, to simplify the interpretation, we neglected downward and upward atmospheric attenuation and only considered results for the first Stokes parameter (which is affected by the Faraday rotation on the upward path across the ionosphere).

Orbit simulations were conducted using the same orbital and instrument configuration anticipated for the actual satellite. SMOS will be placed in a circular sun-synchronous low Earth orbit at a mean flight altitude of 755 km. The local time of the ascending node will be 6:00 A.M., and the inclination of the orbital plane will be 98.42°. In addition, the antenna array plane will be tilted from the horizontal by 32°. To produce one orbit simulation, we first established a fixed Earth grid by propagating the satellite through one orbit at a time step of 24 s and projecting onto the Earth’s surface a set of points along a cross-track line at η = 0.0 in the instrument frame DC coordinates. As illustrated in Fig. 7(a), this procedure establishes a 21 × 250 point fixed grid E_{ij} on the Earth’s surface, with 250 rows (with index i) of 21 projected η = 0 points (with index j). Having established this grid, we then propagated the satellite through the same orbit but with a 2.4-s time step, producing a set of snapshots S_k. At each of the 21 × 250 grid points in a given SMOS FOV, we recorded parameters such as the incidence and azimuth angles at target and the location in antenna frame (ξ_s, η_s). The result is a grid of dwell lines, D_{ijk}, where a dwell line at grid point (i, j), which is denoted as D_{ijk}, consists of a set of all k for which S_k contains the point E_{ijk} together with the corresponding set of positions in those snapshots, i.e.,

\[ D_{ijk} = \{ k, \xi(i, j, k), \eta(i, j, k) : (\xi(i, j, k), \eta(i, j, k)) \in S_k \}. \]  

(17)

Fig. 7(b) shows examples of dwell lines in DC coordinates.

In the following experiments, we configured each orbit in the monthly sequence of orbits so that the grid points do not change location from one month to the next. Since we only consider constant geophysical conditions, this has no significance beyond the fact that the geographical locations of the grid points remain the same from one month to the next.

B. Perfectly Smooth Sea Surface Contamination

Before evaluating the impact of rough surface scattered sky noise, we establish the impact of flat surface reflected noise as a baseline. In Fig. 8, we show for each orbit the fraction of measurements contaminated by unpolarized reflected celestial noise exceeding 4 K. Most notable is the fact that the reflected celestial sky noise is quite different for the ascending and descending swaths, with generally larger contamination in the descending swaths, which is to be expected since the specular points tend to be closest to the galactic equator. A significant portion of the dwell lines for the September 28 descending swath have nearly 90% of their reconstructed brightness temperatures exceeding 4 K. Most notable is the fact that the specular points tend to be nearest the galactic equator. The contamination is greatest during the northern hemisphere autumn when the specular points tend to be lowest in the FOV.
Fig. 8. Fraction of dwell line brightness temperature measurements contaminated by perfectly smooth sea surface reflected celestial noise \( \left( \frac{1}{2}(T_v + T_h) \right) \) greater than 4 K, for each month of the year. (a) Descending swaths. (b) Ascending swaths. The orbit dates (month–day) are indicated above each swath.

Fig. 9. Maximum unpolarized scattered celestial noise \( \left( \frac{1}{2}(T_v + T_h) \right) \) over all measurements of each dwell line for (a) descending and (b) ascending swaths. The wind speed is 7 m \( \cdot \) s\(^{-1}\), and the downwind direction is 0°. The Kudryavtsev wave spectrum and the KA scattering model are used to compute the scattered signal. Solutions are expressed in kelvin.

C. Rough Surface Contamination

Although examination of the flat surface reflected celestial noise provides some indication of expected contamination patterns, it does not provide a realistic picture of the true magnitude of the contamination, since at any given time only about 5% of the Earth’s ocean surface is nearly perfectly smooth [21]. As discussed in Part I, the differences between the smooth surface reflected and rough surface scattered signals may be large, even at wind speeds below 7 m \( \cdot \) s\(^{-1}\). In this section, we examine the expected contamination pattern for SMOS in idealized rough sea surface conditions with a constant wind speed of 7 m \( \cdot \) s\(^{-1}\) and downwind direction of 0°. Using the Kudryavtsev equilibrium wave spectrum [22] and the Kirchhoff scattering model (see Part I), we computed the expected rough surface scattered celestial noise over the same dwell lines considered in the previous section. In Fig. 9, we show the maximum predicted unpolarized scattered sky noise \( \left( \frac{1}{2}(T_v + T_h) \right) \) for both descending and ascending passes throughout the year. Both the spatial and temporal structures of the contamination are similar to those of the flat surface contamination, but in the rough surface case, the patterns tend to be smoother with significantly lower maximum contamination, as one would expect from the results presented in Part I. The strong maximum contamination first appears in late June and propagates across the FOV toward higher \( \xi \) in DC coordinates as time progresses. By September 28, the peak contamination is situated near the middle of the swath and nearly extends from pole to pole on Earth. By late November, this maximum has shifted off the right-hand side of the FOV.

In the ascending swaths, the time of year of maximum contamination is different. The peak contamination, which is slightly smaller in magnitude (approximately 4.7 K) than for the descending swaths, begins to enter the swath in early January and propagates toward the west and north within the swath, reaching the domain center in March, when it extends from the South Pole to near the Earth’s equator. By the end of May, the peak has nearly left the swath.

The results for all of the orbits show that the time periods of maximum contamination for the ascending and descending swaths are nearly disjoint. At any given time of the year, either the ascending or descending swaths, but not both, will suffer...
contamination. Given the relative ease with which contamination of reconstructed brightness temperatures by flat surface reflected celestial noise may be evaluated, it is important to determine if a more involved computation of the rough surface scattered celestial noise will yield significantly different results. Therefore, we assessed the overall difference between results based on flat surface reflection and those based on rough surface scattering calculations. Fig. 10(a) shows, at each dwell line of the descending swaths, the fraction of measurements for which the absolute difference between the perfectly smooth and rough surface solutions exceeds 0.5 K. The maxima in this fraction generally exceed 10% and tend to coincide with the maxima in the contamination. Similar results were obtained for the ascending swaths [Fig. 10(b)]. The fraction of measurements for which the difference between the flat and rough solutions exceeds 0.1 K approaches 100% for a substantial number of dwell lines, as shown in Fig. 10(c) and (d). Given that the maxima in the difference between the flat and rough surface solutions tend to coincide with the maxima in the flat surface reflected noise, one might hope to be able to develop a correction and error flagging strategy based upon the flat surface solution. Unfortunately, the differences between the flat and rough surface solutions do not exactly coincide with the flat surface solution because the rough surface scattering solutions depend strongly upon the spatial structure of the source in the vicinity of the specular direction; therefore, it is not possible to determine a universal threshold based on the flat surface solution alone.

V. PROCESSING ISSUES

Given the significant and systematic impact of ocean surface roughness on the contamination of reconstructed brightness temperatures by celestial sky glitter, it is certainly desirable to have a practical correction and flagging strategy for operation purposes that incorporates the effect of surface roughness. Unfortunately, it is not practical to perform per-measurement integrations of (16) to obtain scattering solutions, particularly given the proposed SMOS level 2 iterative SSS inversion scheme in which the surface wind speed is adjusted until convergence to a solution for the salinity is achieved. The proposed solution for SMOS level 2 processing involves precomputing the scattered celestial noise for a range of wind speeds, incidence angles, specular sky locations, and the incidence plane orientation angle $\psi_{uh}$ introduced in Part I. The precomputed results are stored in a lookup table from which solutions are obtained during the salinity inversion procedure by interpolation. The change of variables introduced in Part I involving the incidence plane orientation angle $\psi_{uh}$ allows the separation of the impact of viewing geometry at the target from the impact of specular sky location, thereby enabling the creation of a lookup table.
with practical discretizations in all dimensions. For the results presented in this paper, we have generated scattered celestial noise solutions for wind speeds of 3, 5, 7, 10, 15, and 25 m · s⁻¹ on a regular 3.75° × 3.75° grid in specular right ascension and declination. The grid spacing in ψh is 22.5°, and the incidence angles range from 0° to 60° by 5° and from 60° to 80° by 10°. To evaluate the performance of this lookup table solution, we considered the same uniform geophysical conditions as in the previous section and computed, for each measurement of each dwell line, the difference between the solution obtained from numerical integration of (16) and that obtained from multilinear interpolation from the lookup table. The solid curve in Fig. 11 shows the cumulative distribution function of the absolute difference between these two computation methods for ascending swath on April 1, when we expect the worst contamination for ascending passes. The absolute difference in the unpolarized signal (1/2(Th + Tν)) is less than 0.1 K in about 95% of measurements, which is far better than that obtained with the perfectly smooth surface solution. For comparison, the dashed curve in the same figure shows the cumulative distribution function of the difference between the WEF-weighted flat surface reflection model and the per-measurement integration results.

Similar results were found for the September 28 descending swath. Therefore, the lookup table approach represents the per-measurement integration with sufficient fidelity that it is appropriate for use in an operation processor. Moreover, the approach easily accommodates alternative scattering models. The strong directional spreading effect of the rough ocean surface permits the creation of the lookup table by integration over a reduced resolution sky map with a grid spacing of 3.75° × 3.75° in right ascension and declination. Presently, the rough surface scattering solutions are implemented for wind speeds greater than 3 m · s⁻¹. At zero wind speed, the WEF-weighted smooth surface solution given in (15) is implemented using the nominal high-resolution sky map (with 0.25° × 0.25° grid spacing and strong sources excluded). For nonzero wind speeds below 3 m · s⁻¹, surface roughness spectral descriptions are known to be inaccurate, and the approach we take to estimate the celestial sky glitter contamination in this low wind speed range is to linearly interpolate between solutions for the perfectly smooth and 3 m · s⁻¹ rough surfaces. Although this approach lacks physical basis, it is proposed here as a practical solution in the absence of an adequate rough surface statistical description for low surface wind speeds. It is anticipated that the algorithm will be refined based on SMOS data obtained after launch.

VI. SUMMARY AND DISCUSSION

In this paper, we have examined how the rough sea surface scattering of L-band celestial sky radiation might affect SMOS measurements.

We began by presenting the nominal celestial sky brightness temperature map at L-band that was generated for SMOS using an approach similar to that described in [2]. The nominal map includes the appropriately integrated impact of the hydrogen line emission, but the impact of highly localized strong sources is neglected. Since omission of these strong sources from this nominal sky map may introduce errors into the scattering calculations, we also derived a map of strong sources and their brightness temperatures using high-resolution surveys. We found that, for wind speeds greater 3 m · s⁻¹ and for the two rough surface scattering models (KA and SSA-1) considered in this paper, the scattered sources associated with these localized strong sources are extremely small. We neglected the directional spreading of the scattered signal by the rough surface. Therefore, in the scattering calculations, we neglected the impact of such sources.

Next, we established expressions for the expected signals at the SMOS antenna array for both flat (perfectly smooth) and rough seas. Using an approximate isotropic (in DC coordinates) synthetic antenna weighting function (i.e., WEF), we obtained expressions for the contribution of the scattered celestial sky radiation to the total reconstructed brightness temperatures. In this theory, to properly assess the impact of celestial glitter in the presence of surface roughness on the reconstructed brightness temperatures, the scattered noise must be computed over the instrument FOV and then integrated over the synthetic antenna weighting function. Given the extreme computational burden of this approach, we evaluated the impact of computing only the synthetic boresight signal and avoiding the WEF integration entirely. We found that, in general, the rough surface scattered signal is sufficiently smooth that, even in the vicinity of a strong (i.e., 50 K) localized source, the scattered signal is not modified by more than approximately 0.05 K by integration over the WEF, so that this step may be avoided in rough ocean surface conditions. Although the WEF impact might be nonnegligible for surface roughness at wind speeds lower than 3 m · s⁻¹ or 799 highly heterogeneous rough surfaces, we do not consider it, except for the perfectly smooth surface conditions.

The sampling characteristics of the instrument are important factors in determining the overall impact of scattered celestial noise for a particular mission. Both the Aquarius/SAC-D and SMOS satellites will maintain sun-synchronous orbits, so that the specular reflection of the antenna pattern on the celestial
sphere will slowly evolve with time, making one complete cycle in a year. As compared with Aquarius/SAC-D, the large FOV of MIRAS is associated with a much larger specular domain in the celestial sphere, thus the reconstructed brightness temperatures derived from MIRAS will suffer from a large range of contamination at any given dwell line on Earth. The results presented here indicate that the contamination exhibits a strong seasonal cycle that is different for the ascending and descending swaths. The largest contamination occurs in the descending swath in September and October, when the specular projection of the FOV is aligned with a strip of strong noise in the vicinity of the North Polar frequency of 1.4 GHz have become available [23]–[26]. These third and fourth Stokes parameters) over the northern sky at a frequency exceeding 4 K in the descending swath during September and October (and to a large extent in August and November). Considering moderate wind speed conditions, a larger portion of the dwell lines will suffer from contamination in which surface roughness modifies the flat surface specularly reflected signal by more than 0.1 K.

Given this potential for strong contamination in a large fraction of measurements and the computational burden of the rough surface scattering calculations, we examined a strategy for computing the rough surface scattered signal using a precomputed lookup table expressed in terms of the specular sky location, incidence angle, wind speed, and the incidence plane orientation angle \( \psi_{uh} \) introduced in Part I. For the monthly orbits considered here, results obtained by interpolation from the lookup table differ from per-measurement scattering calculations by less than 0.1 K in 95% of measurements for April and September, during which we expect the worst contamination and the largest impact of surface roughness.

In the monthly orbit calculations with a moderate surface wind speed, only 70% of the rough surface scattered signals differ from the smooth surface counterparts by less than 0.1 K. Importantly, the numerically integrated scattered signals are based on asymptotic scattering models, and the statistical description of the rough surface is based on an idealized wave model. Although we found that in the vicinity of the strongest noise source the results obtained from two electromagnetic models do not differ by more than 0.02 K, the amplitudes and phases of the wind direction dependence can exhibit large differences between models. In addition, we have not considered here the dependence of the results on the wave model, which may have a significant impact on the contamination and its relative wind direction dependence.

Finally, we have not considered polarized source radiation. Recently, new surveys of linearly polarized radiation (i.e., the third and fourth Stokes parameters) over the northern sky at a frequency of 1.4 GHz have become available [23]–[26]. These maps reveal a highly variable polarized intensity that can reach 444 500 mK in magnitude (e.g., in the vicinity of the North Polar Spur). This polarization, which is neglected in our formulation, might impact the celestial noise contamination to an extent that is significant for SSS retrieval. Unfortunately, incorporation of this polarization complicates the formulation. In particular, the polarimetric algorithm must account for polarization rotation from the celestial basis to the usual target basis as well as 850 downward Faraday rotation. Generalizing the expression for the total scattered brightness temperature in the antenna frame (3) to the fully polarized case, we obtain

\[
T^{\text{e}} = \frac{1}{\Omega_{a} \Omega_{b}} \int_{\Omega_{a}} \frac{(GM_{0})}{4 \pi} e^{-a \sec \theta_{a}} \times \int_{\Omega_{b} \Omega_{a}} M_{s}(\Omega_{a}, \Omega_{b}) M_{d}(\Omega_{a}, t) M_{\Psi} T_{q} e^{-a \sec \theta_{0} \sin \theta_{0} d\Omega_{a} d\Omega_{b}}
\]

where \( T_{q} = (T_{h}, T_{v}, U, V)^{T} \) is the full Stokes vector of 854 incident radiation, and \( T_{p} \) is the Stokes vector of the WEF-weighted signal in the instrument polarization basis. In contrast with the formulation given in [3], which is approximated for unpolarized sky radiation, \( M_{s}(\Omega_{a}, \Omega_{b}) \) is the fully 858 polarimetric Mueller matrix (with the obvious dependence on the rough surface omitted). The incoming Stokes vector \( T_{q} \) is 860 transformed before scattering by a change in polarization basis \( M_{\Psi}(\Omega_{b}) \) and the time-dependent Faraday rotation \( M_{d}(\Omega_{a}, t) \), 862 which in turn depends upon the incident radiation direction. The transformation matrix \( M_{\Psi}(\Omega_{b}) \) implicitly depends on the target location and radiometer incidence and azimuth angles, but these additional dependencies may be accounted for by the incidence plane orientation angle \( \psi_{uh} \) introduced in Part I, so that more explicitly \( M_{\Psi} = M_{\Psi}(\psi_{uh}, \Omega_{b}) \). Therefore, without 868 Faraday rotation, no additional difficulties are encountered in the formulation of the lookup table. When we perform the integration of the fully polarimetric scattering cross sections over the upper hemisphere for each \( \{ \alpha_{s}, \delta_{s}, \theta_{s}, \psi_{uh} \} \), the set 872 of polarization basis rotations (one for each point in the upper hemisphere integration) is completely determined. In practice, source polarization may increase the sensitivity of the scattered Stokes vector components to the orientation angle \( \psi_{uh} \) and 876 therefore require a lookup table with finer resolution than for the unpolarized case.

A more difficult problem involves the downward Faraday rotation. Were it not for the time dependence in the Faraday rotation, no additional problem would arise, since the ad-ditonal rotation could be incorporated into the polariza-tion basis transformation as an additional rotation. However, Faraday rotation is strongly time dependent, and this necessitates further approximation to be able to use a pregenerated time-independent lookup table of scattered noise. One possible approach is to approximate the downward Faraday rotation by the time-dependent value evaluated only in the specular direction. Unfortunately, in general, the scattering matrix does not commute with the downward Faraday rotation matrix, so that \( M_{s}(\Omega_{a}, \Omega_{b}) M_{d}(\Omega_{a}, t) \neq M_{d}(\Omega_{a}, t) M_{s}(\Omega_{a}, \Omega_{b}) \), and thus, it is not possible to bring \( M_{d}(\Omega_{a}, t) \) 892
outside the integral, even with the time-dependent specular approximation.

The effect of Faraday rotation is analogous to the effect of the polarization basis rotation. Without Faraday rotation, the polarization basis rotation is strictly a function of the specular location and incidence plane orientation angle $\psi$, so that it will be accounted for implicitly. Unfortunately, Faraday rotation is an additional (seventh) independent variable. If, however, we neglect the portion of the $\psi$ dependence related to variations in the mapping from the upper hemisphere to the reduced-resolution celestial map, which conserves the energy counting for the (possibly more subtle) effect of sky noise orientation on the upper hemisphere. According to the models considered in this paper, the maximum peak-to-peak variability (with respect to $\psi$) in the scattered unpolarized signal was on order of 0.5 K; therefore, this approach may be worth considering.

**APPENDIX**

**GENERATION OF A REDUCED-RESOLUTION CELESTIAL MAP**

Here, we describe the method that we used to generate a reduced-resolution celestial map, which conserves the energy flux of the full-resolution map.

Letting $\delta$ denote declination and $\alpha$ right ascension, the original discrete celestial map provides data on a grid of cells such that the brightness temperature field is piecewise constant within each cell and has the form

$$ T^f_{\text{sky}} = T^f_{\text{sky}} \left( \delta^f_0 + (j^f - 1)(\Delta \delta)^f, \alpha^f_0 + (k^f - 1)(\Delta \alpha)^f \right) $$

where $j^f$ and $k^f$ are positive integer indices that satisfy

$$ 1 \leq j^f \leq n^f_\delta $$

$$ 1 \leq k^f \leq n^f_\alpha. $$

Here, the grid spacing is $(\Delta \delta)^f = 0.25^\circ$ in declination and $(\Delta \alpha)^f = 0.25^\circ$ in right ascension. A reduced-resolution celestial map was produced by applying an energy-flux-conserving averaging operator $R(\cdot)$ to the original celestial map to produce a celestial noise map on a low-resolution grid $G_r(j^r(\Delta \delta)^c, k^r(\Delta \alpha)^c)$, where $(\Delta \delta)^c = (2n^c + 1)(\Delta \delta)^f$, and $(\Delta \alpha)^c = (2n^c + 1)(\Delta \alpha)^f$. The integer rescaling factor $n^c$ is set to 7 for this paper, which yields a 15-fold increase in grid spacing in right ascension and declination. The integer indices of the coarse grid, i.e., $j^c$ and $k^c$, satisfy

$$ 1 \leq j^c \leq \left( \frac{n^c_\delta - 1}{2n^c + 1} \right) + 1 $$

$$ 1 \leq k^c \leq \left( \frac{n^c_\alpha - 1}{2n^c + 1} \right) + 1 $$

and the discrete low-resolution brightness temperature field is

$$ T^c_{\text{sky}}(j^c, k^c) = \frac{1}{N} \sum_{j^f = j^c_0}^{j^c + 1} \sum_{k^f = k^c_0}^{k^c + 1} \sin \left( \delta^f_0 + j^f(\Delta \delta)^f \right) \times T^f_{\text{sky}}(\delta^f_0 + (j^f - 1)(\Delta \delta)^f, \alpha^f_0 + (k^f - 1)(\Delta \alpha)^f) $$

where $T^f_{\text{sky}}$ is the (piecewise constant) fine grid brightness field, and $T^c_{\text{sky}}$, is the coarse grid field. The index limits of summation over the fine grid may be expressed in terms of coarse grid indices and the resolution reduction factor $n^c$ by

$$ j^c_0(j^c) = 1 + (2n^c + 1)(j^c - 1) - n^c $$

$$ j^c_1(j^c) = 1 + (2n^c + 1)(j^c - 1) + n^c $$

$$ k^c_0(k^c) = 1 + (2n^c + 1)(k^c - 1) - n^c $$

$$ k^c_1(k^c) = 1 + (2n^c + 1)(k^c - 1) + n^c. $$

The factor

$$ N = \sum_{j^f = j^c_0}^{j^c + 1} \sum_{k^f = k^c_0}^{k^c + 1} \sin \left( \delta^f_0 + j^f(\Delta \delta)^f \right) $$

is a normalization factor for the averaging operator. The first cell in the reduced grid (in both declination and right ascension) is always aligned with the first cell in each dimension in the original grid, and the celestial brightness temperature values assigned to each of the reduced-resolution grid cells is the weighted average of the brightness temperatures in all original grid cells contained within the encompassing coarse grid cell.

In the averaging procedure, the weight given to a particular 948 fine grid cell is proportional to the solid angle subtended by that cell.

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